

Kaishangi Teivedi - 82

Homework - 1

Lagrange's Interpolation

1)	x	y	$x_k = 9.5$	
	7	3	$x_0 = 7$	$f(x_0) = 3$
	8	1	$x_1 = 8$	$f(x_1) = 8$
	9	1	$x_2 = 9$	$f(x_2) = 1$
	10	9	$x_3 = 10$	$f(x_3) = 9$

$$\begin{aligned} f(x_k) = & \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} f(x_0) + \\ & \frac{(x_k - x_0)(x_k - x_2)(x_k - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} f(x_1) + \\ & \frac{(x_k - x_0)(x_k - x_1)(x_k - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} f(x_2) + \\ & \frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} f(x_3) \end{aligned}$$

$$f(x_k) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3 +$$

$$\frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1 +$$

$$\frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1 +$$

$$\frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$= 3 \left(\frac{1.5 \times 0.5 \times -0.5}{-1 \times -2 \times -3} \right) + \left(\frac{2.5 \times 0.5 \times -0.5}{1 \times -1 \times -2} \right) +$$

$$\frac{2.5 \times 1.5 \times -0.5}{2 \times 1 \times -1} + \frac{9(2.5 \times 1.5 \times 0.5)}{3 \times 2 \times 1}$$

$$= -0.1875 - 0.3125 + 0.9375 + 2.1875$$

$$= 3.625$$

$$f(9.5) = 3.625$$

Q.2.

$$\alpha_k = 6$$

x	y
3	168
7	120
9	72
10	63

$$x_0 = 3$$

$$x_1 = 7$$

$$x_2 = 9$$

$$x_3 = 10$$

$$f(x_0) = 168$$

$$f(x_1) = 120$$

$$f(x_2) = 72$$

$$f(x_3) = 63$$

$$f(x) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 +$$

$$\frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120 +$$

$$\frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 +$$

$$\frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$\frac{(-1x - 3x - 4)168}{-4 - 6 - 7} + \frac{(3x - 3x - 4)120}{4x - 2x - 3}$$

$$\frac{72(3x - 1x - 4)}{6 \times 2x - 1} + \frac{63(3x - 1x - 3)}{4 \times 2x - 1}$$

$$= 12 + 480 + 72 + 27$$

$$= 147$$

$$f(6) = 147$$

Q.5.

$$x_k = 6$$

$$x_0 = 1$$

$$x_1 = 2$$

$$x_2 = 7$$

$$x_3 = 8$$

$$1$$

$$2$$

$$2$$

$$5$$

$$7$$

$$5$$

$$8$$

$$4$$

$$f(x_0) = 4$$

$$f(x_1) = 5$$

$$f(x_2) = 5$$

$$f(x_3) = 4$$

~~Q.5.~~

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 +$$

$$\frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5 +$$

$$\frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5 +$$

$$\frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$4 \frac{(4x-1x-2)}{(-1x-6x-7)} + 5 \frac{(5x-1x-2)}{(1x(-5)x(-6))} + 5 \frac{(5x4x-2)}{(6x5x-1)}$$

$$+ 4 \frac{(5x4x-1)}{7x6x1}$$

$$= 0.76190476 + 1.66667 + 6.66667$$

$$= 1.904761905$$

$$= 5.666670$$

4.

x	y	$n_k = 5$	
1	2	$n_0 = 1$	$f(x_0) = 2$
2	4	$n_1 = 2$	$f(x_1) = 4$
3	8	$n_2 = 3$	$f(x_2) = 8$
4	16	$n_3 = 4$	$f(x_3) = 16$
7	128	$n_4 = 7$	$f(x_4) = 128$

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 +$$

$$\frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4 +$$

$$\frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 +$$

$$\frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16 +$$

$$\frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128.$$

$$= \frac{3 \times 2 \times 1 \times (-2)}{-1 \times -2 \times -3 \times -6} \times 2 + \frac{4(4 \times 2 \times 1 \times (-2))}{1 \times -1 \times -2 \times -5} +$$

$$\frac{8(4 \times 3 \times 2 \times (-2))}{2 \times 1 \times -1 \times -4} + \frac{16(4 \times 3 \times 2 \times (-4))}{3 \times 2 \times 1 \times -3} +$$

$$\frac{128(4 \times 3 \times 2 \times 1)}{6 \times 5 \times 4 \times 3} = -0.66667 + 6.4 - 24$$

$$+ 42.66667 + 8.53334$$

$$= 32.933334$$

(5)

	x	y	
(x_0)	10	0.25	$f(x_0)$
(x_1)	20	0.2	$f(x_1)$
(x_2)	14	0.15	$f(x_2)$
(x_3)	16	0.10	$f(x_3)$

$$f(15) = \frac{(15-20) \cdot (15-14) \cdot (15-16)}{(10-20) \cdot (10-14) \cdot (10-16)} \times 0.25$$

$$\frac{(15-10) \cdot (15-14) \cdot (15-16)}{(20-10) \cdot (20-14) \cdot (20-16)} \times 0.2$$

$$\frac{(15-10) \cdot (15-20) \cdot (15-16)}{(14-10) \cdot (14-20) \cdot (14-16)} \times 0.15$$

$$\frac{(15-10) \cdot (15-20) \cdot (15-14)}{(16-10) \cdot (16-20) \cdot (16-14)} \times 0.10$$

$$= \frac{(-3 \times 1 \times -2)}{-10 \cdot -4 \cdot -6} \times 0.25 + \frac{0.2 \cdot (5 \times -1 \times -1)}{10 \times 6 \times 4}$$

$$0.15 \frac{[5 \times (-5) \times (-1)]}{6 \times 4 \times (-6) \times (-2)} + 0.10 \frac{(5 \times (-5) \times 1)}{6 \times (-4) \times 6 \times 2}$$

$$= -0.00520334 - 0.00416667 + 0.073125 + 0.0520323$$

$$= 0.1208334$$

Homework - 2.

Regular Falsi.

Q.1. Let $y = x^3 + x - 1 = 0$
between 0.5 & 1.

when $x = 0.5$ $y = -0.375$

$x_1 = 0.6$ $y_1 = -0.184$

$x_2 = 0.7$ $y_2 = 0.043$

0.8 0.312

0.9 0.629

1

1

$$\bar{x} = \frac{0.6 \times 0.043 + 0.7 \times -0.184}{0.043 - (-0.184)}$$

$$= \cancel{0.68} 0.6810523$$

$$\bar{y} = -0.00304177$$

Q.2. $x^2 - 2x + 0.5 = 0$

between 0 and 1.

x

y

0

0.5

1

-0.5

$$\bar{x} = \frac{0 \times -0.5 - 1 \times 0.5}{-0.5 - 0.5}$$

$$= \frac{-0.5}{-1} = 0.5$$

$$\bar{y} = \cancel{x^2} (0.5)^2 - 2(0.5) + 0.5 =$$

$$= -0.375$$

$$\begin{array}{cc} x & y \\ 0 & 0.5 \\ 0.5 & -0.375 \end{array}$$

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0 \times 0.375 - 0.5 \times 0.5}{-0.375 - 0.5}$$

$$= -0.25$$

$$-0.375$$

$$\bar{x} = 0.2857$$

$$\bar{y} = (0.2857)^2 - 2(0.2857) + 0.5$$

$$= -0.048105$$

$$\begin{array}{cc} x & y \\ 0 & 0.5 \end{array}$$

$$x \quad 0.2857 \quad -0.048105$$

$$\bar{x} = \frac{-0.5 \times 0.2857}{-0.048105 - 0.5}$$

$$\bar{x} = 0.260625245$$

$$\therefore \bar{y} = -0.003547305$$

$$\therefore 0.260625245 \text{ is the root of } x^2 - 2x + 0.5 = 0$$

Q.3.

$$x^3 + 2x - 20 = 0$$

~~1.4310~~

x	y
x_1 2	-8 y_1
x_2 2.5	0.625 y_2

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{2 \times 0.625 - 2.5 \times -8}{0.625 - (-8)}$$

$$= 2.46378116$$

$$y = x^3 + 2x - 20$$

$$= (2.46378116)^3 + 2(2.46378116) - 20$$

$$= -0.1161760$$

x	y
x_1 2.46378116	-0.1161760 y_1
x_2 2.5	0.625 y_2

$$\bar{x} = \frac{2.46378116 \times 0.625 - 2.5 \times -0.1161760}{0.625 - (-0.1161760)}$$

$$= 2.4695$$

$$y = -0.000926497675$$

2.4695 is a root of $x^3 + 2x - 20 = 0$

Q. 4. 32

$$3x - \cos x - 1 = 0$$

x	y
x_1 0	-2 y_1
x_2 1	1.000152305 y_2

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0 \cdot 1.000152305 - 1 \cdot (-2)}{1.000152305 - (-2)}$$

$$\bar{y} = -0.0000654$$

$$\therefore x \approx 0.6666$$

Q. 5. $x^3 - 3x - 5 = 0$

x	y	$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$
2	-3	
2.5	3.125	

$$= \frac{2 \cdot 3.125 - 2.5 \cdot (-3)}{3.125 - (-3)}$$

$$= 2.2447$$

$$\bar{y} = -0.4214$$

x	y
2.2447	-0.4214
2.5	3.125

$$\bar{x} = 2.449(3.125) - (-0.4214)$$

$$3.125 - (-0.4214)$$

$$= 2, 2752$$

$$\begin{aligned} \bar{y} &= x^2 - 3x - 5 \\ &= (2.2752)^2 - 3(2.2752) - 5 \\ &= -0.0479 \end{aligned}$$

x	y
2.2752	-0.0479
2.5	3.125

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{2.2752(3.125) - (0.0479 \times 2.5)}{3.125 - (-0.0479)}$$

$$\begin{aligned} \bar{y} &= x^2 - 3x - 5 \\ &= (2.2786)^2 - 3(2.2786) - 5 \\ &= -0.0052 \end{aligned}$$

$$x \approx 2.2786$$

$$Q.6. \quad x \log_{10} x = 1.2 = 0$$

x	y
2	-0.5979
3	0.2314

$$\begin{aligned} \bar{x} &= \frac{2(0.2314) - 3(-0.5979)}{0.2314 + 0.5979} \\ &= 2.720 \end{aligned}$$

$$\bar{y} = -0.0171$$

x	y
2.7210	-0.0171
3	0.2314

$$\begin{aligned} \bar{x} &= 2.7402 \\ \bar{y} &= -0.000389 \\ \therefore x &\approx 2.7402 \end{aligned}$$

Q.7.

$$x^3 - 4x + 15 = 0$$

x	y
x_1 0	1 y_1
x_2 0.3	-0.173 y_2
0.6	-1.184
0.9	-1.871
1	-2

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0(-0.173) - 0.3 \times 1}{-0.173 - 1}$$

$$= \frac{-0.3}{-1.173}$$

$$= 0.255784478$$

$$\therefore x = 0.006288913$$

x	y
0.0	1
0.255784478	-0.006288913

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.006288913$$

$$\therefore \bar{x} = 0.25415611$$

$$\bar{y} = -0.000207143$$

$$\bar{x} \approx 0.25415611$$

$$8. \quad x^3 + 2x^2 - 8 = 0$$

x	y
0	-8
1	-5
2	2

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0 \cdot 2 - 1 \cdot (-5)}{-5 - (-8)} = \frac{5}{3} = 1.6667$$

x	y
0	-8
1	-5
2	2

$$\bar{x} = \frac{18}{13} = 1.3846$$

$$\bar{y} = -1.5113$$

x	y
1.3846	-1.5113
2	2

$$\bar{x} = 1.4824$$

$$\bar{y} = -0.3474$$

x	y
1.4824	-0.3444
2	8

$$\bar{x} = 1.5037$$

$$\bar{y} = -0.0752$$

x	y
1.5039	-0.0752
2	8

$$\bar{x} = 1.5035$$

$$\bar{y} = -0.062$$

$$\therefore x \approx 1.5035$$

Q.9. $x^2 - 9x + 1 = 0$

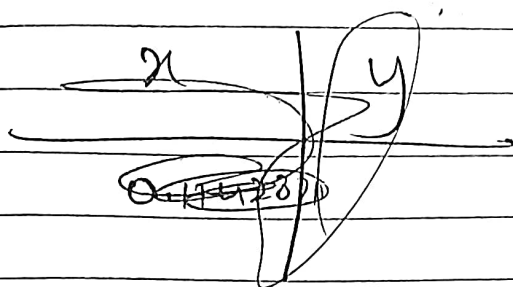
x	y
x_1 0	1
x_2 0.5	-3.375

$$\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$$

$$= \frac{0 \times -3.375 - 1 \times 0.5}{-3.375 - 1}$$

$$= 0.114285714$$

$$\bar{y} = -0.027078717$$



x	y
0	1
0.112285714	-0.027078717

$$= \text{~~0.112285714~~}$$

$$\bar{x} = 0.11272595$$

$$\bar{y} = -0.000075622$$

$$\therefore x \approx 0.11272595$$

$$[0.10] \quad x^3 - x - 4 = 0$$

x	y
0	-4
x ₁ 1	-4 y ₁
x ₂ 2	2 y ₂

$$\begin{aligned} \bar{x} &= \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1} \\ &= \frac{2 + 8}{6} = 1.66667 \end{aligned}$$

$$\begin{aligned} \bar{y} &= (1.66667)^2 - (1.66667) - 4 \\ &= -1.03703703 \end{aligned}$$

x	y
1.66667	-1.03702703
2	2

$$\begin{aligned} \bar{x} &= \frac{2(1.66667) - 2(-1.03703)}{2 - (-1.03703)} \\ &= 1.780484807 \end{aligned}$$

$$\bar{y} = -0.136017822$$

x	y
1.780488	-0.13097
2	2

$$\bar{x} = 1.793978329$$

$$\bar{y} = -0.02031323 -$$

$$x \approx 1.793978327$$