ASSIGNMENT - 2

1) $\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix} \begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix} \begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 3-8 & b+2 \\ 3a+0 & ab+0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & b+2 \\ 3a & ab \end{bmatrix}$$

$$\therefore \begin{bmatrix} -5 & b+2 \\ 3a & ab \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

$\therefore b+2 = 6$

$3a = 12$

$ab = 16$

$\therefore a = \frac{\cancel{12}^{4}}{\cancel{3}} = 4$

$b = 6-2 = 4$

2) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A^{20} = ?$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\therefore A^{20} = (A^2)^{10} = I^{10} = I$

3] $\begin{bmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

∴ The Matrix is not invertible if the determinant is zero.

$$\therefore \begin{vmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1(2) - \lambda(3-0) = 0$$

$$= 2 - 3\lambda = 0$$

$$-3\lambda = -2$$

$$\lambda = 2/3$$

4] $S = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} W.B & C.F & F.M \end{array}$

$$S = \begin{array}{c|c} \begin{array}{ccc} W.B & C.F & F.M \\ \end{array} & \\ \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix} & \begin{array}{l} \text{Mumbai} \\ \text{Pune} \end{array} \end{array}$$

$$P = \begin{array}{cc} \begin{array}{cc} \text{undiscount} & \text{Discount} \end{array} & \\ \begin{bmatrix} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{bmatrix} & \begin{array}{l} W.B \\ C.F \\ F.M \end{array} \end{array}$$

Revenue $= S \times P$

$$\therefore \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix} \begin{bmatrix} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 20(700)+10(300)+8(1200) & 20(600)+10(200)+8(1000) \\ 15(700)+12(300)+4(1200) & 15(600)+12(200)+4(1000) \end{bmatrix}$$

$$= \begin{bmatrix} 26600 & 22000 \\ 18900 & 15400 \end{bmatrix} \rightarrow \text{ASR}$$

5) Let the first term be a and the common difference be d.

The n^{th} term is $a + (n-1)d$.

$$a + 3d = 3a$$
$$a + 6d = 2(a+2d) + 1$$

$$\therefore 2a = 3d$$
$$a + 6d = 2a + 4d + 1$$
$$2d = a + 1$$
$$\therefore a = 2d - 1$$

$$\therefore \frac{3d}{2} = 2d - 1$$
$$3d = 4d - 2$$
$$2 = d$$
$$\therefore d = 2$$
$$\therefore a = 4 - 1 = 3$$

6) ~~7(a+6d)~~

$$7(a+6d) = 11(a+10d)$$

$$7a+42d = 11a+110d$$

$$\therefore 11a-7a+110d-42d = 0$$
$$4a+68d = 0$$
$$4(a+17d) = 0$$

$$\therefore a+17d = 0$$

$\downarrow$

18^{th} term

Hence proved, The 18^{th} term is 0.

7) The sum to n terms is given by

$$\frac{n}{2}(2a+(n-1)d)$$

$$\therefore \frac{20}{2}(2a+(20-1)d) = \frac{22}{2}(2a+(22-1)d)$$
$$10(2a+19d) = 11(2a+21d)$$
$$20a+190d = 22a+231d$$
$$-2a = 41d$$

$d = -2$ (given)

$$\therefore -2(a) = 41(-2)$$
$$a = 41$$

8) $S_n = 2n^2 + 5n$

The sum to n terms is given by

$$\frac{n}{2}(2a+(n-1)d)$$

$$\therefore \frac{n}{\cancel{2}} \times \cancel{2}a + \frac{n(n-1)(d)}{2}$$

$$= na + \frac{(n^2-n)(d)}{2}$$

$$= na + \frac{n^2 d}{2} - \frac{nd}{2}$$

$$= n^2\left(\frac{d}{2}\right) + n\left(a - \frac{d}{2}\right)$$

Comparing the co-efficients of n and n^2

$$\frac{d}{2} = 2 \qquad a - \frac{d}{2} = 5$$

$$d = 4 \qquad a - \frac{4}{2} = 5$$

$$a - 2 = 5$$

$$a = 7$$

9) $111\ldots\ldots1$ } (91 times)

$= 1 + 10 + 100 + 1000 + \ldots\ldots 10^{90}$

This is the sum of G.P with

$a = 1,\ n = 91,\ r = 10$

$\therefore \text{Sum} = \dfrac{1(10^{91} - 1)}{10 - 1}$

$= \dfrac{(10^{7})^{13} - 1}{10^{7} - 1} \times \dfrac{10^{7} - 1}{10 - 1}$

$= (1 + 10^{7} + 10^{14} + \ldots 10^{91}) \times (1 + 10 + 100 + \ldots 10^{7})$

Since both the terms are natural no.s, the given no. is a multiple of two no.s, Therefore, the given no. is composite and not a prime.

10) Let the nos be a, ar, ar^2

$\therefore \quad a + ar + ar^2 = 31$

$\therefore \quad a(1+r+r^2) = 31 \quad -(i)$

$a^2 + (ar)^2 + (ar^2)^2 = 651$

$a^2(1+r^2+r^4) = 651 \quad -(ii)$

Squaring eq (i)

$a^2(1+r^2+r^4+2r+2r^2+2r^3) = 961$

OR

$a^2(1+r^2+r^4) + a^2(2r+2r^2+2r^3) = 961$

$a^2(1+r^2+r^4) + (2ar)(a)(1+r+r^2) = 961$

Substituting from (i) and (ii) we get

$651 + (2ar)(31) = 961$

$\cancel{2ar}$

OR

$(2ar)(31) = 310$

$ar = 5$

$a = \frac{5}{r}$

Substituting the value in eq (i) we get,

$$\frac{5}{r}(1+r+r^2) = 31$$

$$\frac{5}{r} + 5 + 5r = 31$$

$$5r + \frac{5}{r} = 26$$

$$5r^2 + 5 = 26r$$

$$5r^2 - 26r + 5 = 0$$

$$5r^2 - 25r - r + 5 = 0$$

$$5r(r-5) - 1(r-5) = 0$$

$$(r-5)(5r-1) = 0$$

$r - 5 = 0$ or $5r - 1 = 0$

$r = 5$ or $r = 1/5$

$\therefore$ $a = 25$ or $a = 1$