Numerical Methods of Algebra

Q1.)

→ Measured value of radius = 12.65 cm.
Actual Value of radius = 12.5 cm.

Absolute Error = Measured Value − Actual Value.
= 12.65 − 12.50

(i) Absolute Error. = ~~1.5~~ 0.15 cm.

Relative Error = $\frac{\text{Absolute error}}{\text{Actual value}}$

$= \frac{0.15}{12.50} \cancel{\times 100}$

(ii) Relative Error = 0.012 ~~×100 = 1.2%~~

Percentage error = $\frac{\text{Measured Value} - \text{Actual value}}{\text{Actual Value}} \times 100$

$= \frac{12.65 - 12.50}{12.50} \times 100$

$= \frac{0.15}{12.50} \times 100$

$= \frac{0.15 \times 100}{12.50}$

$\cancel{= \frac{15 \times 100}{1250}}$

$= \frac{3}{25} \cancel{\times 100}$

(iii) Percentage Error = 0.012 ~~×10~~ = 1.2%

Q2.)

~~Ans.~~

x	$f(x) = \log(x)$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

Ans. a) $x_0 = 4$

$x_1 = 6$

$f[x_1, x_0] = [f(6) - f(4)/(6-4)] = 0.0880046$

$f(5) \approx f(4) + f(5-4)\, 0.0880046 = 0.690106 \quad \varepsilon_t = 1.27\%$

b) $x_0 = 4.5$

$x_1 = 5.5$

$f[x_1, x_0] = [f(5.5) - f(4.5)/(5.5 - 4.5)] = 0.0871502$

$f(5) \approx f(4.5) + (5-4.5)\, 0.0871502 = 0.696788 \quad \varepsilon_t = 0.3\%$

Q3.)

Ans. $y = x^4 - x - 10$

$a = 1.5, b = 2$

x	y
1.5	-6.4375
2	4

$x_1 = 1.5 \;, y_1 = -6.4375$

$x_2 = 2 \;, y = 4$

$x_3 = \frac{x_1 + x_2}{2} = \frac{1.5 + 2}{2} = \frac{3.5}{2} = 1.75$

$y_3 = (1.75)^4 - (1.75) - 10 = -2.371$

$$x_4 = \frac{x_3+x_2}{2} = \frac{1.75+2}{2} = \frac{3.75}{2} = 1.875$$

$$y_4 = (1.875)^4 - (1.875) - 10 = 0.4846$$

$$x_5 = \frac{x_3+x_4}{2} = \frac{1.75+1.875}{2} = 1.8125$$

$$y_5 = (1.8125)^4 - (1.8125) - 10 = -1.020$$

$$x_6 = \frac{x_5+x_4}{2} = \frac{1.875+1.8125}{2} = 1.84375$$

$$y_6 = (1.84375)^4 - (1.84375) - 10 = -0.1972$$

$$x_7 = \frac{x_4+\cancel{x_5}}{2} = \frac{1.875+1.84375}{2} = 1.859375$$

$$y_7 = (1.859375)^4 - (1.859375) - 10 = 0.093$$

Root of $x^4 - x - 10 = 0$ is $x = 1.859375$. i.e $\underline{\underline{x = 1.86}}$.

Q4)

Ans. $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	0	1	2
$f(x)$	-1	-1	5

$$x_{n} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 1$, $f(x_0) = -1$, $f'(x_0) = -2$

$$x_1 = 1 - \left(\frac{-1}{2}\right) = 1.5$$

$f(x_1) = (1.5)^3 - (1.5) - 1 = 0.87$

$f'(x_1) = 3(1.5)^2 - 1 = 5.75$

$$x_2 = 1.5 - \left(\frac{0.87}{5.75}\right) = 1.3487$$

$f(x_2) = (1.3487)^3 - (1.3487) - 1 = 0.1046$

$f'(x_2) = 3(1.3487)^2 - 1 = 4.457$

$$x_3 = 1.3487 - \left(\frac{0.1046}{4.457}\right) = 0.9765$$

$f(x_3) = (0.9765)^3 - (0.9765) - 1 = -1.045$

$f'(x_3) = \cancel{3(0.5383} \; 3(0.9765)^2 - 1 = \cancel{6.0991} \; 1.8607$

$$x_4 = 0.9765 - \left(\frac{-1.0454}{1.8607}\right) = 1.5383$$

$f(x_4) = (1.5383)^3 - (1.5383) - 1 = 1.1019$

$f'(x_4) = 3(1.5383)^2 - 1 = 6.0991$

$$x_5 = (1.5383)^3 - \left(\frac{1.1019}{6.0991}\right) = 1.3576.$$

$f(x_5) = (1.3576)^3 - (1.3576) - 1 = 0.1446$

$f'(x_5) = 3(1.3576)^2 - 1 = 4.5292$

$$x_6 = 1.3576 - \left(\frac{0.1446}{4.5292}\right) = 1.3257$$

$f(x_6) = (1.3257)^3 - (1.3257) - 1 = 0.00419$

$f'(x_6) = 3(1.3257)^2 - 1 \doteq 4.2724$

$x_7 = 1.3257 - \left(\frac{0.00419}{4.2724}\right) = 1.3247$

$f(x_7) = (1.3247)^3 - (1.3247) - 1 = -0.0000766$

$f'(x_7) = 3(1.3247)^2 - 1 = 4.2645$

Root of the equation $f(x) = x^3 - x - 1$, is $x = 1.3247$.

Q5)

Ans. $A^2 = A$,

$7A - (I+A)^2 = 7A - (I^3 + 3I^2A + 3IA^2 + A^3)$

$= 7A - I^3 - 3A - 3A^2 - A^2 \cdot A$

$(\because A^2 = A)$

$= 7A - I - 3A - 3A - A \cdot A$

$= \cancel{A} - I - \cancel{A}$

~~7A(~~ $= -I$

$\therefore 7A - (I+A)^2 = -I$

Q6.)

Ans. $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{D} C^T$

$M_{11} = -6$, $M_{12} = -4$, $M_{13} = 4$

$M_{21} = -1$, $M_{22} = -1$, $M_{23} = 1$

$M_{31} = -6$, $M_{32} = -2$, $M_{33} = 4$

$$C = \begin{bmatrix} -6 & 4 & 4 \\ -1 & -1 & -1 \\ -6 & -2 & 4 \end{bmatrix} \Rightarrow C^T = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$D = 1(-6) - (-1)(-4) + 2(4) = -6 - 4 + 8 = -2$$

$$A^{-1} = \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

Q7.)

Ans. $A = 8i - 9j \Rightarrow |A| = \left|\sqrt{(8)^2 + (-9)^2}\right| = 12.042$

$B = 7i - 9j \Rightarrow |B| = \left|\sqrt{(9)^2 + (-9)^2}\right| = 11.402$

$$A \cdot B = (8i - 9j) \cdot (7i - 9j)$$
$$= (8)(7) + (-9)(-9)$$
$$= 56 + 81$$

$$A \cdot B = 137$$

$$\cos\theta = \frac{A \cdot B}{|A| \cdot |B|} = \frac{137}{(12.042)(11.402)} = 0.998$$

$$\cos\theta = 0.998$$
$$\theta = \cos^{-1}(0.998)$$
$$\theta = 3.62^\circ$$

Q8)

Ans. Magnitude $= \sqrt{(75)^2 + (25)^2}$

$= \sqrt{5625 + 625}$

$= \sqrt{6250}$

$= 79.057$

$\therefore$ Magnitude $= 79.057$

Q9.)

Ans. 101, 104, ... , 998.

$a = 101,\ d = \cancel{8}3,\ a_n = 998$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)\cancel{8}3$

$998 - 101 = 3\cancel{8}(n-1)$

$\frac{897}{\cancel{8}3} = (n-1)$

$(n-1) = 299$

$n = 299 + 1$

$n = 300$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{300}{2}[2(101) + (300-1)3]$

$= 150[202 + (299)3]$

$= 150[202 + 897]$

$= 150[1099]$

$S_{300} = 164850$

Q10.)

Ans. $T_6 = 32, \; T_8 = 128$

$T_n = a(r^{n-1})$

$T_6 = ar^5$ $\qquad$ $T_8 = ar^7$

$32 = ar^5$ $\qquad$ $128 = ar^7$

$$\frac{T_8}{T_6} = \frac{\cancel{a}r^{\cancel{7}\,2}}{\cancel{a}r^{\cancel{5}}}$$

$$\frac{128}{32} = r^2$$

$r^2 = 4$

$r = 2$

Q11.)

Ans. $(4+3x)^5 = {}^5C_0(4)^5(3x)^0 + {}^5C_1(4)^4(3x)^1 + {}^5C_2(4)^3(3x)^2 + {}^5C_3(4)^2(3x)^3 + {}^5C_4(4)(3x)^4 + {}^5C_5(4)^0(3x)^5$

$= 1024 + 5(256)(3x) + 10(64)(9x^2) + 10(16)(27x^3) + 5(4)(81x^4) + 243x^5.$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5.$

$(4+3x)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024.$

Q12)

Ans. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$|A - \lambda I_3| = 0$

$$\left|\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}\right| = 0$$

$$(2-\lambda)\begin{vmatrix} -5-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} - (-3)\begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} + 0\begin{vmatrix} 2 & -5-\lambda \\ 0 & 0 \end{vmatrix} = 0$$

$$(2-\lambda)(-5-\lambda)(3-\lambda) + 3(2)(3-\lambda) = 0.$$
$$(3-\lambda)[(-10-2\lambda+5\lambda+\lambda^2)+6] = 0$$
$$(3-\lambda)(\lambda^2+3\lambda-4) = 0.$$
$$(3-\lambda)(\lambda^2+4\lambda-\lambda-4) = 0.$$
$$(3-\lambda)(\lambda+4)(\lambda-1) = 0$$
$$\lambda = -4, 1, 3$$

Eigen values of ~~a~~ A are $-4, 1, 3$.

Eigen vectors of A when $\lambda = -4$.

$(A - \lambda I)X = 0$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

at $\lambda = -4$

$$\begin{bmatrix} 2+4 & -3 & 0 \\ 2 & -5+4 & 0 \\ 0 & 0 & 3+4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_1 \to R_1 - 2(R_2)$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2x_1 - x_2 \\ 7x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 - x_2 = 0$

$2x_1 = x_2 \to (1)$

$7x_3 = 0.$

$x_3 = 0. \to (2)$

So the Eigen vector for $\cancel{\lambda} = -4$ are $\begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix}$

When $\lambda = 1$,

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 3x_2 \\ 0 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 3x_2 = 0$

$2x_3 = 0$

$x_1 = 3x_2$ — (1)

$x_3 = 0$ — (2)

So the Eigen vectors when $\lambda = 1$ are $\begin{bmatrix} 3x^2 \\ x^2 \\ 0 \end{bmatrix}$

When $\lambda = 3$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-x_1 - 3x_2 = 0$.

$x_1 = -3x_2 \rightarrow$ (1)

$2x_1 - 8x_2 = 0$

$\cancel{2(x_1 - 4x_2) = 0}$.

$2(-3x_2) - 8x_2 = 0$

$-6x_2 - 8x_2 = 0$.

$-14x_2 = 0$

$x_2 = 0 \rightarrow$ (2)

Substitute (2) in (1).

$x_1 = -3x_2$

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$x_1 = -3(0)$

$x_1 = 0 \longrightarrow (3)$

So the Eigen vector when $\lambda = 3$ are $\begin{bmatrix} 0 \\ 0 \\ U.D \end{bmatrix}$

Q13.)

Ans. $f(x) = x^3 - 7x^2 + 8x - 3,\ x_0 = 5,\ x_2 = ?$

$f'(x) = 3x^2 - 14x + 8$

$f(x_0) = f(5) = (5)^3 - 7(5)^2 + 8(5) - 3 = -13$

$f'(x_0) = f'(5) = 3(5)^2 - 14(5) + 8 = 13$

$$x_N = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 5 - \left(\frac{-13}{13}\right) = 6$$

$f(x_1) = f(6) = (6)^3 - 7(6)^2 + 8(6) - 3 = 9$

$f'(x_1) = f'(6) = 3(6)^2 - 14(6) + 8 = 32$

$$x_2 = 6 - \left(\frac{9}{32}\right) = 5.719$$

$\therefore x_2 = 5.719.$

Q14.)

Ans. $(1-x+x^2)^4 = [(x^2-x)+1]^4$

Using binomial theorem,

$[(x^2-x)+1]^4 = {}^4C_0(x^2-x)^4(1)^0 + {}^4C_1(x^2-x)^3(1) + {}^4C_2(x^2-x)^2(1)^2$
$+ {}^4C_3(x^2-x)(1)^3 + {}^4C_4(x^2-x)^0(1)^4.$

$= (x^2-x)^4 + 4(x^2-x)^3 + 6(x^2-x)^2 + 4(x^2-x) + 1$

$= [{}^4C_0(x^2)^4(-x)^0 + {}^4C_1(x^2)^3(-x) + {}^4C_2(x^2)^2(-x)^2 + {}^4C_3(x^2)^1(-x)^3$
$+ {}^4C_4(x^2)^0(-x)^4] + 4[{}^3C_0(x^2)^3(-x)^0 + {}^3C_1(x^2)^2(-x)^1 + {}^3C_2(x^2)$
$(-x)^2 + {}^3C_3(x^2)^0(-x)^3] + 6[x^4-2x^3+x^2) + 4x^2 - 4$

$= x^8 - 4x^7 + 6x^6 - 4x^5 + x^4 + 4x^6 - 12x^5 + 12x^4 - 4x^3$
$+ 6x^4 - 12x^3 + 6x^2 + 4x^2 - 4.$

$(1-x+x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4$

Q15.) $e^{-x} = 3\log(x).$

Ans. $e^{-x} - 3\log(x) = 0.$

x	1	2
y	0.368	-0.768

$x_1 = 1, \quad y_1 = 0.368$

$x_2 = 2, \quad y_2 = -0.768$

$x_3 = \frac{x_1 + x_2}{2} = 1.5$

$y_3 = e^{-1.5} - 3\log(1.5) = -0.305$

$x_4 = \frac{x_1 + x_3}{2} = \frac{2.5}{2} = 1.25.$

$y_4 = e^{-1.25} - 3\log(1.25) = -0.004$

$\therefore x = x_4 = 1.25.$

—— x ——