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① $\int_{\frac{1}{50}}^{2} \frac{e^{2/w}}{w^2} dw$

= Putting $e^{2/w} = t$

$e^{2/w} \cdot \frac{w \cdot \frac{d}{dw} 2 - 2 \cdot \frac{d}{dw} w}{w^2} = \frac{dt}{dw}$

$e^{2/w} \left[\frac{0 - 2}{w^2} \right] = \frac{dt}{dw}$

$dw \cdot e^{2/w} \left[\frac{-2}{w^2} \right] = dt$

$dw = \frac{dt}{e^{2/w}(-2/w^2)}$

$\int_{\frac{1}{50}}^{2} \frac{t}{\cancel{w^2} \times e^{2/w}(-2/\cancel{w^2})} dw$

$= \frac{1}{-2} \int_{\frac{1}{50}}^{2} \frac{t}{t} \cdot dw \qquad e^{2/w} = t$

$= -\frac{1}{2} \int_{\frac{1}{50}}^{2} 1 \, dw$

$= -\frac{1}{2} \left[w \right]_{\frac{1}{50}}^{2} = -\frac{1}{2} \times \left[2 - \frac{1}{50} \right]$

$= -\frac{1}{2} \times \left[\frac{100 - 1}{50} \right] = -\frac{1}{2} \left[\frac{99}{50} \right] = \frac{-99}{100}$

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(2) $\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

Putting $t^4+2t = t$

$4t^3+2 = \frac{dt}{dx}$

~~$dx = \frac{dt}{}$~~ $4t^3+2\, dx = dt$

$\int \frac{2t^3+1}{(t)^3 \quad (4t^3+2)\, dx}$

$= \int \frac{\cancel{2t^3+1}}{(t)^3 \cdot 2\cancel{(2t^3+1)}\, dt}$

$= \frac{1}{2}\int \frac{1}{t^3}\, dt$

$= \frac{1}{2}\int t^{-3}\, dt$

$= \frac{1}{2}\int \frac{t^{-3+1}}{-3+1}$

$= \frac{1}{2} \times \frac{t^{-2}}{-2}$

$= \frac{1}{-4t^2}$

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(3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int x^4 + 3x - 9 = \int x^4 dx + \int 3x\,dx - \int 9\,dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x$

(4)

(5) $\int 3(8y-1)e^{4y^2-y}\,dy$

$=$ Let $4y^2 - y = t$

$(8y-1)dy = dt$

$= 3\int e^t \cdot dt = 3e^{4y-y^2} + C$

(6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$, $f(1) = \frac{-5}{4}$, $f(4) = 404$

$= f(x) = \iint (15\sqrt{x} + 5x^3 + 6)\,dx\,dx$

$= \int\left(\frac{15 \times x^{3/2}}{3/2} + \frac{5}{4}x^4 + 6x\right) dx$

$= \int\left(10x^{3/2} + \frac{5x^4}{4} + 6x\right) dx$

$= \frac{2}{5} \times 10x^{5/2} + \frac{5}{4} \times x^5/5 + 6x^2/2 + C$

$= 4x^{5/2} + x^5/4 + 3x^2 + C$

(9) $\frac{dv}{dt} = 9.8 - 0.196v$

$= \frac{dv}{dt} + 0.196v = 9.8$

$P = 0.196$

$Q = 9.8$

$\boxed{I.F = e^{\int P\, dv}}$

$I.F = e^{\int 0.196\, dv}$

$= e^{0.196v}$

$=$ Soln $= t \times I.F = \int Q \times I.F\, dx$

$= t \times e^{0.196v} = \int 9.8 \times e^{0.196v}\, dv$

$= t \times e^{0.196v} = 9.8 \int e^{0.196v}\, dv$

$= t \times e^{0.196v} = 9.8 \cdot \frac{e^{0.196v}}{0.196} + C$

Q7) $f(x+iy) + g(x-iy)$

$f(x) + f(iy) + g(x) - g(iy) = c$

$\therefore f(x) + g(x) = c + g(iy) - f(iy)$

Integrating both sides

$\int f(x) + g(x)\, dx = c \int g(iy) - f(iy)\, dy$

Q8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1) = 2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

Integrating both sides

$\int r^{-2}\, dr = \int \theta^{-1}\, d\theta$

$-\frac{1}{r} = \log|\theta| + c$

$r(1) = 2$

$\therefore \frac{d(1)}{d\theta} = \frac{(1)^2}{\theta} = 2 \quad = \frac{1}{\theta} = 2 \quad \theta = \frac{1}{2}$

(12) $f(x) = x^3 - 10x^2 + 6$, $x = 3$

$f(x) = x^3 - 10x^2 + 6$ $\quad$ $f(3) = 27 - 90 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $\quad$ $f'(3) = 27 - 60 = -33$

$f''(x) = 6x - 20$ $\quad$ $f''(3) = 18 - 20 = -2$

$f'''(x) = 6$ $\quad$ $f'''(3) = 6$

$$x^3 - 10x^2 + 6 = (-57) + (x-3)(-33) - (x-3)^2(-2) + (x-3)^3 \ldots$$

(13) Maclaurin Series $(1+x)^{\mu}$

$f(x) = (1+x)^{\mu}$ $\quad$ $f(0) = 1$

$f'(x) = \mu(1+x)^{\mu-1}$ $\quad$ $f'(0) = \mu$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$ $\quad$ $f''(0) = \mu(\mu-1)$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$ $\quad$ $f'''(0) = \mu(\mu-1)(\mu-2)$

$f''''(x) = \mu(\mu-1)(\mu-2)(\mu-3)(1+x)$ $\quad$ $f''''(0) = \mu(\mu-1)(\mu-2)(\mu-3)$

$$f(x) = 1 + \mu x + \mu(\mu-1)\frac{x^2}{2} + \mu(\mu-1)(\mu-2)\frac{x^3}{6} + \mu(\mu-1)(\mu-3)\frac{x^4}{24} \ldots$$

(14) $f(x) = \sqrt{1+x}$, Maclaurin series

$f(x) = \sqrt{1+x}$ $\quad$ $f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\quad$ $f'(0) = 1/2$

$f''(x) = \frac{-1}{4(1+x)^{3/2}}$ $f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{+1}{8(1+x)^{5/2}}$ $f'''(0) = 3/8$

$f(x) = 1 + x/_2 + x^2/2! \times -\frac{1}{4} + x^3/3! \times 3/8$

$= 1 + x/_2 - \frac{x^2}{8} + \frac{x^3}{16} + \ldots\ldots$

(15) $f(x) = e^{-6x}$ $x = -4$

$f(x) = e^{-6x}$ $f(-4) = e^{24}$
$f'(x) = (-6)e^{-6x}$ $f'(-4) = -6e^{-24}$
$f''(x) = 36e^{-6x}$ $f''(-4) = 36e^{-24}$

$f(x) = e^{24} - (x+4)\,6e^{24} + (x+4)^2 \times 18e^{24} + \ldots$

(16) Taylor series for $f(x) = \log(3+4x)$, $x = 0$

$f(x) = \log(3+4x)$ $f(0) = \log(3)$
$f'(x) = \frac{1}{3+4x}$ $f'(0) = \frac{1}{3}$

$f''(x) = \frac{-4}{(3+4x)^2}$ $f''(0) = -4/9$

$f'''(x) = \frac{8}{(3+4x)^3}$ $f'''(0) = 8/27$

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$$f(x) = \log(3) + x/3 - \frac{4}{2!} \times x^2/9 + \frac{8x^3}{27\times 6} \dots$$

$$= \log(3) + x/3 - 2\cdot x^2/9 + 4x^3/81 + \dots$$

Q4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2 + \int_{1}^{3} 6\,dx$$

$$= (6x)_1^3 + (x^3)_{-2}^{1}$$

$$= (18-6) + (1-(-8))$$

$$= 12 + (9) = 21$$

Q10) $ty' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

~~Putting $y = 1/2$~~

$$\cancel{t.\frac{dy}{dt} + 2\left(\frac{1}{2}\right) = t^2 - t + 1}$$

$$\cancel{t.\frac{dy}{dt} = t^2 - t + 1}$$

$$\cancel{\frac{dy}{dt} = \frac{t^2 - t + 1}{t}}$$

$$t \times \frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{dy}{dt} = \frac{t^2 - t + 1 - 2y}{t}$$

$$\frac{dy}{dt} + \frac{2y}{t} = t^2 - t + 1$$

$$P = \frac{2}{t} \qquad Q = t^2 - t + 1$$

$$\text{I.F.} = e^{\int P\,dt}$$
$$= e^{\int \frac{2}{t}\,dt}$$
$$= e^{2\log|t| + c}$$
$$= e^{\log t^2}$$
$$= t^2$$

Soln. $y \times \text{I.F} = \int Q \times \text{I.F}\,dt$

$= y \times t^2 = \int (t^2 - t + 1) \times t^2\,dt$

$= yt^2 = \int t^4 - t^3 + t^2\,dt$

$= yt^2 = \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} + c$