CALCULUS ASSIGNMENT

Q1.] $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw$ $\qquad e^{2/w}$

$e^{2/w}\left(\frac{2}{w}\right)$

$\int_{1/50}^{2}$ Let $t = e^{2/w}$ $\qquad e^{2/w} \, 2w^{-1}$

$\frac{dt}{dw} = e^{2/w} \frac{d}{dw}\left(\frac{2}{w}\right)$ $\qquad \frac{dt}{dw} \; e^{2/w} - 2w^{-2}$

$\frac{dt}{dw} = e^{2/w} \cdot -2w^{-2}$

$\frac{dt}{-2e^{2/w}} = \frac{1}{w^2} \, dw$

$\frac{dt}{-2e^{2/w}} = \frac{dw}{w^2}$

$\cancel{\int_{1/50}^{2} \frac{dt}{-2e^{2/w}}}$ $\qquad \frac{dt}{-2} = \frac{e^{2/w}}{w^2} \, dw$

$\int_{1/50}^{2} \frac{dt}{-2}$

$\therefore \frac{1}{-2} \int_{1/50}^{2} 1 \cdot dt$

$\therefore \frac{1}{-2} \left[x \right]_{1/50}^{2}$

$\therefore \frac{1}{-2} \left[2 - \frac{1}{50} \right]$

$\therefore \frac{1.98}{-2} = -0.99$

Q2.] $\int \frac{2t^3 + 1}{(t^4 + 2t)^3} \, dt$ $\qquad x = t^4 + 2t$

$\frac{dx}{dt} \; 4t^3 + 2$

Let, $x = t^4 + 2t$

$\frac{dx}{dt} = 4t^3 + 2$ $\qquad \frac{dx}{2} = 2t^3 + 1 \, dt$

$\frac{dx}{2} = (2t^3 + 1) \, dt$

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$$\int \frac{dt/2}{t}$$

$$\frac{1}{2}\int \frac{1}{t}\,dt$$

$$\frac{1}{2}\log|t| + c$$

$$\frac{1}{2}\log|t^4 + 2t| + C$$

05.]

3.] $f'(x) = x^4 + 3x - 9$

$$f(x) = \int (x^4 + 3x - 9)\,dx$$

$$f(x) = \int f'(x)\,dx = \int (x^4 + 3x - 9)\,dx$$

$$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$$

Q4.] $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx$$

$$\cancel{\int_{-2}^{3}} \quad \int_{1}^{\infty} 6\,dx + \int_{-\infty}^{1} 3x^2\,dx$$

06.]

$$[6x]_1^{\infty} + \left[\frac{3x^3}{3}\right]_{-\infty}^{1} \cancel{+C}$$

$$[0 - 6] \quad \cancel{6(0)} + [1 - 0]$$

$$-6 + 1$$

$$-5$$

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Q5.] $\int 3(8y-1)e^{4y^2-y}\,dy$

$\int (24y-3)e^{4y^2-y}\,dy$

$\int 24ye^{4y^2-y} - 3e^{4y^2-y}\,dy$

$24\int ye^{4y^2-y} - 3\int e^{4y^2-y}\,dy$

$24\left[y\int e^{4y^2-y}\,dy - \int\left[\frac{d}{dy}(y)\int e^{4y^2-y}\,dy\right]dy\right.$

$-3\frac{e^{4y^2-y}}{4} + C$

$24\left[y\frac{e^{4y^2-y}}{4} - \int\left[\frac{e^{4y^2-y}}{4}\right]dy - \frac{3}{4}e^{4y^2-y} + C\right.$

$\cancel{24y}$

$6y\,e^{4y^2-y} - \frac{e^{4y^2-y}}{16} - \frac{3}{4}e^{4y^2-y} + C$

$e^{4y^2-y}\left[6y - \frac{1}{16} - \frac{3}{4}\right] + C$

$e^{4y^2-y}\left[6y - \frac{1-12}{16}\right] + C$

$e^{4y^2-y}\left[6y - \frac{11}{16}\right] + C$

Q6.] $f''(x) = 15\sqrt{x} + 5x^3 + 6$, $f(1) = \frac{-5}{4}$, $f(4) = 40$

$\int f''(x)\,dx = f'(x)$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6\,dx$

$= \cancel{15}^{5}\,\frac{2}{\cancel{3}}x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$

$= 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$

$\int f'(x)\,dx = f(x)$

$\frac{3}{2}+1$

$\frac{5}{2}$

~~356~~

$$f(x) = \int 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1 \, dx$$

$$= 10 \times 2 \frac{x^{5/2}}{3} + \frac{5}{4}\frac{x^5}{5} + \frac{6x^2}{2} + C_1x$$

$$= \frac{20x^{5/2}}{3} + \frac{x^5}{4} + 3x^2 + C_1x + C_2$$

Let $f(1) = \frac{-5}{4}$

$$\frac{-5}{4} = 20\frac{1^{5/2}}{3} + \frac{1^5}{4} + 3\,1^2 + C_1(1) + C_2$$

$$\frac{-5}{4} = \frac{20}{3} + \frac{1}{4} + 3 + C_1 + C_2$$

$$-\frac{5}{4} = \frac{80+3}{12} + 3 + C_1 + C_2$$

$$\cancel{\frac{-5}{4}} = -\frac{5}{4} - 3 = \frac{83}{12} + C_1 + C_2$$

$$\frac{-5-12}{4} - \frac{83}{12} = C_1 + C_2$$

$$-\frac{17}{4} - \frac{83}{12} = C_1 + C_2$$

$$-\frac{51-83}{12} = C_1 + C_2$$

$$\frac{-134}{12} = C_1 + C_2 \quad -(1)$$

$$C_1 = -\frac{134}{12} - C_2$$

$f(4) = 404$

$$404 = \frac{20}{3}(4)^{5/2} + \frac{1}{4}(4)^5 + 3(4)^2 + C_1(4) + C_2$$

$$= \frac{640}{3} + \frac{1024}{4} + 48 + 4C_1 + C_2$$

$$= \frac{2560 + 3072}{12} + 48 + 4C_1 + C_2$$

$$404 - 48 = \frac{5532}{12} + 4C_1 + C_2$$

$\therefore f$

$\cancel{356} = 356 - \frac{5532}{12} = 4C_1 + C_2$

$\frac{4152 - 5532}{12} = 4C_1 + C_2$

$\frac{-1380}{12} = 4C_1 + C_2$ — (2)

$\frac{-1380}{12} = 4\left(\frac{-134}{12} - C_2\right) + C_2$

$\frac{-1380}{12} = -\frac{536}{12} - 4C_2 + C_2$

$\frac{-1380}{12} + \frac{536}{12} = -3C_2$

$\frac{-844}{12} = -3C_2$

$\frac{-844}{-36} = C_2$

$C_2 = \frac{211}{9}$

put $C_2 = \frac{211}{9}$ in eq (1)

$\frac{-134}{12} = C_1 + \frac{211}{9}$

$C_1 = \frac{-134}{12} - \frac{211}{9}$

$= \frac{+1206 - 2532}{108}$

$C_1 = -\frac{3738}{108}$

$C_1 = -\frac{623}{18}$

$\therefore f(x) = \frac{20}{3}x^{5/3} + \frac{x^5}{4} + 3x^2 - \frac{623}{18}x + \frac{211}{9}$

$\frac{\cancel{844}}{\cancel{36}}$ $\cancel{12}$ $211\ \frac{\cancel{844}}{\cancel{36}}$ $\cancel{18}$ 9

623 $\cancel{1869}$ $\frac{\cancel{3738}}{\cancel{108}}$ $\cancel{54}$ 18

8.] $\frac{dr}{d\theta} = \frac{r^2}{\theta} \quad r(1) = 2$

12.] $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f(x) = x^3 - 10x^2 + 6 \qquad f(3) = 3^3 - 10(3)^2 + 6$

$= -57$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$

$f''(x) = 6x - 20 \qquad f''(3) = -2$

$f'''(x) = 6 \qquad f'''(3) = 6$

$f^{iv}(x) = 0 \qquad f^{iv}(3) = 0$

$\therefore x^3 - 10x^2 + 6 = \cancel{-57(x-3)} + -57 - 33(x-3) + \frac{(-2)}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3 + 0$

$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3$

13.] $(1+x)^{\mu}$

$f(x) = (1+x)^{\mu}$

$f'(x) = \mu(1+x)^{\mu-1} \qquad f(0) = (1+0)^{\mu} = 1$

$f'(0) = \mu(1+0)^{\mu-1}$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2} \qquad = \mu$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3} \qquad f''(0) = \mu(\mu-1)$

$f^{iv}(x) = \mu(\mu-1)(\mu-2)(\mu-3)(1+x)^{\mu-4} \qquad f'''(0) = \mu(\mu-1)(\mu-2)$

$f^{iv}(0) = \mu(\mu-1)(\mu-2)(\mu-3)$

$(1+x)^{\mu} = 1 + \mu x + \frac{\mu(\mu-1)}{2!}x^2 + \frac{\mu(\mu-1)(\mu-2)}{3!}x^3 + \frac{\mu(\mu-1)(\mu-2)(\mu-3)}{4!}x^4 + \dots$

$$\frac{1}{2}-1$$

$$-\frac{1}{2}-1 \quad \frac{-1-2}{2} = \frac{1}{2}\times -\frac{1}{2} \quad -\frac{1}{4}\times -\frac{3}{2}$$

$$-\frac{1}{4} \qquad \frac{3}{8}$$

14] $f(x) = \sqrt{1+x}$

$f(x) = \sqrt{1+x}$ $\quad f(0) = 1$

$f''(x) = (1+x)^{1/2}$ $\quad f'(0) = 1$

$= \frac{1}{2}(1+x)^{-1/2}$ $\quad f''(0) = \frac{1}{2}(1+0)^{-1/2}$

$= \frac{1}{2}$

$f'''(x) = \frac{-1}{4}(1+x)^{-3/2}$ $\quad f'''(0) = -\frac{1}{4}$

$f''''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\quad f''''(0) = \frac{3}{8}$

$\sqrt{1+x} = 1 + x + \cancel{\frac{\frac{1}{2}}{2!}x^2}\ x^2 + \left(\frac{-\frac{1}{4}}{3!}\right)x^3$

$\sqrt{1+x} = 1 + x + x^2 - \frac{3}{2}x^3 + \frac{39}{4}x^4 \ldots$

15] $f(x) = e^{-6x}$ $\quad x = -4$

$f(x) = e^{-6x}$ $\quad f(0) = 1 \quad f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\quad f'(0) = -6 \quad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\quad f''(0) = 36 \quad f''(-4) = -216 e^{24}$

$f'''(x) = -216e^{-6x}$ $\quad f'''(0) = -216 \quad f'''(-4) = -216e^{24}$

$f^{IV}(x) = 1296e^{-6x}$ $\quad f''''(0) = 1296 \quad f^{IV}(-4) = 1296e^{24}$

$e^{-6x} = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4) - 36e^{24}$
$+ 54e^{24}(x+4)$

16.] $f(x) = \ln(3+4x)$ $\quad x = 0$

$f(x) = \ln(3+4x)$ $\quad f(0) = \ln(3)$

$f'(x) = \frac{4}{3+4x}$ $\quad f'(0) = \frac{4}{3}$

$f''(x) = -4 \cdot \frac{1}{(3+4x)^2}$ $\quad f''(0) = \frac{-4}{9}$

$f'''(x) = 8 \cdot \frac{1}{(3+4x)^2}$ $\quad f'''(0) = \frac{8}{27}$.

Series

$\ln(3+4x) = \ln(3) + x \cdot \frac{4}{3} + x^2\left(-\frac{2}{9}\right) + \frac{x^3}{3!}\left(\frac{4}{81}\right.$ 10.]

8.] $\frac{dr}{d\theta} = \frac{r^2}{\theta} \quad r(1) = 2$

$\int \frac{1}{r^2}\, dr = \int \frac{1}{\theta}\, d\theta$

$\Rightarrow -\frac{1}{r} = \log\theta + C$

$\therefore \frac{-1}{(1)} = \log(2) + C$

$C = -3.3219280$

$\Rightarrow \frac{-1}{r} = \log\theta - 3.3219280$ 11.]

9.] $\frac{dv}{dt} = 9.8 - 0.196v$

$\left(\frac{1}{9.8 - 0.196v}\right) dv = 1\, dt$

$\int \frac{1}{\left(\frac{7\sqrt{5}}{5}\right)^2 - \left(\frac{7\sqrt{10v}}{50}\right)}\, dv = \int 1\, dt$

$\frac{1}{2 \times \frac{7}{\sqrt{5}}} \ln\left(\frac{7\sqrt{5}/5 + 7\sqrt{10v}/50}{7\sqrt{5}/5 - 7\sqrt{10v}/50}\right) = t + c$

$\Rightarrow \frac{\sqrt{5}}{14} \ln\left(\frac{\sqrt{5} + \sqrt{v/10}}{\sqrt{5} - \sqrt{v/10}}\right) = t + c$

$\Rightarrow \frac{\sqrt{5}}{14} \ln\left(\frac{\sqrt{50} + \sqrt{v}}{\sqrt{50} - \sqrt{v}}\right) = t + c$

$\Rightarrow \ln\left(\frac{\sqrt{50} + \sqrt{v}}{\sqrt{50} - \sqrt{v}}\right) = \frac{14t}{\sqrt{5}} + c$

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10] $ty' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$

$t\frac{dy}{dt} + 2y = t^2 - t + 1$

$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$

$\frac{dy}{dt} = t - 1 + \frac{1}{t} - \frac{2y}{t}$

11] $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1$

$\int \frac{1}{y^3}\,dy = \int \frac{x}{\sqrt{1+x^2}}\,dx \qquad 1+x^2 = t$

$2x\,dx = dt$

$dx = \frac{1}{2}dt$

$-\frac{1}{2y^2} = \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt$

$-\frac{1}{2} \cdot \frac{1}{y^2} = 2 \times \frac{1}{2}\sqrt{1+x^2} + c$

$-\frac{1}{2} \cdot \frac{1}{(-1)^2} = \sqrt{1+(0)^2} + c$

$-\frac{1}{2} = 1 + c$

$-\frac{3}{2} = c$

$\Rightarrow -\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$