Name Ronit shah R. No. 67 Class / Div. B Date

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Assignment 2

1) $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$

Let $t = e^{2/w}$

$\frac{dt}{dw} = -2 e^{2/w} \cdot \frac{1}{w^2}$

$= \int_{1/50}^{2} \frac{t w^2}{-2w^2 e^{2/w}} dt$

$= -\frac{1}{2} \int_{1/50}^{2} dt$

$= -\frac{1}{2} \left[t \right]_{1/50}^{2}$

$= -\frac{1}{2} [2 - 0.02]$

$= -0.99$

2) $\int \frac{2t^3+1}{(t^4+2t)^3} dt$

Let $x = (t^4+2t)^3$

$\frac{dx}{dt} = 3(t^4+2t)^2(4t^3+2)$

$= \int \frac{2t^3+1}{x(3)(t^4+2t)^2(4t^3+2)} dx$

Multiply and divide by 2

$= \int \frac{4t^3+2}{x(2)(3)\cancel{(t^4+2t)^2}(4t^3+2)(x)^{2/3}} dx$

$= \frac{1}{6} \int \frac{dx}{x^1 \cdot x^{2/3}}$

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2) $\frac{1}{6} \int x^{-5/3} \, dx$

$= \frac{1}{6} \left[\frac{x^{-5/3+1}}{-5/3+1} \right] + c$

$= \frac{1}{6} \times -\frac{2}{3} \left[x^{-5/3+1} \right] + c$

$= -\frac{1}{9} x^{-\frac{2}{3}} + c$

3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int x^4 + 3x - 9 \, dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

4) $\int_{-2}^{3} f(x) \, dx$

$= \int_{-2}^{1} f(x) \, dx + \int_{1}^{3} f(x) \, dx$

$= \int_{-2}^{1} 3x^2 \, dx + \int_{1}^{3} 6 \, dx$

$= \left[x^3 \right]_{-2}^{1} + \left[6x \right]_{1}^{3}$

$= 1 + 8 + 18 - 6$

$= 9 + 12$

$= 21$

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5) $\int 3(8y-1)\, e^{4y^2-y}\, dy$

Let $x = e^{4y^2-y}$

$\frac{dx}{dy} = (e^{4y^2-y})(8y-1)$

$= \int \frac{3(8y-1)(e^{4y^2-y})}{(e^{4y^2-y})(8y-1)}\, dx$

$= \int 3\, dx$

$= 3x + c$

$= 3(e^{4y^2-y}) + c$

6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6\, dx$

$= 15\frac{x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + K$

$f(x) = \int \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + K\, dx$

$= 4x^{5/2} + \cancel{\frac{5x^4}{4}}\ \frac{5x^5}{5.4} + 6\frac{x^2}{2} + Kx + c$

$f(1) = -5/4$

$4(1)^{5/2} + \frac{(1)^5}{4} + 6\frac{(1)^2}{2} + K(1) + c = -\frac{5}{4}$

$K + c = -8.5$

$f(4) = 404$

$4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + K(4) + c = 404$

$4K + c = 404 - 32 - 256 - 48$

$4K + c = 68$

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6) $4k + c = 68$ — (1)

$k + c = -8.5$ — (2)

$4k + c = 68$

$k + c = -8.5$

$(-) \quad (-) \quad (-)$

$3k + 0 = 76.5$

$k = 25.5$

Substitute $k = 25.5$ in equation (2)

$k + c = -8.5$

$25.5 + c = -8.5$

$c = -34$

$\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + 25.5x - 34$

9) $\frac{dV}{dt} = 9.8 - 0.196V$

$\int \frac{dV}{9.8 - 0.196V} = \int dt$

$\log(9.8 - 0.196V) = -0.196t + c$

12) Taylor series for $f(x) = x^3 - 10x^2 + 6$ at $x = 3$

$$f(x) = x^3 - 10x^2 + 6 \qquad f(3) = \cancel{-33} \; -57$$
$$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$$
$$f''(x) = 6x - 20 \qquad f''(3) = -2$$
$$f'''(x) = 6 \qquad f'''(3) = 6$$

$$f(x) = f'(3) + (x-3)f'(x) + \frac{(x-3)^2}{\lfloor 2} f''(3) + \frac{(x-3)^3}{\lfloor 3} f'''(3) + \ldots\ldots$$

$$x^3 - 10x^2 + 6 = -57 + (x-3)(-33) + \frac{(x-3)^2}{\lfloor 2}(-2) + \frac{(x-3)^3}{\lfloor 3}(6) + \ldots\ldots$$

13) Maclaurin series for $(1+x)^{\mu}$

$$f'(x) = \mu(1+x)^{\mu-1}$$
$$f''(x) = (\mu)(\mu-1)(1+x)^{\mu-2}$$
$$f'''(x) = (\mu)(\mu-1)(\mu-2)(1+x)^{\mu-3}$$

$$f(x) = f(0) + \frac{x^1}{\lfloor 1} f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \frac{x^3}{\lfloor 3} f'''(0) + \ldots\ldots$$

$$(1+x)^{\mu} = 1 + \mu + (\mu-1)(\mu) + (\mu)(\mu-1)(\mu-2) \ldots\ldots$$

14) $f(x) = \sqrt{1+x}$

$f'(x) = \frac{1}{2} \cdot (1+x)^{-1/2}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4} (1+x)^{-3/2}$ $\qquad f''(0) = -\frac{1}{4}$

$f'''(x) = -\frac{3}{8} (1+x)^{-5/2}$ $\qquad f'''(0) = -\frac{3}{8}$

$f(x) = f(0) + \frac{x^1}{L_1} f'(0) + \frac{x^2}{L_2} f''(0) + \frac{x^3}{L_3} f'''(0) + \ldots\ldots$

$\sqrt{1+x} = \sqrt{1+x} + \frac{x^1}{L_1}\left(\frac{1}{2}\right) + \frac{x^2}{L_2}\left(\frac{-1}{4}\right) + \frac{x^3}{L_3}\left(\frac{-3}{8}\right) + \ldots$

15) $f(x) = e^{-6x}$ at $x = -4$

$f(x) = e^{-6x}$ $\qquad f(-4) = e^{24}$

$f'(x) = e^{-6x} \cdot (-6)$ $\qquad f'(-4) = e^{24} (-6)$

$f''(x) = (-6) e^{-6x} (-6)$ $\qquad f''(-4) = e^{24} (-6)^2$

$f'''(x) = (-6)^3 e^{-6x}$ $\qquad f'''(-4) = e^{24} (-6)^3$

$f(x) = f(-4) + \frac{x^1}{L_1} f'(-4) + \frac{x^2}{L_2} f''(-4) + \frac{x^3}{L_3} f'''(-4) + \ldots\ldots$

$e^{-6x} = e^{24} + \frac{(x-4)}{L_1} (e^{24})(-6) + \frac{(x-4)^2}{L_2} (e^{24})(-6)^2 + \frac{(x-4)^3}{L_3} (e^{24})$

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16) $f(x) = \ln(3+4x)$ at $x = 0$

$f(x) = \ln(3+4x)$ $\qquad f(0) = \ln(3)$

$f'(x) = \dfrac{4}{3+4x}$ $\qquad f'(0) = \dfrac{4}{3}$

$f''(x) = (-4)(4x+3)^{-2}(4)$ $\qquad f''(0) = (-16)(3)^{-2}$

$f'''(x) = (8)(4x+3)^{-3}(4)^2$ $\qquad f'''(0) = (128)(3)^{-3}$

$$f(x) = f(0) + \frac{x^1}{\lfloor 1} f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \frac{x^3}{\lfloor 3} f'''(0) + \cdots\cdots$$

$$\ln(3+4x) = \ln(3) + \frac{x^1}{\lfloor 1}\left(\frac{4}{3}\right) + \frac{x^2}{\lfloor 2}(-16)(3)^{-2} + \frac{x^3}{\lfloor 3}(128)(3)^{-3}$$