Assignment 2

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Subject: Calculus.

① $\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw.$

Sol^n let $e^{2/w} = t$

$w = 1/50 \Rightarrow t = 100$

$w = 2 \Rightarrow t = 1$

$\frac{-2}{w^2} = \frac{dt}{dw}$

$\frac{dw}{w^2} = -\frac{dt}{2}$

$= \int_{100}^{1} -e^t \frac{dt}{2}$

$= -\frac{1}{2}\left[e^1 - e^{100}\right]$

$= \boxed{\frac{1}{2}\left[e^{100} - e\right]}$

② $\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

Sol^n $t^4 + 2t = x$

$4t^3 + 2 = dx/dt$

$(2t^3+1)dt = dx/2$

$= \int \frac{dx}{2x^3} = \frac{1}{2}\left[\frac{x^{-2}}{-2}\right]$

$$\boxed{= \frac{-1}{4(t^4+2t)^2}}$$

(3) $f(x) = \int f'(x)\,dx$

Solⁿ $f(x) = \int (x^4 + 3x - 9)\,dx$

$$\boxed{f(x) = \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + c}$$

(4) $f(x) = \begin{cases} 6 & , \text{ if } x>1 \\ 3x^2 & , \text{ if } x \le 1 \end{cases}$

Solⁿ $\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

$= [x^3]_{-2}^{1} + [6x]_{1}^{3}$

$= 9 + 12$

$\boxed{= 21}$

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(5) $\int 3(8y-1)e^{4y^2-y}\,dy$.

Sol^n let $4y^2-y=t$

$(8y-1)\,dy = dt$

$= 3\int e^t\,dt$

$\boxed{= 3e^{4y\cdot y^2} + c}$

(6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

Sol^n $f(x) = \iint (15\sqrt{x} + 5x^3 + 6)\,dx\,.\,dx$

$= \int \left(\frac{15 \times x^{3/2}}{3/2} + \frac{5}{4}x^4 + 6x \right) dx$

$= \int \left(10x^{3/2} + \frac{5}{4}x^4 + 6x \right) dx$

$= \frac{2 \times 10^2}{5} x^{5/2} + \frac{5}{4}\frac{x^5}{5} + \frac{6x^2}{2} + c$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + c_1 + c_2$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - 65x - 2$

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(7) Formulate PDE from $f(x+iy)+g(x-iy)$

Sol^n $f(x+iy)+g(x-iy)=z$

Using langergiar Multiplier.

$\Delta d(x,y)=\lambda\nabla g(x,y)$

$P=\frac{dz}{dx}=f'(x+iy)+g(x-iy)$

$Q=\frac{dz}{dy}=if'(x+iy)-ig(x-iy)$

$R=\frac{d^2z}{dx^2}=f''(x+iy)+g''(x-iy)$

$T=\frac{d^2z}{dy^2}=i^2f''(x+iy)+i^2g''(x-iy)$
$=-f''(x+iy)-g''(x-iy)$

$R+T=0$ is the required P.D.E //

(8)

Sol^n

(9)

Sol^n

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8) $\frac{dr}{d\theta} = r^2/\theta$, $r = 1$.

Sol^n

$$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$$

$$-\frac{1}{r} = \log\theta + c$$

$r = 1$, $\theta = 2$

$$\frac{-1}{(1)} = \log(2) + c$$

$$c = -3.321928$$

$$\boxed{\frac{-1}{r} = \log\theta - 3.321928}$$

9) Find the solution for the $\frac{dv}{dt} = 9.8 - 0.0196v$.

Sol^n

$$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\int \frac{dv}{9.8 - 0.196v} = \int dt$$

$$\int \frac{1}{(7\sqrt{5}/1)^2 - (\frac{7}{10}\sqrt{v})^2}\, du = t + c$$

$$\frac{1}{2 \cdot 7\sqrt{5}/\sqrt{5}} \ln \left[\frac{7\sqrt{5}/5 + 7\sqrt{10v}/50}{7\sqrt{5}/5 - 7\sqrt{10v}/50} \right) = t + c$$

$$\boxed{\log\left[\frac{\sqrt{50} + \sqrt{v}}{\sqrt{50} - \sqrt{v}}\right] = \frac{14}{\sqrt{5}} t + c}$$

(10) $ty' + 2y = t^2 + t + 1$ $\quad y(1) = 1/2$

$y' + \frac{2y}{t} = t + 1 + \frac{1}{t}$

$\mu(t) = \int \frac{2}{t}\, dt = 2\ln(t).$

$y(t) = e^{2\ln(t)} \int e^{-2\ln t}(t - 1 + 1/t)$

$= t^2 \int \frac{1}{t^2}\left(t - 1 + \frac{1}{t}\right) dt$

$= t^2 \left[\ln(t) + \frac{1}{t} - \frac{1}{2t^2} + c\right]$

$y(1) = 2$

$1 - \frac{1}{2} + c = 2 \Rightarrow c = 3/2.$

$\boxed{\therefore y(t) = t^2\left[\ln(t) + \frac{1}{t} - \frac{1}{2t^2} + \frac{3}{2}\right]}$

(11) Solve the following IVP $\quad y' = \frac{xy^3}{\sqrt{1+x^2}}$, $y(0) = -1$

Solⁿ

$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\int \frac{dy}{y^3} = \int \frac{x\,dx}{\sqrt{1+x^2}}$, $\quad$ let $1 + x^2 = t$

$x\,dx = dt/2$

$\frac{-1}{2y^2} = \frac{1}{2} \int \frac{t}{t^{1/2}}$

$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$ $\quad y(0) = -1$

$\frac{-1}{2(-1)^2} = 1 + c$

$\boxed{-3/2}$

$\boxed{\frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}}$

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(12) $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

Sol^n

$f(x) = x^3 - 10x^2 + 6$ $\quad$ $f(3) = 27 - 90 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $\quad$ $f'(3) = 3^3 - 20(3) = 27 - 60 = -33$

$f''(x) = 6x - 20$ $\quad$ $f''(3) = 18 - 20 = -2$

$f'''(x) = 6$ $\quad$ $f'''(3) = 6$

$$x^3 - 10x^2 + 6 = (-57) + (x-3)(-33) - (x-3)^2(2) + 6(x-3)^3 + \ldots$$

(13) Maclaurin series for $(1+x)^u$

Sol^n

$f(x) = (1+x)^u$ $\quad$ $f(0) = 1$

$f'(x) = u(1+x)^{u-1}$ $\quad$ $f'(0) = u$

$f''(x) = u(u-1)(1+x)^{u-2}$ $\quad$ $f''(0) = u(u-1)$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$ $\quad$ $f'''(0) = u(u-1)(u-2)$

$$f(x) = 1 + ux + u(u-1)\frac{x^2}{2} + u(u-1)(u-2)\frac{x^3}{6} + \ldots$$

$$\boxed{f(x) = 1 + \sum_{i=1}^{n} \frac{u x^n (u-n)}{n!}}$$

(14) $f(x) = \sqrt{1+x}$, Maclaurian series

Sol^n

$f(x) = \sqrt{1+x}$ $\quad$ $f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\quad$ $f'(0) = \frac{1}{2}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}}$ $\quad$ $f''(0) = -\frac{1}{4}$

$f'''(x) = \dfrac{+3}{8(1+x)^{5/2}}$ $\quad$ $f'''(0) = \frac{3}{8}$

$$\boxed{f(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \ldots\ldots}$$

(15) Taylor series for $f(x) = e^{-6x}$ for $x = -4$

Sol^n

$f(x) = e^{-6x}$ $\qquad$ $f(-4) = e^{24}$

$f'(x) = (-6)e^{-6x}$ $\qquad$ $f'(x) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\qquad$ $f''(x) = 36e^{24}$

$$\boxed{f(x) = e^{24} - (x+4)\,6e^{24} + (x+4)^2 \times 18e^{24} + \ldots\ldots}$$

(16) Taylor series for $f(x) = \ln(3+4x)$ for $x = 0$

Sol^n

$f(x) = \ln(3+4x)$ $\qquad$ $f(0) = \ln(3)$

$f'(x) = \frac{1}{3+4x}$ $\qquad$ $f'(0) = \frac{1}{3}$

$f''(x) = \frac{-4}{(3+4x)^2}$ $\qquad$ $f''(0) = -\frac{4}{9}$

$f'''(x) = \frac{8}{(3+4x)^3}$ $\qquad$ $f'''(0) = \frac{8}{27}$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{\cancel{4}^{2}x^2}{\cancel{2}\times 9} + \frac{8x^3}{27\times 6} + $$

$$\boxed{f(x) = \ln(3) + \frac{x}{3} - \frac{2x^2}{9} + \frac{4x^3}{81} + \ldots\ldots}$$