

Numerical Methods & Algebra

Assignment

Q/ i) $A = \pi r^2$

$$A = \pi \times (12.5)^2 \text{ (Actual value)}$$

$$A = \pi \times (12.65)^2 \text{ (changed value)}$$

$$\begin{aligned} \text{Change in Area} &= 160.022\pi - 156.25\pi \\ &= 3.7725\pi \text{ cm}^2 \end{aligned}$$

$$\text{Absolute Error} = \text{Actual Value} - \text{Approx value}$$

$$= 3.7725\pi - 3.7\pi$$

$$= 0.0725\pi \text{ cm}^2$$

i) Relative Error

$$= \frac{\text{Actual value} - \text{Approximate value}}{\text{Actual value}}$$

$$= \frac{3.772\pi - 3.75\pi}{3.7725\pi}$$

$$= \frac{0.0225\pi}{3.7725\pi}$$

$$= 0.006 \text{ cm}^2$$

ii) Percentage error

$$0.006 \times 100$$

$$= 0.6\%$$

82 a) Linear Interpolation

$$f(5) = f(4) + \frac{f(6) - f(4)}{6-4} (5-4)$$

$$f(5) = 0.60206 + \frac{0.7781513 - 0.60206}{6-4} (5-4)$$

$$= 0.690106$$

$$\text{error} = \frac{0.69897 - 0.690106}{0.6987}$$

$$= 12.7\%$$

$$b) f(5) = f(4.5) + \frac{f(5.5) - f(4.5)}{5.5-4.5} (5-4.5)$$

$$f(5) = \frac{0.653229 - 0.740362}{5.5-4.5} (5-4.5)$$

$$= 0.69877$$

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$$\text{error} = \frac{0.69897 - 0.696788}{0.69897} \times 100\%$$

$$= 0.3\%$$

Q3 Iteration table is given as follows

No.	a	b	c	$f(a) * f(c)$
1	1.5	2	1.75	15.264
2	1.75	2	1.875	2.419
3	1.75	1.875	1.812	2.419
4	1.812	1.875	1.844	-0.303
5	1.844	1.875	1.85	-0.02

Solving the eqn = 1.86

84 $f(x) = x^3 + x - 1$
 $f'(x) = 3x^2 + 1$

$$x = (-1) \rightarrow f(-1) = (-2)$$

$$x = 0 \rightarrow f(0) = (-1)$$

$$x = +1 \rightarrow f(1) = 1$$

this indicate root occurs between (0, 1)

Let $f(x) = x^3 + x - 1$

$x_0 = 1$ initial approx

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x^3 + x - 1)}{(3x^2 + 1)}$$

$$x_1 = x_0 - \frac{(x^3 + x - 1)}{(3x^2 + 1)}$$

$$x_2 = 0.6823278$$

Q8 $7A - (I + A)^3$

$$= 7A - (I^3 + A^3 + 3I^2A + 3IA^2)$$

$$= 7A - (I + A^3 + 3A + 3A^2)$$

$$= 7A - (I - A^2A + 3A + 3A^2)$$

$$= 7A - (I - A - A + 3A + 3A)$$

$$= 7A - (I - A + 6A)$$

$$= 7A - (I + A^2 + 6A)$$

$$= 7A - (I + A + 6A)$$

$$= 7A - (I + 7A)$$

$$\therefore 7A - (I + A^3) = -I$$

Q6 Inverse of $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 3 & -\frac{1}{2} & 3 \\ -2 & \frac{1}{2} & -1 \\ -2 & \frac{1}{2} & -2 \end{bmatrix}$$

Q7 a) $A = 8i - 9j$ and $B = 7i - 9j$

$$\theta = \cos^{-1} \left(\frac{A \cdot B}{|A| |B|} \right)$$

$$\theta = \cos^{-1} \left(\frac{(8i - 9j) \cdot (7i - 9j)}{(8i - 9j) \cdot (7i - 9j)} \right)$$

$$= \cos^{-1} \left[\frac{(8)(7) + (-9)(9)}{\sqrt{8^2 + (-9)^2} \sqrt{7^2 + (-9)^2}} \right]$$

$$\theta = 30.76$$

$$Q8 \ R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$= \sqrt{75^2 + 25^2 + 2(75)(25) \cos 30}$$

$$= 97.46 \text{ m}$$

$$Q9 \ A^P = 101, 104, 107 \sim \sim 999$$

$$= 164,850$$

Q10 6th term = 32
8th term = 128

$$a_n = ar^{n-1}$$

$$ar^5 = 32 \text{ eq}^n \text{ (1)}$$

$$ar^7 = 128 \text{ eq}^n \text{ (2)}$$

$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$r = 2$$

$$\therefore r = 2$$

Q11 $(4 + 3n)^5$

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2$$

$$= 243n^5 + 1620n^4 + 432n^3 + 5760n^2 + 3840n + 1024$$

Q12 $\begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

Q13 $f(x) = x^3 - 7x^2 + 8x - 3$ if $x_0 = 5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 14x + 8$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \frac{f(5)}{f'(5)}$$

$$= 5 - \frac{-13}{13}$$

$$= 6$$

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$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 6 - \frac{f(6)}{f'(6)}$$

$$= 6 - \frac{9}{32}$$

$$= 5.71875$$

$$\therefore x_2 = 5.71875$$

Q14 $(1-x+x^2)^4$

Let $1-x=y$

$$(1-x+x^2)^4 = (y+x^2)^4$$

$$= {}^4C_0 y^4 (x^2)^0 + {}^4C_1 y^3 (x^2)^1 + {}^4C_2 y^2 (x^2)^2 + {}^4C_3 y (x^2)^3 + {}^4C_4 (x^2)^4$$

$$= y^4 + 4y^3 x^2 + 6y^2 x^4 + 4y x^6 + x^8$$

$$= (1-x)^4 + 4x^2 (1-x)^3 + 6x^4 (1-x)^2$$

$$+ 4x^6 (1-x) + x^8$$

$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$$

Q.5 $e^{-n} = 3 \log(n)$

No	a	b	c	fa	fb
1	0.5	1.5	1	0.555	
2	1	1.5	1.25	-1.555	10
3	1	1.25	1.25		
4	1.125	1.25	1.187	0.013	0.44

$n = 1.187$