(1) $\lim_{h \to 0} \frac{2(h-3)^2 - 18}{h}$

$\lim_{h \to 0} \frac{2(h^2+9-6h) - 18}{h}$

$\lim_{h \to 0} \frac{2h^2 + \cancel{18} - 12h - \cancel{18}}{h}$

$\lim_{h \to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}}$

$\lim_{h \to 0} 2(0-6) = -12$

(2) (a) $x = 4$.

$g(x) \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$ $\boxed{\text{continuous}}$

$\lim_{x \to 4^-} = \lim_{x \to 4^+} = \lim_{x \to 4} f(4) \Rightarrow 2(4) \Rightarrow 8$

(b) at $x = 6$.

$\lim_{x \to 6^-} \Rightarrow 2(6) \Rightarrow 12$

$\lim_{x \to 6^+} \Rightarrow 6-1 \Rightarrow 5$ So, $\boxed{\text{discontinuous}}$

$\lim f(6) \Rightarrow 5$

(3) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \Rightarrow \frac{(\sqrt{x})^2 - (3)^2}{3-\sqrt{x}}$

$\lim_{x \to 9} \Rightarrow \frac{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}{\cancel{(\sqrt{x}-3)}}$

ree:

$\Rightarrow -6$ when $\sqrt{9} = +3$

$\sqrt{9} = -3$

(4) $f(x) = \frac{4x+5}{9-3x}.$

(1) $x = -1$

$f(-1) \Rightarrow \frac{4(-1)+5}{9-3(-1)} \Rightarrow \frac{-4+5}{9+3} \Rightarrow \frac{-1}{12}$

(2) $x = 0$

$f(0) \Rightarrow \frac{4(0)+5}{9-3(0)} \Rightarrow \frac{5}{9}$

(3) $x = 3$

$f(3) = \frac{4x+5}{9-3(0)} \rightarrow$ not defined

$f \lim_{x \to 3^-} \rightarrow$ ~~4...~~ $-\infty$

$\lim_{x \to 3^+} \rightarrow +\infty$.

(5) $\lim_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$$\cancel{\left[\dfrac{6e^{4x} - \dfrac{1}{e^{2x}}}{8e^{4x} - e^{2x} + \dfrac{3}{e^{x}}} \Rightarrow \dfrac{\dfrac{6e^{6x} - 1}{e^{2x}}}{8e^{5x} - e^{3x} + 3} \right.}$$

$\Rightarrow \lim_{x \to \infty} \dfrac{6 - e^{-2x-4x}}{8 - e^{2x-4x} + 3e^{-x-4x}}$ Dividing num & deno.

$\Rightarrow \lim_{x \to \infty} \dfrac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}} \Rightarrow \dfrac{6}{8} = \dfrac{3}{4}$ Ans.

(6) $g(y) = \begin{cases} y^2+5 & y < -2 \\ 1-3y & y \geq -2 \end{cases}$

(1) $\lim_{y \to 6} g(y)$

$\Rightarrow \lim_{y \to 6^-} = \lim_{y \to 6^+} = f(6) \Rightarrow 1-3(6) \Rightarrow -17$

(2) $\lim_{y \to -2} g(y)$ $\lim_{y \to -2^-} \Rightarrow y^2+5 \Rightarrow (-2)^2+5$

$\Rightarrow 9$

$\lim_{y \to -2^+} \Rightarrow 1-3(-2) \Rightarrow 1+6 = 7$

~~Since~~ $\lim_{y \to (-2)} = f(-2) = 7.$

(7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$

$\Rightarrow 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - \ldots$

$\Rightarrow 20t^4 - 132t^3 + 24t^2 + 12.$

-67
64
131

(8) $R(w) = \dfrac{3w + w^4}{2w^2+1}$

$R'(w) = \dfrac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$

$\Rightarrow \dfrac{6w^2+3+8w^5+4w^3 - [12w^2+4w^5]}{(2w^2+1)^2}$

$\Rightarrow \dfrac{6w^2+3+8w^5+4w^3-12w^2-4w^5}{(2w^2+1)^2}$

$\Rightarrow \dfrac{-6w^2+4w^5+4w^3+3}{(2w^2+1)^2} \Rightarrow \dfrac{4w^5+4w^3-6w^2+3}{4w^4+4w^2+1}$

(9)

$$f(x) = 2e^{x} - 8^{x}$$

$$f'(x) = 2e^{x} - 8^{x}\log_e 8$$

(10)

$$y = z^{5} - e^{2}\ln(z)$$

$$y' = 5z^{4} - \left[e^{z}\ln(z) + e^{2}\times\frac{1}{z}\right]$$

$$\Rightarrow 5z^{4} - e^{z}\ln(z) + e^{2}z^{-1}$$

(11) (a)

$$f(x) = (6x^{2}+7x)^{4}$$

$$f'(x) = 4(6x^{2}+7x)^{3}(12x+7)$$

(b) ~~f(t)=5~~

$$f(t) = 5 + e^{4t+t^{7}}$$

$$f'(t) = e^{4t+t^{7}} \times (4+7t^{6})$$

$$\boxed{f'(t) \Rightarrow e^{4t+t^{7}}(4+7t^{6})}$$

(12) (a)

$$z = \ln(7-x^{3})$$

$$\frac{dz}{dx} = \frac{1}{(7-x^{3})}(-3x^{2}) \Rightarrow \frac{-3x^{2}}{7-x^{3}}$$

$$\frac{d^{2}z}{dx^{2}} \Rightarrow \frac{(7-x^{3})(-6x) + 3x^{2}(-3x^{2})}{(7-x^{3})^{2}} \Rightarrow \frac{-42x + 6x^{4} - 9x}{(7-x^{3})^{2}}$$

$\Rightarrow \dfrac{42x - 3x^4}{(7-x^3)^2}$ Ans.

(b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$Q'(x) \Rightarrow$

$Q(v) = 2(6+2v-v^2)^{-4}$

$\Rightarrow 2(-4)(6+2v-v^2)^{-5}(2-2v)$

$Q'(v) \Rightarrow -16(1-v)(6+2v-v^2)^{-5}$

$Q''(v) \Rightarrow -16\left[(1-v)\left[(-5)(6+2v-v^2)^{-6}(2-2v)\right] + (6+2v-v^2)^{-5}(-1)\right]$

$\Rightarrow (+16)(6+2v-v^2)^{-5}\left[(10)(1-v)^2 + 1\right]$

$Q''(v) \Rightarrow (16)(6+2v-v^2)^{-5}\left[10(1-v)^2 + 1\right]$. Ans

(c) $f(t) = \ln(1+t^2)$

$f'(t) = \dfrac{2t}{1+t^2}$

$f''(t) \Rightarrow \dfrac{(1+t^2)(2) - (2t)^2}{(1+t^2)^2} \Rightarrow \dfrac{2+2t^2-4t^2}{(1+t^2)^2}$

$\Rightarrow \dfrac{2-2t^2}{(1+t^2)^2}$

Sol (13) $f(x) = x^3 - 3x^2 - 9x + 12$.

$f'(x) = 3x^2 - 6x - 9$

$f'(x) = 0$

$3x^2 - 6x - 9 = 0$.

P = -3

S = -2

-3, 1

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$x = -1, 3$.

$f''(x) = 6x - 6 \Rightarrow$ Put $x = -1$

$-12 < 0$ Maxima.

Put $x = 3$:

$18 - 6 > 0$ Minima.

$\Rightarrow$ Maximum value:

$f(-1) \Rightarrow (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$\Rightarrow -1 - 3 + 9 + 12$

$\Rightarrow 17$

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$\Rightarrow -15$.

(14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

Put $f'(x) = 0$

$x^2 - 3x + 2 = 0$ $\qquad$ $P = 2$

$x^2 - 2x - x + 2 = 0$ $\qquad$ $S = -3$

$x(x-2) - 1(x-2) = 0$ $\qquad$ $2, 1$

$1, 2$

$f''(x) = 24x - 36$

Put $x = 1 \Rightarrow 24 - 36 < 0$ Maxima

$\Rightarrow 24(2) - 36 > 0$ Minima

Maxima value $\Rightarrow f(1) = 4 - 18 + 24 - 7$

$\Rightarrow -16 - 7 + 24$

$\Rightarrow 3$

48
36
24

$f(2) = 4(8) - 18(4) + 24(2) - 7$

$\Rightarrow 36 - 72 + 48 - 7$

$\Rightarrow 84 - 79$

$\Rightarrow -39$ $\quad$ 1

18
24
32

(15)

$f(x) = x^2(x+3)$

$= x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

Put $f'(x) = 0$ $\Rightarrow$ $3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0, \quad x = -2$

$f''(x) = 6x + 6$

Put $x = 0$ $\quad f''(x) > 0$

$x = -2$ $\quad f''(x) < 0$

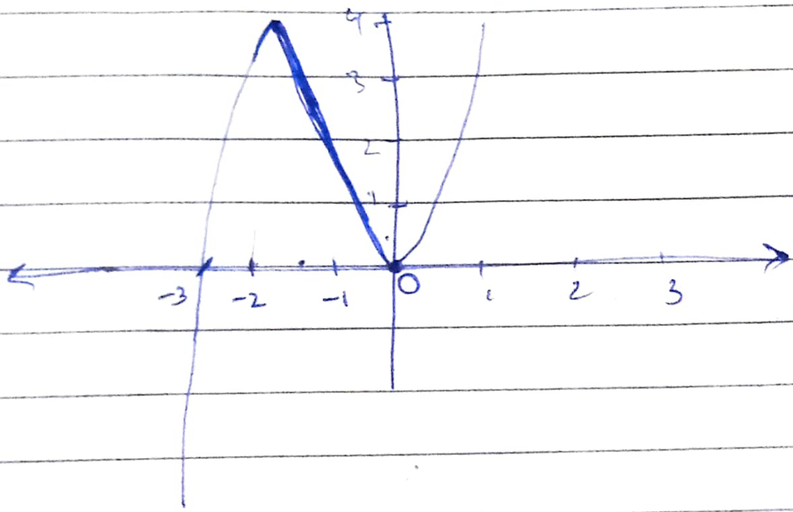
So, $x = 0$, $f(x)$ attains ~~max~~ min value

$x = -2$ $f(x)$ " max value

+ − +

$f(0) = 0$

$f(-2) = (-2)^3 + 3(-2)^2 \Rightarrow -8 + 12 = 4.$



$3x^2 - 6x = 0$
$3x(x-2) = 0$
$0, 2$

16

$f(x) = 3x^2 - 6x$

decreasing | increasing

$f'(x) = 6x - 6$

$f'(x) = 0 \Rightarrow 6x - 6 = 0$

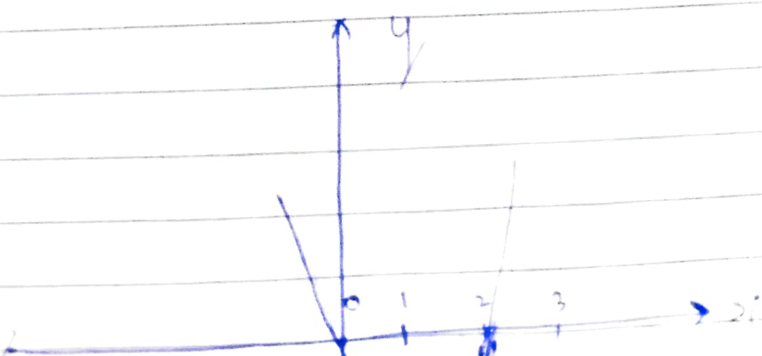
$6(x-1) = 0$

$\boxed{x = 1}$

$f''(x) = 6 > 0$

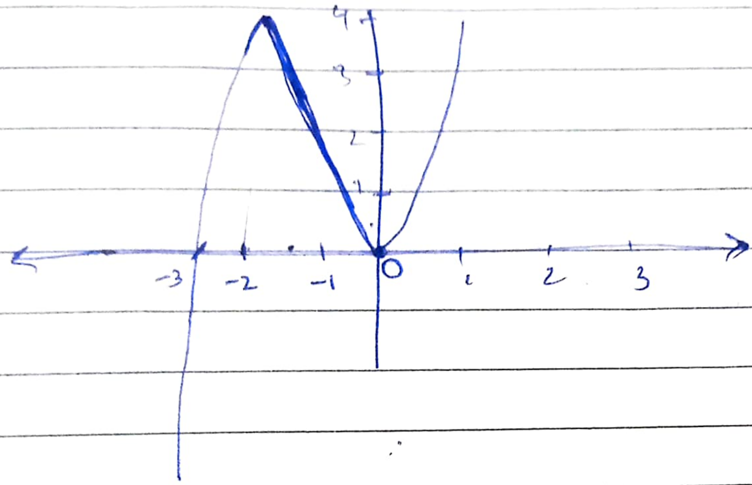
So, at $\boxed{x=1}$, $f(x)$ attains minimum value.

$f(1) = 3(1)^2 - 6 \Rightarrow 3 - 6 = \boxed{-3}$



$f(0) = 0$

$f(-2) = (-2)^3 + 3(-2)^2 \Rightarrow -8 + 12 = 4$



$3x^2 - 6x = 0$
$3x(x-2) = 0$
0, 2

16 $f(x) = 3x^2 - 6x$

decreasing ~~increasing~~ increasing ~~decreasing~~

$f'(x) = 6x - 6$

$f'(x) = 0 \Rightarrow 6x - 6 = 0$

$6(x-1) = 0$

$\boxed{x = 1}$

$f''(x) = 6 > 0$

so, at $\boxed{x=1}$, $f(x)$ attains minimum value.

$f(1) = 3(1)^2 - 6 \Rightarrow 3 - 6 = \boxed{-3}$