SRM 4 Assignment 1
Unit 1 & Unit 2

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1] P(default) = 0.1

(i) Gumbel copula:

$\alpha = 2$

$$P(T_1 \le 1, T_2 \le 1, T_3 \le 1) = C(u_1, u_2, u_3)$$

$$= \exp\left\{ -\left((-\ln u_1)^{\alpha} + (-\ln u_2)^{\alpha} + (-\ln u_3)^{\alpha} \right)^{1/\alpha} \right\}$$

since probability of default of each bond is same

$$= \exp\left\{ -\left(3(-\ln u)^{\alpha} \right)^{1/\alpha} \right\}$$

$$= \cancel{\exp\left\{ -3(-\ln(0.1)) \right\}}$$

$$= 0.0185$$

$$= 1.85\%$$

(ii) Gumbel copula exhibits upper tail dependence and no lower tail dependence. Hence, for earlier duration, Gumbel copula might be appropriate. But if we are observing probability of default, gumbel copula might not be the best function to be used.

2] i) Three parameters of Generalized Extreme Value distribution are-
→ location parameter α
→ Scale parameter β
→ shape parameter γ

ii) Parameters α & β rescale the distribution. Parameter γ determines the overall shape of distribution.

iii) ~~α & β~~ α is analogous to mean, β is analogous to ~~var~~ standard variation and γ is analogous to skewness.

3] i) Four ways of measuring tail weight of a particular distribution.
→ Existence of moments (raw & central)
→ Limiting density ratio
→ Hazard Rate
→ Mean Residual Time

(ii) $\alpha_1 = 0.2$ & $\alpha_2 = 0.3$

According limiting density ratio method,

$$\lim_{x \to \infty} \frac{f_{x_1}(x)}{f_{x_2}(x)}$$

$$\lim_{x\to\infty} \frac{\dfrac{\alpha_1 \lambda^{\alpha_1}}{(\lambda+x)^{\alpha_1+1}}}{\dfrac{\alpha_2 \lambda^{\alpha_2}}{(\lambda+x)^{\alpha_2+1}}} = \lim_{x\to\infty} \frac{0.2\,\lambda^{0.2}}{(\lambda+x)^{1.2}} \times \frac{(\lambda+x)^{1.3}}{0.3\,\lambda^{0.3}}$$

$$= \frac{0.2}{0.3} \times \lambda^{0.2-0.3} \lim_{x\to\infty} (\lambda+x)^{1.3-1.2}$$

$$= \frac{2}{3} \frac{1}{\lambda^{0.1}} \lim_{x\to\infty} (\lambda+x)^{0.1} = \infty$$

Here, numerator has heavy tail i.e. distribution with $\alpha = 0.2$ has a heavier tail.

4] P(survival of Male) = 0.6 $\quad {}_{30}p_{50}^{M}$
P(survival of Female) = 0.65 $\quad {}_{30}p_{50}^{F}$

P(Male dies) = 0.4 $\quad {}_{30}q_{50}^{M}$
P(Female dies) = 0.35 $\quad {}_{30}q_{50}^{F}$

Let X be the ~~rando~~ future lifetime of Male & Y be the future lifetime.

$P(X \le 30) = 0.4$

$P(Y \le 30) = 0.35$

P(paying death benefit) = $P(X \le 30, Y \le 30)$

i) Gumbel, $\alpha = 3.5$

$$C(u_1, u_2) = \exp\left[-\left[(-\ln u_1)^{\alpha} + (-\ln u_2)^{\alpha}\right]^{1/\alpha}\right]$$

$u_1 = 0.4$

$u_2 = 0.35$

$$C(u_1, u_2) = \exp\left[-\left[(-\ln 0.4)^{3.5} + (-\ln 0.35)^{3.5}\right]^{1/3.5}\right]$$

$$= 0.29963$$

iii) ~~Clayton~~ Frank, $\alpha = 3.5$

$$C[u_1, u_2] = \frac{-1}{\alpha} \ln\left[1 + \frac{(e^{-\alpha u_1} - 1)(e^{-\alpha u_2} - 1)}{(e^{-\alpha} - 1)}\right]$$

$$= \frac{-1}{3.5} \ln\left[1 + \frac{(e^{-3.5 \times 0.4} - 1)(e^{-3.5 \times 0.35} - 1)}{(e^{-3.5} - 1)}\right]$$

$$= 0.22739$$

(iii) ~~Frank~~ clayton, $\alpha = 3.5$

$$c[U_1, U_2] = \left[U_1^{-\alpha} + U_2^{-\alpha} - 1\right]^{-\frac{1}{\alpha}}$$

$$= \left[0.4^{-3.5} + 0.35^{-3.5} - 1\right]^{\frac{-1}{3.5}}$$

$$= 0.30595$$

iv) Clayton copula gives us the highest probability and it also exhibits lower tail dependence which suggests that if one life does not survive for long then the other life will also not survive for long.

The independent probability of paying the benefit is $0.4 \times 0.35 = 0.14$.

Here, Clayton copula would be more appropriate as the two lives covered under the contract are related & hence probability of paying a benefit would be higher than calculated by assumption of independent.

5] Let X denote lifetime of new mobile phone

$X \sim Exp(0.125)$

(i) Mean Residual life of 2 year old phone

$$= \frac{\int_x^\infty (1-F(y))\,dy}{1-F(x)}$$

$$= \frac{\int_x^\infty e^{-\lambda y}\,dy}{e^{-\lambda x}}$$

$$= \frac{1}{\lambda} = \frac{1}{0.125} = 8 \text{ years.}$$

(ii) Exponential distribution follows memoryless property. Hence, mean residual life of a 2 year old phone and new phone $\left(\frac{1}{\lambda} = 8\right)$ is the same.

Hence, Exponential distribution is not suitable to model Mean Residual life.