Probability & Statistics
Assignment

Q1. $X = 200$

$X \sim Poi(\theta)$

$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$

$P[x_1 < \theta < x_2] = 0.95$

$P[-1.196 < \theta < 1.96] = 0.95$

95% CI $= \theta \pm 1.96\sqrt{\theta}$

$= 200 \pm 1.96\sqrt{200}$

$= (172.281,\ 227.718)$

2. $x_i = 1, 2, \ldots n$ iidi RV

mean $= \mu$

variance $= \sigma^2$

$\bar{x} = \sum_1^n \frac{x_i}{n}$

$s^2 = \frac{1}{n-1} \sum_1^n (x_i - \bar{x})^2$

(i) Mean of $\bar{x}$

$E(\bar{x}) = E\left(\frac{\sum x_i}{n}\right)$

$= \frac{1}{n} E(\sum x_i) = \frac{\sum [E(x_i)]}{n}$

$= \frac{\cancel{n}\mu}{\cancel{n}} = \boxed{\mu}$

$V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right) = \frac{1}{n^2} V(\sum x_i) = \frac{\sum V(x_i)}{n^2} = \frac{n\sigma^2}{n^2} =$

$= \boxed{\frac{\sigma^2}{n}}$

(ii) $E(s^2) = E\left[\frac{1}{(n-1)} \Sigma (x_i - \bar{x})^2\right]$

$$= \frac{1}{(n-1)} E\left[\Sigma (x_i - \bar{x})^2\right]$$

$$= \frac{1}{(n-1)} \Sigma\left[E\left(x_i^2 + \bar{x}^2 - 2x_i\bar{x}\right)\right]$$

$$= \frac{1}{(n-1)} \Sigma\left[E(x_i^2) + E(\bar{x}^2) - 2\bar{x}^2\right]$$

$$= \frac{1}{(n-1)} \Sigma\left[E(x_i^2) - E(\bar{x}^2)\right]$$

$$= \frac{1}{n-1} \Sigma\left[\sigma^2 + \mu^2 - \left(\frac{\sigma^2}{n} + \mu^2\right)\right]$$

$$= \frac{1}{(n-1)} \Sigma\left[\sigma^2\left(\frac{n-1}{n}\right)\right]$$

$$= \frac{\cancel{n}\sigma^2}{\cancel{n}} = \underline{\underline{\sigma^2}}$$

There

(iii) $x_i \sim$ Norm

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$v(\chi^2_{n-1}) = \frac{n-1}{2\left(\frac{1}{n}\right)} = 2(n-1)$$

$$v\left[\frac{(n-1)s^2}{\sigma^2}\right] = v\left[\chi^2_{n-1}\right]$$

$$\frac{(n-1)^2}{\sigma^4} v(s^2) = 2(n-1)$$

$$v(s^2) = \frac{2\sigma^4}{n-1}$$

(iv) $n = 10$

$\sigma^2 = 100$

$$P[50 < s^2 < 150] = P[s^2 < 150] - P[s^2 < 50]$$

$$= P\left[\frac{(n-1)s^2}{\sigma^2} < \frac{9 \times 150}{100}\right]$$

$$- P\left[\frac{(n-1)s^2}{\sigma^2} < \frac{9 \times 50}{100}\right]$$

$$= P\left[\chi^2_9 < 13.5\right] - P\left[\chi^2_9 < 4.5\right]$$

$$= 0.8587 - 0.1245$$

$$= 0.7342$$

3. X = no. of claims

$X \sim Bin(n, p)$

No. of claims	0	1	2	3	4
No. of policies	86	75	16	2	1

(i) $$L = \prod_{i=1}^{180} P(X = n)$$

$$= [P(X=0)]^{86} \cdot [P(X=1)]^{75} \cdot [P(X=2)]^{16} \cdot [P(X=3)]^2 \cdot [P(X=4)]$$

$$= ({}^4C_0 p^0 (1-p)^4)^{86} \cdot [{}^4C_0\, p(1-p)^3)^{75} \cdot [{}^4C_2 p^2 (1-p)^2)^{16} \cdot ({}^4C_3 p^3 (1-p))^2 \cdot ({}^4C_4 p^4)$$

$$L = \text{constant} \times (1-p)^{603} p^{117}$$

$$\ln L = \text{constant} + 603 \ln(1-p) + 117 \ln p$$

$$\frac{d}{dp} \ln L = -\frac{603}{1-p} + \frac{117}{p} = 0$$

$$603p = 117(1-p)$$

$$720p = 117$$

$$p = 0.1625$$

$$\frac{d^2}{dp^2} \ln L = -\frac{603}{(1-p)^2} - \frac{117}{p^2} < 0 \text{ — minima}$$

$$\hat{p} = \underline{\underline{0.1625}}$$

(ii) goodness of fit test

H_0 : no. of claims confirms to the bin. model

H_1 : no. of claims does not con. to the bin. model

	0	1			
O_i	86	75	16	2	1
E_i	88.542	68.724	19.998	2.574	0.108

$p = 0.1625$

$P(X=0) = {}^4C_0\, p^0 (1-p)^4 = 0.4919$

$P(X=1) = {}^4C_1\, p^1 (1-p)^3 = 0.3818$

$P(X=2) = 0.1111$

$P(X=3) = {}^4C_3\, p^2 (1-p)^1 = 0.0143$

$P(X=4) = 0.0006$

O_i	86	75	19
E_i	88.542	68.724	22.68

$$T.S = \frac{\Sigma (O_i - E_i)^2}{E_i} \sim \chi^2_{\cancel{5-1-1-2}}$$

$$1.24322 \sim \chi^2_1$$

We do not reject H_0 at 5% level of significance no. of claims on each policy conform to the binomial model

4. $f(x) = \dfrac{C\beta^3}{(x+\beta)^4}$; $x > 0$ $\beta > 0$

According to the table book.

Pareto distribution : $f(x) = \dfrac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$

$\therefore$ β corresponds to α

comparing both the equations

$C = 3$

Mean of x:

$E(x) = \dfrac{\beta}{C-1} = \dfrac{\beta}{2}$

variance of x:

$V(x) = \dfrac{C\beta^2}{(C-1)^2(C-2)} = \dfrac{3\beta^2}{4}$

(ii) $E(x) = \dfrac{\beta}{2}$

$E(x) = \bar{x}$

$\dfrac{\beta}{2} = \bar{x}$

$\hat{\beta} = 2\bar{x}$

verifying unbiasness

$$\begin{aligned}\text{Bias} &= E(\hat{\beta}) - \beta \\ &= E(2\bar{x}) - \beta \\ &= 2E(\bar{x}) - \beta \\ &= \frac{2}{n}\sum E(x_i) - \beta \\ &= \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}\beta}{\cancel{2}} - \beta \\ &= 0\end{aligned}$$

$\therefore$ Bias = 0, thus unbiased

$MSE = V(\hat{\beta}) + (Bias)^2$

$= V(2\bar{x})$

$= 4V(\bar{x})$

$= 4V\left(\frac{\sum x_i}{n}\right)$

$= \frac{4}{n^2} \sum V(x_i)$

$= \frac{\cancel{4}}{n^{\cancel{2}}} \times \frac{3\beta^2 \cancel{n}}{\cancel{4}}$

$MSE = \frac{3\beta^2}{n}$

As the $n \to \infty$, MSE tends to 0

$\hat{\beta}$ is consistent.

(iii) Estimators are of the form $b\bar{x}_n$ $b \to$ const.

$\hat{\beta} = b\bar{x}_n$

$MSE = V(\hat{\beta}) + Bias^2$

$V(\hat{\beta}) = V(b\bar{x}_n) = b^2 V(\bar{x}_n)$

$= \frac{b^2}{n^2} \sum V(x_i^2)$

$= \frac{b^2}{n^2}\left(\frac{n3\beta^2}{4}\right)$

$= \frac{3b^2\beta^2}{4n}$

$E(\hat{\beta}) = E(b.\bar{x})$

$= \frac{b}{n} \sum E(x_i)$

$= \frac{b\beta}{2}$

$$\text{Bias} = \frac{b\beta}{2} - \beta = \frac{b\beta - 2\beta}{2} = \frac{\beta(b-2)}{2}$$

$$MSE = \frac{3b^2\beta^2}{4n} + \left[\frac{\beta(b-2)}{2}\right]^2$$

$$= \frac{3b^2\beta^2}{4n} + \frac{\beta^2}{4}\cdot[b^2 + 4 - 4b]$$

$$= \frac{3b^2\beta^2 + \beta^2 n[b^n + 4 - 4b]}{4n}$$

$$= \frac{\beta^2}{4n}[3b^2 + nb^2 + 4n - 4nb]$$

(iv) For unbiasedness

$$\text{Biass} = E(g(x)) - \theta$$

$$= E(\hat{\beta}) - \beta$$

$$= \frac{b\beta}{2} - \beta$$

$$= \beta\left[\frac{b-2}{2}\right)$$

$$\text{At } b = \frac{2}{1+3/n}$$

$$\text{Bias} = \beta\left[\frac{2n/3+n/2 - 2}{2}\right]$$

$$= \beta\left[\frac{2n - 6 - 2n}{2(n+3)}\right]$$

$$= \frac{-3\beta}{n+3}$$

since ~~β~~ Bias < 0 $\hat{\beta}$ is an underestimate of the true parameter value

$$MSE = var(\hat{\beta}) + Bias^2$$

$$= \frac{\beta^2}{4n}[3b^2 + nb^2 + 4n - 4nb]$$

$$= \frac{\beta^2}{4n}[b^2(3+n) + 4n(1-b)]$$

$$= \frac{\beta^2}{4n}\left[\frac{4n^2}{(n+3)^2}(n+3) + 4n\left(\frac{1-2n}{n+3}\right)\right]$$

$$= \frac{\beta^2}{4n}\left[\frac{4n^2}{n+3} + \frac{4n(3-n)}{n+3}\right]$$

$$= \frac{\beta^2(4n^2 + 12n - 4n^2)}{4n(n+3)}$$

$$MSE = \frac{3\beta^2}{n+3}$$

As $n \to \infty$, $MSE \to 0$

Hence it is consistent.

5(i) $X \sim U(a,b)$

$$E(X) = \frac{a+b}{2}$$

$$\bar{X} = \frac{a+b}{2}$$

$$a = 2\bar{X} - b \quad \text{— (1)}$$

$$V(X) = \frac{(b-a)^2}{12}$$

$$S^2 = \frac{(b-a)^2}{12}$$

$$12S^2 = (b-a)^2$$

$$2\sqrt{3}\,S = b - a$$

$$2\sqrt{3}S = 2\bar{X} - a - a \qquad [b = 2\bar{X} - a]$$

$$\sqrt{3}\,S = \bar{X} - a$$

$$\hat{a} = \bar{X} - \sqrt{3}S$$

(ii) $b = 2\bar{x} - a$

$b = 2\bar{x} - \bar{x} + \sqrt{3}S$

$\hat{b} = \bar{x} + \sqrt{3}S$

(iii) Sample of observations = 5, 12, 20, 35, 50

$\bar{x} = 24.4, \quad S = 18.477$

$\hat{a} = -7018 \quad \hat{b} = 55.87$

(iv) Sample = $x_1, x_2, \ldots x_n$

$X \sim U(0, b)$

$$L = \prod_{i}^{n} f(x)$$

$$L = \left(\frac{1}{b-a}\right)^n$$

$$\ln L = -n \ln b$$

$$\frac{d}{db}(\ln L) = -\frac{n}{b} = 0$$

Thus $b = \infty$, which is not possible

Also $\frac{d^2}{db^2}(\ln L) = \frac{n}{b^2} > 0$, minima

$$\hat{b} = \operatorname{Max}(x_i)$$

Q6 done in last

7. (i) Sample mean:

Private hospital $(\bar{x}_{Pr}) = \frac{\Sigma x_i}{n} = 35.056$

Public hospital $(\bar{x}_{Pv}) = \frac{270}{9} = 30$

Sample variance

$$S^2_{Pr} = \frac{\Sigma x_i^2 - n\bar{x}}{n-1} = 67.403$$

$$S^2_{Pv} = 64.3125$$

(ii) For testing equal mean, we can use a 2 sided t test assuming that the population variation of both is equal

To verify this assumption

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad ; \quad H_1 : \sigma_1^2 \neq \sigma_2^2.$$

$$\frac{S^2_{Pr} / S^2_{Pv}}{\sigma^2_{Pr} / \sigma^2_{Pv}} \sim F_{n_{Pr}-1,\, n_{Pv}-1}$$

For 95 % CI

$$P\left[\frac{1}{n} < F_{8,8} < x\right] = 0.95$$

$$P\left[\frac{1}{4.43} < \frac{67.403}{64.3125} \frac{\sigma^2 P_V}{\sigma^2 P_r} < 4.433\right]$$

95 % of CI = (0.2152, 4.229)

Since at 5 % Cos, CI include 1, we don't reject H_0. Our assumption of equal popn variance holds.

(iii) $H_0 : \mu_{pr} = \mu_{pu}$; $H_1 = \mu_{pr} > \mu_{pu}$

This is a one-sided ten

Since popn var. is unknown we use t-distn

$$t_{cor} = \frac{(\bar{x}_{pr} - \bar{x}_{pu}) - (\mu_{pr} - \mu_{pu})}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{5.056}{8.6756\sqrt{2/9}} = 1.2363$$

$$t_{tab} = t_{16, 5\%} = 1.746$$

Since $t_{cor} < t_{tab}$, we have insufficient evidence to reject H_0 at 5% level of significance.

8. (i) An unbiased estimator is an estimator whose mean value $E(\hat{\theta})$), is equal to the parameter value (θ).

It is an estimator where expected value is equal to the parameter

(ii) Bias is the measure of the difference between the expected value of the estimator & the true parameter value.

(iii) Mean square error is the second moment the estimator abt the true parameter value

that is since $\hat{\theta} \rightarrow$ estimator

True parameter is θ

$$MSE = E[(\hat{\theta} - \theta)^2]$$

(vi) Two methods of estimating θ are:
Method of moments
Maximum likelihood expectation

9. t_k is defined as: $\frac{Z}{\sqrt{C/k}}$ wt k
i)
where $Z \sim N(0,1)$
$C \sim \chi^2_{k=n-1}$

k is the degrees of freedom

(ii) Mean of t_k = 0
variance = $\frac{k}{k-2}$, $k>2$

(iii) $n = 10$
$\bar{x} = 50$
$s^2 = 48.667$

(a) TS : $\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1}$

For 99% CI = $\bar{x} \pm t_{n-1, 0.5\%} \, s/\sqrt{n}$
$= 50 \pm 3.250 \times \sqrt{\frac{48.667}{10}}$
$= (42.83, 57.1677)$

MAHAGURU Page No.
Date / /

10. $n = 1000$

$\lambda = 3$

$X \sim Poi(n\lambda)$

$X \sim Poi(3000)$

(i) $P(\text{type 1 error}) = P(\text{reject } H_0 \mid H_0 \text{ is true})$

$= P(X > 3100 \mid X \sim Poi(3000))$

Using normal approx

$$P(X > 31000) = P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$$

$= 1 - P(Z < 1.825)$

$= 3.39\%$

(ii) Type II error : event of accepting the hypothesis when it is false

(iii) Power of test $= P(\cancel{\text{type}}\text{ rejecting } H_0 \mid H_0 \text{ is false})$

$= P[X > 31000 / X \sim Poi(n\lambda) \sim N(n\lambda, n\lambda)]$

$$= 1 - P\left(Z < \frac{31000 - n\lambda}{\sqrt{n\lambda}}\right)$$

(iv) $\hat{\mu}$ is estimator of μ, $\hat{\mu}$ follows $N\left(\mu, \frac{\hat{\mu}}{n}\right)$

Hence $\frac{\hat{\mu} - \mu}{\sqrt{\hat{\mu}/n}} \sim N(0,1)$

$$P\left(-2.5758 < \frac{2.7 - \mu}{\sqrt{2.9/1000}} < 2.5758\right) = 0.99$$

confidence interval for μ is $(2.7613, 3.0387)$

11. $\Sigma x = 213200$
$n = 12$
$\Sigma x^2 = 4,917,860,000$
$\bar{x} = 17,767$
$S = 10.144$

$\Sigma x_{new} = \Sigma x +$ correct values $-$ incorrect values
$= 213200 + (2500 + 31500) - (11000 + 48000)$
$= 213200$

$\bar{x}_{new} = \frac{\Sigma x_{new}}{x} = \frac{213200}{12} = 17767$

$\Sigma x_i^2{}_{new} = \Sigma x_i^2 - (11000^2 + 48000^2) + 2500^2 + 31500^2$
$= 4243360000$

$s^2 = \frac{1}{n-1}(\Sigma x_i^2{}_{new} - n\bar{x}^2{}_{new})$

$= \frac{1}{11}[4243360000 - (12 \times 17767^2)]$

$s^2 = 41396776$
$s = 6434$

12. $X = X_1, X_2, \ldots X_n$
$X \sim U(0, \theta)$

(i) CRLB for variance of unbiased estimator of θ doesnt apply here as the range of distribution involves the parameter, hence the derivative in the formula does not make sense.

(ii) $\hat{\theta}(c) = cY$, $c \to$ constant, $Y = \max X_i$

Since $Y = \max X_i$

$$F(y) = F(X_i \max) = F(\max(x_1, x_2, \dots x_n))$$
$$= 1 - P(X \geq \max(x_1, x_2, \cdots x_n)]$$
$$= 1 - P(X > x_1)\, P(X > x_2) \dots P(X > x_n))$$
$$= 1 - P(X \geq x)^n$$
$$= P(X < x)^n$$
$$= [F(x)]^n$$

$$F(y) = \left(\frac{x-0}{\theta-0}\right)^n = \left(\frac{x}{\theta}\right)^n = \left(\frac{y}{\theta}\right)^n$$

$$f(y) = F'(y)$$
$$= \frac{d}{dy}\left(\frac{y}{\theta}\right)^n$$
$$= \frac{1}{\theta^n}\left[n y^{n-1}\right], \quad 0 < y < \theta$$

$$E(y^k) = \int_0^\theta y^k f(y)\, dy$$
$$= \int_0^\theta y^k \left(\frac{n}{\theta^n} y^{n-1}\right) dy$$
$$= \frac{n}{\theta^n}\left[\frac{y^{n+k}}{k+n}\right]_0^\theta$$
$$= \frac{n\theta^{n+k}}{\theta^n (n+k)}$$
$$= \frac{n \cancel{\theta^n} \theta^k}{\cancel{\theta^n}(n+k)}$$

$$E(Y^k) = \frac{n\theta^k}{n+k} \quad \text{——} \quad (1)$$

(iii) $\text{Bias}[\hat{\theta}(c)] = E[\hat{\theta}(c)] - \theta$

$= E(cY) - \theta$

$= c\,E(Y) - \theta$

using ① at $k = 1$

$\text{Bias} = c\left[\frac{n\theta}{n+1}\right] - \theta$

$= \theta\left[\frac{cn}{n+1} - 1\right]$

$MSE[\hat{\theta}(c)] = \text{Var} + \text{Bias}^2$

$V(\hat{\theta}(c)) = V(cY)$

$= c^2 V(Y)$

$= c^2 [E(Y^2) - [E(Y)]^2]$

$= c^2 \left[\frac{n\theta^2}{n+2} - \left(\frac{n\theta}{n+1}\right)^2\right]$

$= c^2 \left[\frac{n\theta^2}{n+2} - \frac{n^2\theta^2}{(n+1)^2}\right]$

$MSE = c^2\left[\frac{n\theta^2}{n+2} - \frac{n^2\theta^2}{(n+1)^2}\right] + \left[\theta\left(\frac{cn}{n+1} - 1\right)\right]^2$

$= \frac{c^2 n\theta^2}{n+2} - \frac{c^2n^2\theta^2}{(n+1)^2} + \theta^2\left[\frac{cn}{n+1} - 1\right]$

$= \frac{c^2 n\theta^2}{n+2} - \frac{c^2n^2\theta^2}{(n+1)^2} + \theta^2\left[\frac{c^2n^2}{(n+1)^2} + 1 - \frac{2cn}{n+1}\right]$

$MSE = \frac{c^2 n\theta^2}{(n+2)} - \frac{c2n\theta 2}{(n+1)} + \theta^2$

(iv) For $\theta(c)$ to become an unbiased estimator of θ

$$\text{Bias} = 0$$

$$E(\hat{\theta}(c)) - \theta = 0$$

$$E(\hat{\theta}(c)) = \theta$$

$$E(cy) = \theta$$

$$c\,E(y) = \theta$$

$$c\left(\frac{n\theta}{n+1}\right) = \theta$$

$$c_u = \frac{n+1}{n}$$

(v) $$MSE = \frac{c^2 n\theta^2}{n+2} - c\frac{(2n\theta)^2}{n+1} + \theta^2$$

$$\frac{d}{dc}(MSE) = \frac{2cn\theta^2}{n+2} - \frac{2n\theta^2}{n+1} = 0$$

$$\frac{2cn\theta^2}{n+2} = \frac{2n\theta^2}{n+1}$$

$$cn + c = n+2$$

$$c = \frac{n+2}{n+1}$$

$$\frac{d^2}{dc^2}(MSE) = \frac{2n\theta^2}{n+2} > 0$$

Minima at $c = \frac{n+2}{n+1}$

$$c_m = \frac{n+2}{n+1}$$

MAHAGURU Page No.

Date / /

(vi) $\hat{\theta}(cm)$ is preferred over $\hat{\theta}(cu)$ as in the latter case it could so happen that some of the estimators are very large while other small, but on average give a true value

6. $H_0 : p = 0.1$

$H_1 : p > 0.1$

(i) X : no. of hits in 1st step

Y : no. of hits in 2nd step

$P(\text{type I error}) = P(X \geq 3 / p = 0.1) + P(Y \geq 5 / p = 0.1)$
$P(X = 0 / p = 0.1) + P(Y \geq 4 / p = 0.1) P(X = 1 / p = 0.1)$
$+ P(Y \geq 3 / p = 0.1) P(X = 2 / p = 0.1)$

$= P(X > 2 / p = 0.1) + P(Y > 4 / p = 0.1) P(X = 0 / p = 0.1)$
$+ P(Y > 3 / p = 0.1) P(X = 1 / p = 0.1) + P(Y > 2 / p = 0.1)$
$P(X = 2 / p = 0.1)$

$= (1 - 0.8891) + (1 - 0.9957)(0.2824) + (1 - 0.9744)$
$(0.6590 - 0.28424) + (1 - 0.8891)(0.8891 - 0.6590)$
$= 0.1433$
$= 14.33\%$

MAHAGURU Page No.
Date / /

(ii) P(rejecting H_0) when $p = 0.3$

$= P(X=2 \mid p=0.3) + P(Y>4 \mid p=0.3) \cdot P(X=0 \mid p=0.3)$
$+ P(Y>3 \mid p=0.3) P(X=1 \mid p=0.3)$
$+ P(Y>2 \mid p=0.3) P(X=2 \mid p=0.3)$

$= (1-0.2528) + (1-0.07237)(0.0138)$
$+ (1-0.4925)(0.0850-0.0138) + (1-0.2528)$
$(0.2528-0.0850)$

$= 0.9125$
$= 91.25\%$

(iii) $P(\text{type II error}) = 1 - P(\text{type I error})$
$= 1 - 0.9125$
$= 0.0875$
$= 8.75\%$

(iii) $\hat{p} = \frac{40}{120} = 0.3333$

$\bar{x} \sim N\left(p, p\frac{(1-p)}{n}\right)$

Pivotal quantity $= \dfrac{\bar{x} - np}{\sqrt{\frac{p(1-p)}{n}}}$

For 90% one sided C I

$P\left[\dfrac{\bar{x}-p}{\sqrt{\frac{p(1-p)}{n}}} < x_1\right] = 0.1$

90% of CI $= \bar{x} - z_{0.1}\sqrt{\frac{p(1-p)}{n}}$

$= 0.3333 - 1.2816\sqrt{\frac{0.3333(0.6667)}{120}}$

$= 0.2783$

v. $H_0 : p = 0.278$

$H_1 : p > 0.278$

Since $\hat{p} = 0.3333 > p$:- continuity correction $\left(\frac{x - 1/2}{n}\right)$

$$T_s : \frac{[x - 1/2 / n] - p}{\sqrt{p(1-p)/n}} \sim N(0,1)$$

$$T_{cal} = \frac{\frac{40 - 0.5}{120} - 0.278}{\sqrt{\frac{0.278(0.722)}{120}}}$$

$$= 1.251$$

$T_{tab} = Z_{10\%} = 1.2816$

since $Z_{cal} < Z_{tab}$

We do not reject H_0

(vi) Lower bound of confidence ~~int~~ interval implies that they say a 10% change of $P \leq 0.2782$ where from the hypothesis test $6p^?$ coloured be less than or equal to 0.2780 with prob. of more than 10%.

(vii) $n_1 = 12$ $\quad \sigma_1 = 54$

$\bar{X}_1 = 65$ $\quad \sigma_2 = 70$

$\bar{X}_2 = 70$ $\quad n_2 = 15$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Assumption : Pop^n var. of both samples are equal

$$T_s = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{\mu_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$sp = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= 4027.04$$

$$t_{cal} = \frac{(65 - 70)}{\sqrt{4027.04\left(\frac{1}{12} + \frac{1}{15}\right)}} = \frac{-5}{\sqrt{24.571}} = -0.2034$$

$$t_{tab} = (-2.060)$$

Since t_{cal} is in the 95% CI, we donot reject Ho