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ROLL NO: 85
FY SECTION B

1

ASSIGNMENT

NUMERICAL METHODS AND ALGEBRA

1) $\bar{r} = 12.65$ cm

$r = 12.5$ cm

Original area of the plate $= \pi r^2 = \pi \times (12.5)^2 = 490.8738521$

Calculated area of the plate $= \pi \bar{r}^2 = \pi \times (12.65)^2 = 502.7255104$

(i) absolute error $= |$ original area − calculated area $|$

$= |490.8738521 - 502.7255104|$

$= 11.85165831$

(ii) relative error $= \dfrac{11.85165831}{490.8738521} = 0.024144$

(iii) percentage error $= \dfrac{11.85165831}{490.8738521} \times 100 = 2.4144$

2)

(a)

x	$f(x)$	$\Delta f(x)$
4	0.60206	0.1760913
6	0.7781513	

$h = 2$

$u = \dfrac{5-4}{2} = \dfrac{1}{2} = 0.5$

$f(x) = f(x_0) + u\,\Delta f(x_0)$

$\log(5) = 0.60206 + (0.5 \times 0.1760913)$

$= 0.69010565$

(b)

x	$f(x)$	$\Delta f(x)$
4.5	0.6532125	0.0871502
5.5	0.7403627	

$h = 1$

$u = \dfrac{5-4.5}{1} = 0.5$

$f(x) = f(x_0) + u\,\Delta f(x_0)$

$= 0.6532125 + (0.5 \times 0.0871502)$

$= \underline{\underline{0.6967876}}$.

3) $x^4 - x - 10 = 0$

x	y
0	-10
1	-10
2	4

$x_1 = 1$, $y_1 = -10$

$x_2 = 2$, $y_2 = 4$

$x_3 = \frac{x_1 + x_2}{2} = \frac{2+1}{2} = 1.5$

$y_3 = (1.5)^4 - 1.5 - 10 = -6.4375$

$x_3 = 1.5$, $y_3 = -6.4375$

$x_2 = 2$, $y_2 = 4$

$x_4 = \frac{x_2 + x_3}{2} = \frac{1.5+2}{2} = 1.75$

$y_4 = (1.75)^4 - 1.75 - 10 = -2.371$

$x_4 = 1.75$, $y_4 = -2.371$

$x_2 = 2$, $y_2 = 4$

$x_5 = \frac{x_2 + x_4}{2} = \frac{2+1.75}{2} = 1.875$

$y_5 = (1.875)^4 - 1.875 - 10 = 0.4846$

$x_4 = 1.75$, $y_4 = -2.371$
$x_5 = 1.875$, $y_5 = 0.4846$

$x_6 = \frac{x_4 + x_5}{2} = 1.8125$

$y_6 = (1.8125)^4 - 1.8125 - 10$
$= -1.0202$

$x_6 = 1.8125$, $y_6 = -1.0202$
$x_5 = 1.875$, $y_5 = 0.4846$

$x_7 = \frac{x_6 + x_5}{2} = \frac{1.8125 + 1.875}{2} = \underline{1.84375}$

$y_7 = (1.84375)^4 - 1.84375 - 10 = -0.2877$

$\therefore$ root of $x^4 - x - 10 = 0$ is approximately $x = 1.84375$.

4) $f(x) = x^3 - x - 1$
$f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{(1-1-1)}{(3\times1^2-1)} = 1 + \frac{1}{2} = 1.5$

$f(x_1) = (1.5)^3 - 1.5 - 1 = 0.875$
$f'(x_1) = (3\times1.5^2) - 1 = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \frac{0.875}{5.75} = 1.3478$$

$f(x_2) = (1.3478)^3 - 1.3478 - 1 = 0.1005$

$f'(x_2) = (3 \times 1.3478^2) - 1 = 4.4497$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3478 - \frac{0.1005}{4.4497} = 1.3252$$

$f(x_3) = (1.3252)^3 - 1.3252 - 1 = 0.0020$

$f'(x_3) = (3 \times 1.3252^2) - 1 = 4.2685$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.3252 - \frac{0.0020}{4.2685} = 1.3247$$

$f(x_4) = (1.3247)^3 - 1.3247 - 1 = -0.00007$

$\therefore$ the root of $x^3 - x - 1$ is approximately 1.3247

5) Given $A^2 = A$

$7A - (I + A)^3 = 7A - (I+A)(I + 2AI + A^2)$

$= 7A - (I+A)(I+3A)$

$= 7A - (I + 4A + 3A^2)$

$= 7A - I - 7A$

$= -I$

6) $A = IA$

$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \rightarrow R_2 - 4R_1$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & -2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & -2 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \rightarrow R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -4 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -4 & 1 & -3 \\ 4 & -1 & 4 \end{bmatrix} A$$

$R_1 \rightarrow R_1 + \frac{R_3}{2}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1/2 & 3 \\ -4 & 1 & -3 \\ 4 & -1 & 4 \end{bmatrix} A$$

$R_2 \longrightarrow R_2 + \frac{R_3}{2}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ 4 & -1 & 4 \end{bmatrix} A$$

$R_3 \longrightarrow \frac{-R_3}{2}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$$

7) $\vec{A} \cdot \vec{B} = |A||B| \cos\theta$

$\Rightarrow (8i - 9j)(7i - 9j) = |8i - 9j| \, |7i - 9j| \cos\theta$

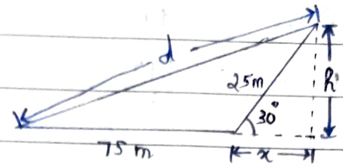
$\Rightarrow 56 + 81 = \sqrt{64 + 81} \times \sqrt{49 + 81} \times \cos\theta$

$\Rightarrow 137 = 137 \cos\theta$

$\Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 0^\circ$

8) $\sin 30^\circ = \frac{h}{25}$



$\Rightarrow \frac{1}{2} = \frac{h}{25}$

$\Rightarrow h = \frac{25}{2}$ m

$\cos 30^\circ = \frac{x}{25}$

$\frac{\sqrt{3}}{2} = \frac{x}{25}$

$\Rightarrow x = \frac{25\sqrt{3}}{2}$ m.

$\sqrt{(75+x)^2 + h^2}$ = displacement of Sabrina

$\Rightarrow \sqrt{\left(75 + \frac{25\sqrt{3}}{2}\right)^2 + \left(\frac{25}{2}\right)^2} = d$

$\Rightarrow \sqrt{9341.345 + 156.25} = d$

$\Rightarrow d = 97.45$ m

9) The lowest 3 digit number is 101 and the highest is 998.

the AP is 101, 104, 107, ..., 998

where $a = 101$

$d = 3$

$a_n = a + (n-1)d$

$\Rightarrow 998 = 101 + (n-1)3$

$(n-1)3 = 897$

$n-1 = \frac{897}{3}$

$n-1 = 299$

$n = 300$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$= \frac{300}{2}\left(202 + (299 \times 3)\right)$

$= 164850$

$\therefore$ the sum of all 3 digit numbers that leave a remainder of 2 when divided by 3 is 164850.

10) $ar^5 = 32$ —— ①

$ar^7 = 128$ —— ②

Dividing equation ② by equation ①,

$r^2 = \frac{128}{32}$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

11) $(a+b)^n = nC_0 a^n b^0 + nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 + \dots + nC_n a^0 b^n$

$\therefore (4+3x)^5 = 5C_0 4^5 (3x)^0 + 5C_1 4^4 (3x)^1 + 5C_2 4^3 (3x)^2 + 5C_3 4^2 (3x)^3 + 5C_4 4^1 (3x)^4 + 5C_5 4^0 (3x)^5$

$$\Rightarrow (4+3x)^5 = 1024 + (5 \times 256 \times 3x) + (10 \times 64 \times 9x^2) + (10 \times 16 \times 27x^3) + (5 \times 4 \times 81x^4) + 243x^5$$

$$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$$

12) Consider $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda) \begin{vmatrix} -(5+\lambda) & 0 \\ 0 & 3-\lambda \end{vmatrix} + 3 \begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} + 0 = 0$$

$$(2-\lambda)\left[-(5+\lambda)(3-\lambda)\right] + 3(6-2\lambda) = 0$$

$$-(2-\lambda)(5+\lambda)(3-\lambda) + 6(3-\lambda) = 0$$

$$(3-\lambda)\left[-(2-\lambda)(5+\lambda) + 6\right] = 0$$

$$(3-\lambda)\left[(\lambda-2)(\lambda+5) + 6\right] = 0$$

$$(3-\lambda)(\lambda^2 + 3\lambda - 4) = 0$$

$$(3-\lambda)(\lambda-1)(\lambda+4) = 0$$

$\therefore$ The eigen values are $\lambda = 3, 1, -4$

For $\lambda = 3$

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 + 2R_1$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 / -14$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 = R_1 + 3R_2$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 0$, $x_2 = 0$, $x_3 =$ indeterminate $= \alpha$

$\therefore$ for $\lambda = 3$, eigen vector is $\begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$

For $\lambda = 1$,

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) x = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 3x_2 \\ 0 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = 0$

$x_1 - 3x_2 = 0$

$x_1 = 3x_2$

$\therefore$ for $\lambda = 1$, eigen vector is $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} x_2$

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For $\lambda = -4$.

$(A - \lambda I) x = 0$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right) X = 0$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} X = 0$$

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 \\ 2x_1 - x_2 \\ 7x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$7x_3 = 0$

$x_3 = 0$

$2x_1 = x_2$

$\therefore$ for $\lambda = -4$, eigen vector is $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} x_1$

13) $f(x) = x^3 - 7x^2 - 8x - 3$

$f'(x) = 3x^2 - 14x - 8$

$x_0 = 5$

$f(x_0) = 5^3 - (7 \times 5^2) - (8 \times 5) - 3$

$= -93$.

$f'(x_0) = (3 \times 5^2) - (14 \times 5) - 8$

$= -3$.

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 5 - \frac{(-93)}{(-3)}$

$= 5 - \frac{93}{3}$

$= -26$

$f(x_1) = (-26)^3 - (7 \times (-26)^2) - (8 \times -26) - 3$

$= -22103$

$f'(x_1) = 2384$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= -26 - \frac{(-22103)}{2384}$

$= -16.7286$

$\therefore x_2 = -16.7286$

14) $(1 - x + x^2)^4 = 4C_0 (1-x)^4 (x^2)^0 + 4C_1 (1-x)^3 (x^2)^1 + 4C_2 (1-x)^2 (x^2)^2$
$+ 4C_3 (1-x)(x^2)^3 + 4C_4 (1-x)^0 (x^2)^4$

$= (1-x)^4 + 4(1-x)^3 x^2 + 6(1-x)^2 x^4 + 4(1-x) x^6$
$+ x^8$

$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$$

15) $e^{-x} = 3\log x$

$e^{-x} - 3\log x = 0$

x	y
1	0.3678
2	-0.7677

$x_1 = 1$, $y_1 = 0.3678$

$x_2 = 2$, $y_2 = -0.7677$

$$x_3 = \frac{x_1 + x_2}{2} = \frac{1+2}{2} = 1.5$$

$$y_3 = e^{-1.5} - 3\log(1.5) = -0.3051$$

$x_1 = 1$, $y_1 = 0.3678$

$x_3 = 1.5$, $y_3 = -0.3051$

$$x_4 = \frac{x_1 + x_3}{2} = \frac{1 + 1.5}{2} = 1.25$$

$$y_4 = e^{-1.25} - 3\log(1.25) = -0.0042$$

$x_1 = 1$, $y_1 = 0.3678$

$x_4 = 1.25$, $y_4 = -0.0042$

$$x_5 = \frac{x_1 + x_4}{2} = \frac{1 + 1.25}{2} = 1.125$$

$$y_5 = e^{-1.125} - 3\log(1.125) = 0.1711$$

$x_4 = 1.25, \; y_4 = -0.0042$
$x_5 = 1.125, \; y_5 = 0.1711$

$$x_6 = \frac{x_4 + x_5}{2} = \frac{1.25 + 1.125}{2} = 1.1875$$

$$y_6 = e^{-1.1875} - 3\log(1.1875) = 0.0810$$

$\therefore$ the value of x is approximately 1.1875.