SRM - Assignment 2

1 i) $\begin{array}{c|c} B & \\ O & \end{array} \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$

ii) Holding times are exponentially distributed with paramater λ in state B and μ in state O

(iii) $\frac{d}{dt} {}_tP_s^{BB} = -\lambda {}_tP_s^{BB} \; \mu {}_tP_s^{BO}$

$\frac{d}{dt} {}_tP_s^{BO} = \lambda {}_tP_s^{BB} - \mu {}_tP_s^{BO}$

iv) ${}_tP_s^{BB} + {}_tP_s^{BO} = 1$

$\frac{d}{dt} {}_tP_s^{BB} = -\lambda {}_tP_s^{BB} + \mu (1 - {}_tP_s)^{BB}$

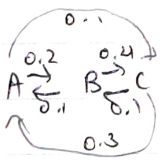
$\frac{d}{dt} \exp((\lambda+\mu)t) {}_tP_s^{BB} = \mu \exp((\lambda+\mu)t)$

$\exp((\lambda+\mu)t) {}_tP_s^{BB} = \frac{\mu}{\lambda+\mu} \exp((\lambda+\mu)t) +$ constanr

Process In state Bid at time so the constant is $\frac{\lambda}{\mu+\lambda}$

Thus, ${}_tP_s^{BB} = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} \exp(-(\lambda+\mu)t)$

2 i)



(ii) $\frac{d}{dt} P_{AA}(t) = -0.3P_{AA}(t) + 0.1P_{AB}(t) + 0.3P_{AC}(t)$

$\frac{d}{dt} P_{AB}(t) = 0.2P_{AA}(t) - 0.5P_{AB}(t) + 0.1P_{AC}(t)$

$\frac{d}{dt} P_{AC}(t) = 0.1P_{AA}(t) + 0.4P_{AB}(t) - 0.4P_{AC}(t)$

(ii) Poisson with parameter $(0.1+0.2) \times 2 = 0.6$
$P(Poi(0.6) = 0) = e^{-0.6} = 0.5488$

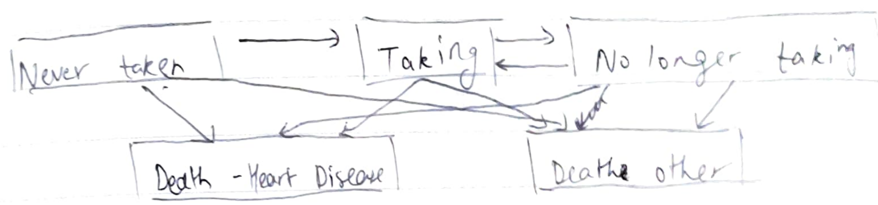
(iv) Possible paths → BAC, CAC, CBC
BAC $= \frac{2}{3} \times \frac{1}{5} \times \frac{1}{3} = \frac{2}{45}$

CAC $= \frac{1}{3} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{12}$

CBC $= \frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{1}{15}$

Sum $= 0.194$

3i)



(ii) ${}_{dt+t}p_x^{34} = {}_tp_x^{31}\,{}_{dt}p_{x+t}^{14} + {}_tp_x^{32}\,{}_{dt}p_{x+t}^{24} + {}_tp_x^{33}\,{}_{dt}p_{x+t}^{34} + {}_tp_x^{34}\,{}_{dt}p_{x+t}^{44}$
$+ {}_tp_x^{35}\,{}_{dt}p_{x+t}^{54}$

${}_{dt}p_{x+t}^{54} = {}_tp_x^{31} = 0$, ${}_{dt}p_{x+t}^{44} = 1$

$\therefore {}_{dt+t}p_x^{34} = {}_tp_x^{32}\,\mu_{x+t}^{24}\,dt + {}_tp_x^{33}\,\mu_{x+t}^{34}\,dt + {}_tp_x^{34} + o(dt)$
${}_{dt+t}p_x^{34} - {}_tp_x^{34} = {}_tp_x^{32}\,\mu_{x+t}^{24}\,dt + {}_tp_x^{33}\,\mu_{x+t}^{34}\,dt + o(dt)$

4i) Mean = Parameter = 3 hrs
(ii) 20 minutes
(iii) $P(0.5) = \frac{e^{-1.5}(1.5)^0}{0!} = 0.2231$

(iv) Average call time $3 \times 7 = 21$ minutes
Probability(engaged) $= \frac{21}{60} = 0.35$

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5 i) $p_{HH}(x, t+dt) = p_{HH}(x,t)\, p_{HH}(t, t+dt) + p_{HS}(x,t)\, p_{SH}(t, t+dt)$
$+ p_{HD}(x,t)\, p_{DH}(t, t+dt)$

Assuming $p_{ij}(t, t+dt) = \lambda_{ij}(t)\, dt \quad o(dt)$

$p_{HH}(x, t+dt) = p_{HH}(x,t)(1 - \sigma(t)\,dt - \mu(t)\,dt) +$
$p_{HS}(x,t)\, \rho(t, C_t) + o(dt)$

~~So that~~ $p_{HH}(x, t+dt) - p_{HH}(x, t) = p_{HH}(x,t)(-\sigma(t) - \mu(t))\,dt$
$+ p_{HS}(x,t)\, \rho(t, C_t)\,dt + o(dt)$

$$\frac{d}{dt} p_{HH}(x,t) = p_{HH}(x,t)(-\sigma(t) - \mu(t)) + p_{HS}(x,t)\, \rho(t, C_t)$$

ii) $$\frac{d}{dt} p_{\overline{HH}}(0,t) = -(\sigma(t) + \mu(t))\, p_{\overline{HH}}(t)$$

$$\frac{1}{p_{\overline{HH}}(0,t)\,dt} \frac{d}{dt} p_{\overline{HH}}(0,t) = -(\sigma(t) + \mu(t))$$

Integrating both sides

$$\left[\ln p_{\overline{HH}}(0,t)\right]_0^t = \int_s^t -(\sigma(s) + \mu(s))\, ds$$

as $p_{\overline{HH}}(0) = 1$

$$p_{\overline{HH}}(0,t) = \exp - \left(\int_0^t (\sigma(s) + \mu(s))\, ds\right)$$

6 All 3 processes have discrete space
A markov chain and a markov jump chain both operate in discrete time but a Markov jump process which is that future increments are independent of past.

7 i) MLE estimates of transition intensity from i to j is the no. of transitions divided by total waiting time.
∴ We need total time spent in each state or entry and exit times for all individuals, and total no. of transitions of each type made

ii) $P_{AA}(s,t) = \exp\left(-\int_s^t \mu \, du\right)$

$$P_{AT}(s,t) = \int_{u=s}^{t} P_{AA}(s,u)\mu \, du$$

iii) $P_{AT}(T) = \int_{s=0}^{T} \exp(-\mu s)\mu \, ds = \left[-\exp(\mu s)\right]_0^T = 1 - \exp(-\mu T)$

And hence expected cost is $C(1-\exp(-\mu T))$

iv)
$$\begin{tikzcd} \boxed{\text{Active policy}} \arrow[r, "\mu"] \arrow[d, "\lambda"] & \boxed{\text{Theft Claim}} \\ \boxed{\text{Lapsed}} & \end{tikzcd}$$

v) $\frac{d}{dt} P_{AA}(t) = -P_{AA}(t)(\mu+\lambda)$

$$P_{AA}(t) = \exp(-(\mu+\lambda t))$$

$$\frac{d}{dt} P_{AT}(t) = P_{AA}(t)\mu = \mu \exp[-(\mu+\lambda)t]$$

∴ Claims become $\frac{\mu}{\mu+\lambda} C\left[1-\exp(-(\mu+\lambda)t)\right]$

8 i) $\{0, 1, 2, 3 \ldots\}$

ii)
$$\begin{tikzcd} 0 & \boxed{1} \arrow[l, "\mu"'] \arrow[r, shift right, "\lambda"'] & \boxed{2} \arrow[l, shift right, "\mu"'] \arrow[r, shift right, "\lambda"'] & \boxed{3} \arrow[l, shift right, "\mu"'] \arrow[r, shift right, "\lambda"'] & \boxed{4} \arrow[l, shift right, "\mu"'] & \cdots \end{tikzcd}$$

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iii)

$$\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \mu & (-\mu+\lambda) & \lambda & 0 & 0 \\ 0 & \mu & (-\mu+\lambda) & 0 & 0 \\ 0 & 0 & \mu & (-\mu+\lambda) & \lambda \\ 0 & 0 & 0 & \mu & (-\mu+\lambda) \end{bmatrix} \end{array}$$

iv) A jump chain is each distinct state visited in the order where the time set is time when sets are moved between

v

$$\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \mu/\mu+\lambda & 0 & \lambda/\mu+\lambda & 0 & 0 \\ 0 & \mu/\mu+\lambda & 0 & \lambda/\mu+\lambda & 0 \\ 0 & 0 & \mu/\mu+\lambda & 0 & \lambda/\mu+\lambda \\ 0 & 0 & 0 & \mu/\mu+\lambda & 0 \end{bmatrix} \end{array}$$

vi) $\left(\dfrac{\mu}{\mu+\lambda}\right)^3$

9 i) $\mu_{ij} \geq 0$ for $i \neq j$

$\mu_{ii} \leq 0$

ii) State space $\{2,1,0\}$ policies in force

iii) $2 \xrightarrow{0.4} 1 \xrightarrow{0.2} 0$

iv) $\frac{d}{dt} P_{22}(t) = -0.4 P_{22}(t)$

$\frac{d}{dt} P_{21}(t) = 0.4 P_{22} - 0.2 P_{21}(t)$

$\frac{d}{dt} P_{20}(t) = 0.2 P_{21}(t)$

10 i) $\frac{d}{dt} P_{AA}(t) = -2t \times P_{AA}(t)$

$\frac{d}{dt} \ln P_{AA}(t) = -2t$

$\ln P_{AA}(s) = -s^2 + k$ (constant)

$P_{AA}(0) = 1 \quad \therefore$ ~~constant~~ $k = 0$

Hence, $P_{AA}(s) = e^{-s^2}$

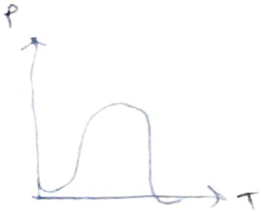
ii) $P_{REQ} = \int_{s=0}^{T} P_{AA}(s) \times 2s \times P_{BB}(s,T)\, ds$

$= \int_{s=0}^{T} e^{-s^2} \times 2s \times e^{-T^2+s^2}\, ds$

$= \int_{s=0}^{T} 2s\, e^{-T^2}$

$= e^{-T^2} \times T^2$

(iii)



b Probability increases and accelaterates as transition rate from A to B increases
However as it goes to B, it is more likely that it jumps back to A. Hence it falls and eventually tends to 0.

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c) $\frac{d}{dt}(e^{-t^2} \times t^2) = 2t \times e^{-t^2} - 2t^3 \times e^{-t^2}$

$e^{-t^2} \times 2t \times (1-t^2) = 0$

This implies $t = 1$

11) i) $P(T_0 \leq y \mid N_s = 1) = \frac{P(T_0 \leq y, N_s = 1)}{P(N_s = 1)}$

$= \frac{P(N_y = 1, N_s - N_y = 0)}{P(N_s = 1)}$

RHS equals $\frac{\lambda y e^{-\lambda y} e^{-\lambda(s-y)}}{\lambda s e^{-\lambda s}} = \frac{y}{s}$

which is cdf of $U(0, s)$

ii) Since holding times are independent

$\lambda^n e^{-\lambda(t_1 + t_2 \cdots t_n)}, \{t_1, t_2 \cdots t_n > 0\}$

(iii) As in part 1,

$P(N_s = k \mid N_t = n) = \frac{P(N_s = k, N_t - N_s = n-k)}{P(N_t = n)}$

From Poisson process

$P(N_s = k \mid N_t = n) = \frac{\frac{e^{-\lambda s}(\lambda s)^k}{k!} \frac{e^{-\lambda(t-s)} \lambda^{n-k} (t-s)^{n-k}}{(n-k)!}}{\frac{e^{-\lambda t}(\lambda t)^n}{n!}}$

$= \frac{e^{-\lambda t} \lambda^n s^k (t-s)^{n-k}}{k!(n-k)!} \cdot \frac{n!}{e^{-\lambda t} \lambda^n t^n}$

$= \frac{n!}{k!(n-k)!} \frac{s^k (t-s)^{n-k}}{t^k t^{n-k}}$

$= \binom{n}{k} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$

which is $Bin(n, s/t)$