Calculus

Q1) $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$= \lim_{h \to 0} \dfrac{2(h^2 - 6h + 9) - 18}{h}$

$= \lim_{h \to 0} \dfrac{2h^2 - 12h \cancel{+18} \cancel{-18}}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{h}(2h - 12)}{\cancel{h}}$

$= -12$

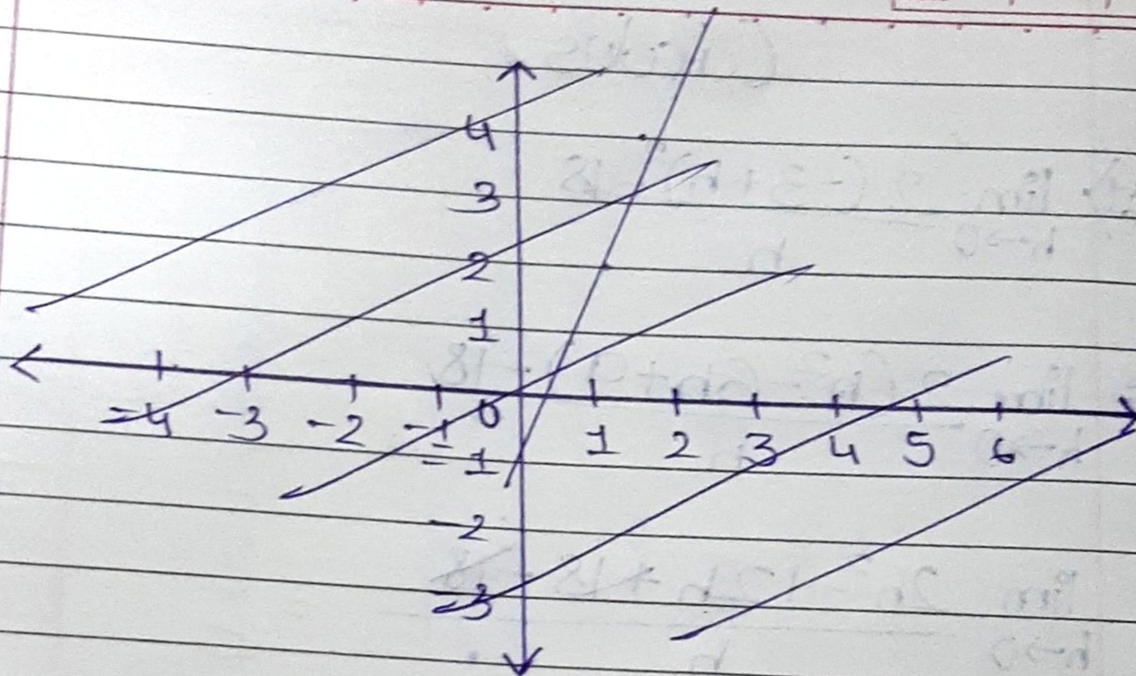
Q2)

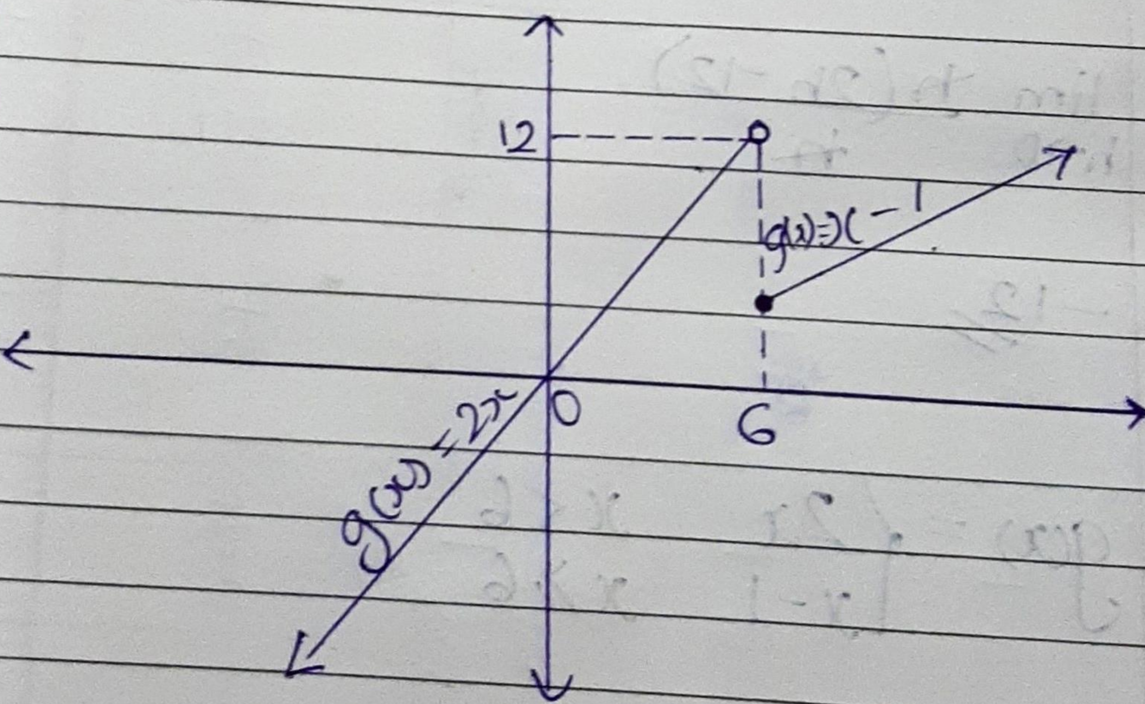
$$g(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$$

ⓐ $x = 4$

$\therefore g(x) = 2x$ for $x < 6$

$\therefore g(4) = 2(4)$
$= 8$





(a) ∴ Continous at $x=4$

(b) ∴ Discontinas at $x=6$

Q3) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$= \lim_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(3-\sqrt{x})}$

$= \lim_{x \to 9} \frac{(-1)\cancel{(3-\sqrt{x})}(\sqrt{x}+3)}{\cancel{(3-\sqrt{x})}}$

$= \lim_{x \to 9} -\sqrt{x}-3$

$= -\sqrt{9}-3$

$= -3-3$

$= -6$

Q4) $f(x) = \dfrac{4x+5}{9-3x}$

(a) $x = -1$

$\therefore f(1) = \dfrac{4(-1)+5}{9-3(-1)}$

$= \dfrac{-4+5}{9+3}$

$= \dfrac{1}{12}$

$\therefore f(x)$ is cont. at $x = \underline{1}$

(b) $x = 0$

$\therefore f(0) = \dfrac{4(0)+5}{9-3(0)}$

$= \dfrac{5}{9}$

$\therefore f(x)$ is cont. at $x = 0$

(c) x

Q5)

(c) $x = 3$

$$\therefore f(3) = \frac{4(3)+5}{9-(3)3}$$

$$= \frac{4(3)+5}{0}$$

$\therefore f(3)$ is not defined

$\therefore f(x)$ is discont. at $x = 3$.

Q5) $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$= \cancel{\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{4x}}}$$

Q6)

$$g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$$

(a) $\lim_{y \to 6} g(y)$

$= \lim_{y \to 6} 1-3y$

$= 1-3(6)$

$= 1-18$

$= -17$

(b) $\lim_{y \to -2^-} g(y)$

$= \lim_{y \to -2^-} y^2+5$

$= (-2)^2+5$

$= 4+5$

$= 9$

$\lim_{y \to +2^-} gy$

$= \lim_{y \to +2^-} 1-3y$

$= 1-3(-2)$

$= 1+6$

$= 7$

$\therefore$ Since $\lim_{y \to -2^-} g(y) \neq \lim_{y \to +2^-} g(y)$

$\therefore \lim_{y \to -2}$ doesn't exist.

Q7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Diff. w.r.t. x

$= (4t^2 - t) \cdot \frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12) \cdot \frac{d}{dt}(4t^2 - t)$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

Q8) $R(\omega) = \dfrac{3\omega + \omega^4}{2\omega^2 + 1}$

Diff w.r.t. ω

$= \dfrac{(2\omega^2 + 1) \cdot \frac{d}{d\omega}(3\omega + \omega^4) - (3\omega + \omega^4)\frac{d}{d\omega}(2\omega^2 + 1)}{(2\omega^2 + 1)^2}$

$= \dfrac{(2\omega^2 + 1)(3 + 4\omega^3) - (3\omega + \omega^4)(4\omega)}{(2\omega^2 + 1)^2}$

$= \dfrac{6\omega^2 + 8\omega^5 + 3 + 4\omega^3 - 12\omega^2 - 4\omega^5}{(2\omega^2 + 1)^2}$

$$= \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$$

Q9) $f(x) = +2e^x - 8^x$

diff. w.r.t. x

$\therefore f'(x) = 2e^x - 8^x \cdot \log 8$

Q10) $y = z^5 - e^z \cdot \log(z)$

diff. w.r.t. z

$$\frac{dy}{dz} = 5z^4 - \left(e^z \cdot \frac{d}{dx}\log z + \log z \cdot \frac{d}{dx} e^z\right)$$

$$= 5z^4 - \left(\frac{e^z}{z} + e^z \cdot \log z\right)$$

$$= 5z^4 - e^z\left(\frac{\log z \cdot z + 1}{z}\right)$$

$$= 5z^4 - \frac{e^z}{z} - e^z \cdot \log z$$

Q11)

(a) $f(x$

(b) $f($

f'

Q12)

(a)

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Q11)

(a) $f(x) = (6x^2+7x)^4$

diff. w.r.t. x

$$f'(x) = 4(6x^2+7x)^3 \cdot \frac{d}{dx}(6x^2+7x)$$

$$= 4(6x^2+7x)^3 \cdot (12x+7)$$

(b) $f(t) = 5 + e^{4t+t^7}$

diff. w.r.t. t

$$f'(t) = e^{4t+t^7} \cdot \frac{d}{dt}(4t+t^7)$$

$$= (7t^6+4) \cdot e^{4t+t^7}$$

Q12)

(a) $z = \ln(7-x^3)$

diff. w.r.t. x

$$\frac{dz}{dx} = \left(\frac{1}{7-x^3}\right) \cdot \frac{d}{dx}(7-x^3)$$

$$= \left(\frac{1}{7-x^3}\right)(-3x^2)$$

$\cdot \frac{d}{dx} e^{z})$

$$\frac{dz}{dx} = \frac{3x^2}{x^3-7}$$

diff. w.r.t. x

$$\frac{d^2z}{dx^2} = \frac{(x^3-7)\cdot \frac{d}{dx}(3x^2) - (3x^2)\cdot \frac{d}{dx}(x^3-7)}{(x^3-7)^2}$$

$$\frac{d^2z}{dx^2} = \frac{(x^3-7)(6x) - (3x^2)(3x^2)}{(x^3-7)^2}$$

$$= \frac{6x^4 - 42x - 9x^4}{(x^3-7)^2}$$

$$= \frac{-3x^4 - 42x}{(x^3-7)^2}$$

(c)

(b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$Q(v) = 2(6+2v-v^2)^{-4}$

diff. w.r.t. v

$$Q'(v) = -8(6+2v-v^2)^{-5} \cdot \frac{d}{dx}(6+2v-v^2)$$

$$= -8(6+2v-v^2)^{-5} \cdot (2-2v)$$

diff w.r.t. v

$$Q''(v) = -8\left[(6+2v-v^2)^{-5}\, d/dx(2-2v) + (2-2v)\, d/dx(6+2v-v^2)^{-5}\right]$$

$$= -8\left[(6+2v-v^2)^{-5}(-2) + -5(2-2v)(6+2v-v^2)^{-6} \cdot (2-2v)\right]$$

$$= -8\left[-2(6+2v-v^2)^{-5} - 5(6+2v-v^2)^{-6}(2-2v)^2\right]$$

(c) $f(t) = \ln(1+t^2)$

diff. w.r.t. t

$$f'(t) = \left(\frac{1}{1+t^2}\right)\frac{d}{dt}(1+t^2)$$

$$= \left(\frac{1}{1+t^2}\right)(2t)$$

diff. w.r.t. t

$$f''(t) = 2\left[\frac{(1+t^2)\cdot d/dt(t) - (t)\cdot d/dt(1+t^2)}{(1+t^2)^2}\right]$$

$$f''(t) = 2\left[\frac{(1+t^2) - 2t^2}{(1+t^2)^2}\right] = 2\left(\frac{1-t^2}{(1+t^2)^2}\right)$$

Q13) $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$\therefore$ If $f'(x) = 0$

$\therefore 3x^2 - 6x - 9 = 0$

$\therefore 3x^2 + 3x - 9x - 9 = 0$

$\therefore 3x(x+1) - 9(x+1) = 0$

$\therefore (3x - 9)(x+1) = 0$

$\therefore 3(x-3)(x+1) = 0$

$\therefore x = 3$ or $x = -1$

$\therefore f''(x) = 6x - 6$

$\therefore f''(3) = 6(3) - 6$

$= 12$ +ve $\therefore$ minima

$\therefore f''(-1) = 6(-1) - 6$

$= -12$ -ve $\therefore$ maxima

$\therefore f(-1) = -1 - 3 + 9 + 12$

$= 17$

$\therefore f(3) = 27 - 27 - 27 + 12$

$= -15$

Q14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$\therefore$ If $f'(x) = 0$

$\therefore 12x^2 - 36x + 24 = 0$

$\therefore 12x^2 - 12x - 24x + 24 = 0$

$\therefore 12x(x-1) - 24(x-1) = 0$

$\therefore (x-1)(12x-24) = 0$

$\therefore (x-1)(x-2)12 = 0$

$\therefore x = 1$ or $x = 2$

$\therefore f''(x) = 24x - 36$

$\therefore f''(1) = 24(1) - 36$

$= -12$ $\quad$ -ve $\therefore$ maxima

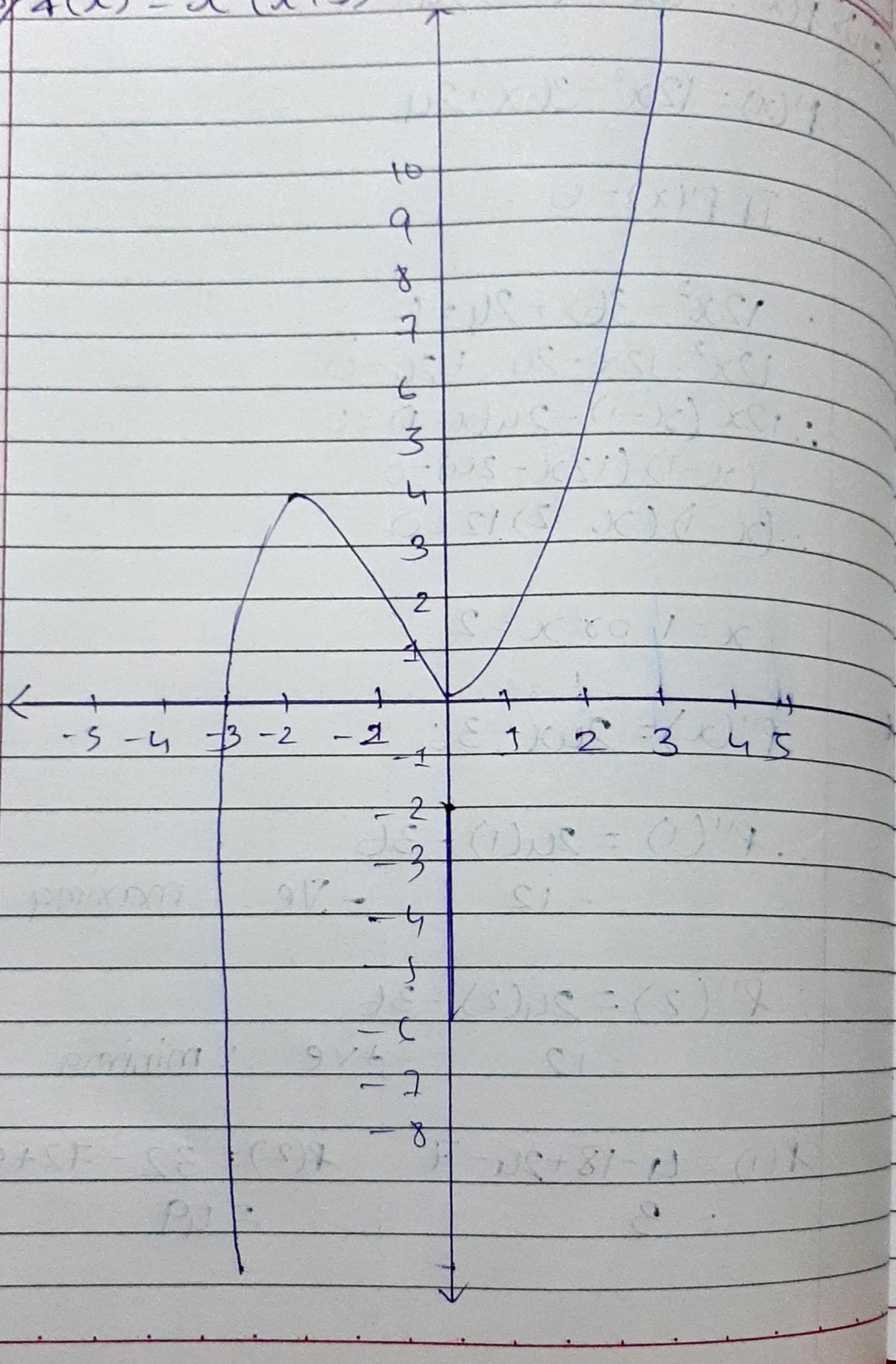
$\therefore f''(2) = 24(2) - 36$

$= 12$ $\quad$ +ve $\therefore$ minima

$f(1) = 4 - 18 + 24 - 7$
$= 3$

$f(2) = 32 - 72 + 96 - 7$
$= 49$

Q15) $f(x) = x^2(x+3)$



$$f(x) = x^2(x+3)$$
$$= x^3 + 3x^2$$

$$f'(x) = 3x^2 + 6x$$

If $f'(x) = 0$

$$3x^2 + 6x = 0$$
$$x^2 + 2x = 0$$
$$x(x+2) = 0$$
$$x = 0 \text{ or } x = -2$$

$f''(x) = 6x + 6$

$f''(0) = 6$ +ve $\therefore$ minima

$f''(-2) = -6$ −ve $\therefore$ maxima

Q16) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

If $f'(x) = 0$

$\therefore 6x - 6 = 0$

$\therefore 6(x-1) = 0$

$\therefore x = \underline{1}$

$\therefore f''(x) = 6$ +ve $\therefore$ minima

$\therefore f''(1) = 6 \longrightarrow$