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Assignment II

Question 1

Let x_i be the claim size of home insurance.

Let y_i be the claim size of car insurance.

$x \sim N(800, 100^2)$ $\quad$ $y \sim N(1200, 300^2)$

$P(y_1+y_2+y_3 > x_1+x_2+x_3+x_4+800)$

$P(y_1+y_2+y_3-(x_1+x_2+x_3+x_4) > 800)$

$y_1+y_2+y_3-(x_1+x_2+x_3+x_4) \sim N(3\times1200-4(800), 300^2(3)+400^2(4))$

$\sim N(400, 310000)$

$$P\left(z > \frac{800-400}{\sqrt{310000}}\right)$$

$P(z > 0.71842) \Rightarrow 1 - P(z < 0.71842)$

$1 - P(z < 0.72)$

$1 - 0.76424$

$\underline{\underline{0.23576}}$

Question 2

2) $N \sim$ Negative Binomial (k, p).

$P(N=n) \Rightarrow {}^{n+2}C_n\,(0.9)^3\,(0.1)^n$

$\Rightarrow {}^{(n+3)-1}C_{n-1}\,(0.9)^3\,(0.1)^n$

So, $k=n$, $p=0.1$

$k=3$, $p=0.9$ $\quad$ $M_y\,t = M_N(\log M_x t)$

$X \sim$ Gamma $(6, 2)$

$y \Rightarrow$ Total claim Amount.

MGF of y $\quad$ $M_y(t) = E(E\,e^{yt}/N=n)$

$E(e^{yt}/N=n) = E(e^{x_1t+x_2t\cdots x_nt})$

$= E(e^{tx_1}) \cdot E(e^{tx_2}) \ldots E(e^{tx_n})$

$\Rightarrow \left(1-\frac{t}{2}\right)^{-6n}$

$$So,\ E(E(e^{yt}/N=n)) = E\left(\left(1-\frac{t}{2}\right)^{-6n}\right)$$

$$= E\left(e^{N\log\left(1-\frac{t}{2}\right)^{-6}}\right)$$

$$= M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k \Rightarrow \left(\frac{pe^{\log\left(1-\frac{t}{2}\right)^{-6}}}{1-qe^{\log\left(1-\frac{t}{2}\right)^{-6}}}\right)^k$$

$$= \left[\frac{p\left(1-\frac{t}{2}\right)^{-6}}{1-q\left(1-\frac{t}{2}\right)^{-6}}\right]^3 = \left[\frac{0.9\left(1-\frac{t}{2}\right)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right]^3$$

ii) $V(y) = E(N)\,V(X) + \left[[E(X)]^2 \cdot V(N)\right]$

$$= \left[\frac{3}{\cancel{0.3}\ 0.9\ 0.1} \times \frac{\cancel{6}\ 3}{\cancel{4}\ 2}\right] + \left[\left(\frac{\cancel{6}\ 3}{2}\right)^2 \times \frac{3\ (0.1)}{(0.9)^2}\right]$$

$$= 1/0.2 + \left[9\ \cancel{8}\ \times 0.3/0.81\right]$$

$$= 5 + \cancel{9.37037037}\ 3.33333$$

$$= \cancel{14.37037037}\ 8.33333$$

$$SD = \sqrt{Var(y)} = \cancel{3.7908\ldots7135}\ 2.886751346$$

Question 3

$$f(x) = \lambda e^{-\lambda(x-\alpha)}$$

$$MGF = E(e^{tx}) = \int_\alpha^\infty e^{tx}\lambda e^{-\lambda(x-\alpha)}dx$$

$$M_X(t) = \lambda\int_\alpha^\infty e^{tx}e^{-\lambda x}e^{\lambda\alpha}dx = e^{\lambda\alpha}\lambda\int_\alpha^\infty e^{-(\lambda-t)x}dx$$

$$= \frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[e^{-(\lambda-t)x}\right]_\alpha^\infty$$

$$= \frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[-e^{-(\lambda-t)\alpha}\right]$$

$$= \frac{\cancel{e^{\lambda\alpha}}\lambda}{\lambda-t}\left[\cancel{e^{-\lambda\alpha}}e^{t\alpha}\right]$$

$$= e^{t\alpha}\left(\frac{\lambda}{\lambda-t}\right)$$

ii) $M_x(t) = \frac{\lambda e^{t\alpha}}{\lambda-t} \Rightarrow \lambda e^{t\alpha}(\lambda-t)^{-1}$

$$M'_x(t) = \lambda\left[e^{t\alpha}(\lambda-t)^{-2}(\times 1) + (\lambda-t)^{-1}e^{t\alpha}(\alpha)\right]$$

$$= \lambda\left[(\lambda-t)^{-1}e^{t\alpha}(\alpha) + e^{t\alpha}(\lambda-t)^{-2}\right]$$

$$= \lambda e^{t\alpha}(\lambda-t)^{-2}(\alpha(\lambda-t)+1)$$

$t=0$ $= \lambda(\lambda)^{-2}(\alpha(\lambda+1))$ $\quad\alpha + 1/\lambda$

$$= \frac{1}{\lambda}(\lambda\alpha+1) = \frac{\lambda\alpha+1}{\lambda}$$

$$M''(t) = \lambda\left[\alpha e^{t\alpha}(\lambda-t)^{-2} + \alpha^2(\lambda-t)^{-1}e^{t\alpha} + e^{t\alpha}(+2)(\lambda-t)^{-3} + (\lambda-t)^{-}e^{t\alpha}\right].$$

Put $t=0$.

$$= \lambda[\alpha(\lambda)^{-2} + \alpha^2(\lambda)^{-1} + 2(\lambda)^{-3} + \alpha(\lambda^{-2})$$

$$= \lambda\left(\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3}\right]$$

$$E(x^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda}\right]^2$$

$$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left(\alpha^2 + \frac{1}{\lambda^2} + \frac{2\alpha}{\lambda}\right]$$

$$= \cancel{\frac{2\alpha}{\lambda}} + \cancel{\alpha^2} + \frac{2}{\lambda^2} - \cancel{\alpha^2} - \frac{1}{\lambda^2} - \cancel{\frac{2\alpha}{\lambda}}$$

$$= \boxed{1/\lambda^2}$$

question 4.

$G_V(t) = \left(\frac{p}{1-qt}\right)^k \qquad p+q=1$

$G_V(t) = \sum t^v \times P(V=v)$

$P(V=4) = \frac{\Gamma(k+4)}{\Gamma 5 \; \Gamma k} p^k q^k \qquad G_V(t) = p^k(1-qt)^k$

$= \frac{\Gamma(k+4)}{4! \; \Gamma k} p^k q^k$

$= \frac{(k+3)!}{(k-1)! \; 4!} p^k (1-p)^k$

Questions.

~~$y(x,y) = ax(x+y)$~~

~~ii) $P(y > 1/3 \; x+10)$~~

i) value of a

$$a \int_{10}^{20} \int_{0}^{5} (x^2 + xy) \, dx \, dy = 1$$

$$\int_{10}^{20} \left[\frac{x^3}{3} + \frac{x^2}{2} y\right]_0^5 dy = 1/a$$

$$\int_{10}^{20} \left(\frac{125}{3} + \frac{25}{2} y\right) dy = 1/a$$

$$\left(\frac{125y}{3} + \frac{25y^2}{4}\right)_{10}^{20} = \frac{1}{a}$$

$$a\left(\frac{2500}{3} + 2500 - \frac{1250}{3} - 625\right) = 1$$

$$a\left(\frac{1250}{3} + 1875\right) = 1$$

$$a\left(6875/3\right) = 1$$

$\boxed{a = 3/6875}$

$a = 0.00043636.$

iii) $E(y|x)$

$$f_{y/y} = \frac{f(x,y)}{f(x)} \Rightarrow f(x) = \frac{3}{6875}\int_{10}^{20} x^2 + xy\,dy$$

$$= \frac{3}{6875}\left[x^2y + \frac{xy^2}{2}\right]_{10}^{20}$$

$$= \frac{3}{6875}\left(20x^2 + x(200) - 10x^2 - 50x\right)$$

$$= \frac{3}{6875}(10x^2 + 150x)$$

$$= \frac{30x}{6875}(x+15)$$

$$f_{y|x} = \frac{\cancel{3}}{\cancel{6875}}\,\frac{\cancel{x}(x+y)}{\cancel{30}_{10}\,(\cancel{x})} \times \frac{\cancel{6875}}{(x+15)}$$

$$= \frac{(x+y)}{10(x+15)}$$

$$\text{so}(E(y|x)) = \int_{10}^{20} y\,\frac{(x+y)}{10(x+15)}\,dy$$

$$= \frac{1}{10(x+15)}\int_{10}^{20} xy + y^2\,dy$$

$$= \frac{1}{10(x+15)}\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}$$

$$= 1/10(x+15)\left[200x + \frac{8000}{3} - 50x - 1000/3\right]$$

$$= 1/10(x+15)\left[150x + \frac{7000}{3}\right]$$

$$= \frac{1}{x+15}\left(15x + \frac{700}{3}\right)$$

$P(y > \frac{1}{3}x + 10)$

$$f(y) = \frac{3}{6875} \int_0^5 x^2 + xy \, dx$$

$$= \frac{3}{6875} \left(\frac{x^3}{3} + \frac{x^2}{2} y \right)_0^5$$

$$f(y) = \frac{3}{6875} \left(\frac{125}{3} + \frac{25}{2} y \right)$$

$$\Rightarrow \frac{3}{6875} \left(\int_{\frac{1}{3}x+10}^{20} \frac{125}{3} + \frac{25}{2} y \, dy \right)$$

$$\Rightarrow \frac{3}{6875} \left[\frac{125y}{3} + \frac{25y^2}{4} \right]_{\frac{1}{3}x+10}^{20}$$

$$= \frac{3}{6875} \left(\frac{125(20)}{3} + \frac{25(400)}{4} - \frac{125}{3} \left(\frac{1}{3}x + 10 \right) - \frac{25}{4} \left(\frac{1}{3}x + 10 \right)^2 \right]$$

$$= \frac{3}{6875} \left(\frac{2500}{3} + 2500 - \frac{125x}{9} - \frac{1250}{3} - \frac{25}{4} \left(\frac{1}{9}x^2 + 100 + \frac{2x}{3} \right) \right)$$

$$= \frac{3}{6875} \left(\frac{2500}{3} + 2500 - \frac{125x}{9} - \frac{1250}{3} - \frac{25x^2}{36} - 625 - \frac{25}{6} x \right).$$

$$\frac{3}{6875} \left(\frac{1250}{3} + 1875 - \frac{25x^2}{36} - \frac{125x}{9} - \frac{25}{6} x \right).$$

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Question 6

$X \sim$ Poisson (λ) $\quad\quad$ $y \sim$ Poisson(M)

$M_x(t) = e^{\lambda(e^t-1)}$ $\quad\quad$ $M_y(t) = e^{M(e^t-1)}$

So $z = x - y$

$M_z(t) = E(e^{tz}) = E(e^{t(x-y)}) = E(e^{tx} \cdot e^{-ty})$

$= E(e^{tx}) \times E(e^{-ty})$

$= \dfrac{E(e^{tx})}{E(e^{ty})}$

$= \dfrac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$

$= e^{(\lambda-\mu)(e^t-1)}$

So by uniquness of MGF of $z = e^{(\lambda-\mu)(e^t-1)}$

$\sim$ Possion $(\lambda - \mu)$

Now Let $z = x+y$

$E(e^{tz}) = e^{\lambda(e^t-1)} \times e^{\mu(e^t-1)}$

$= e^{(\lambda+\mu)(e^t-1)}$

So, By uniquness $z \sim$ Poisson$(\lambda + \mu)$.

Question 7

		x			
		1	2	3	4
y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

i) $V(X/y=2) = E(x^2/y=2) - [E(x/y=2)]^2$

$E(x^2/y=2) = \sum x^2 \dfrac{P(X=x, y=2)}{P(y=2)}$

when $x=1$ + when $x=2$ + when $x=3$ + when $x=4$

$1^2 \dfrac{P(x=1, y=2)}{P(y=2)}$ $\quad \dfrac{4\ (0.3)}{0.6} + \dfrac{9\ (0.1)}{0.6} + \dfrac{16\ (0.2)}{6}$

$\dfrac{1(0)}{0.6}$ $\quad = 8.8333$

$$E(X/y=2) = \sum_{x} x \frac{P(x=x, y=2)}{P(y=2)}$$

$$\frac{1(0)}{0.6} + \frac{2(0.3)}{0.6} + \frac{3(0.1)}{0.6} + \frac{4(0.2)}{0.6}$$

$$= \frac{0.6 + 0.3 + 0.8}{0.6}$$

$$= 2.8333$$

$$V(X/y=2) = 8.8333 - (2.8333)^2$$

$$= \underline{\underline{0.8075711}}$$

ii $$f(u,v) = \frac{48}{67}(2uv - u^2) \qquad 0 < u < 1, \; \frac{u}{2} < v < 2.$$

$$E(U/V=v) = ?$$

$$\frac{f(u,v)}{f(v)} \Rightarrow f_{V}/_{V=v}.$$

$$f(v) = \frac{48}{67}\int_0^1 (2uv - u^2)\,du$$

$$= \frac{48}{67}\left[u^2 v - \frac{u^3}{3}\right]_0^1$$

$$= 48/67\left(v - 1/3\right)$$

$$= \frac{\overset{16}{\cancel{48}}}{67}\left(\frac{3v-1}{\cancel{3}}\right)$$

$$f(v) = \frac{16}{67}(3v-1)$$

$$\frac{f(u,v)}{f(v)} = \frac{\cancel{48}^{3}}{\cancel{67}} \frac{(2uv^2 - u^2)}{\cancel{16}(3v-1)} \times \cancel{67}$$

$$= \frac{3}{3v-1}(2uv^2 - u^2) \qquad (2u^2v^2 - u^3)$$

Now $E(U/V=v) = \frac{3}{3v-1}\int_0^1 u(2uv^2 - u^2)\,du$

$$= \frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{u^4}{4}\right]_0^1$$

$$= 3/_{3v-1}\left[\frac{2v^2}{3} - 1/4\right]$$

$$= \frac{\cancel{3}^{1}}{3v-1}\left[\frac{8v^2-3}{\cancel{12}4}\right]$$

$$= \frac{8v^2-3}{4(3v-1)}$$

question8.

$X \sim Gamma(3,2) \qquad E(X) = 3/2 \qquad V(X) = 3/4.$

$E(Y/X=x) = 3x+1 \qquad Var(Y/X=x) = 2x^2+5$

$E(Y) = E(E/Y=x) \Rightarrow E(3x+1)$

$= 3E(x)+1$

$= 3 \times 3/2 + 1$

$= \frac{9}{2} + 1$

$= 11/2.$

$V(Y) = E(V(X/Y)) + V(E(X/Y))$

$= E(2x^2+5) + V(3x+1)$

$= 2E(x^2) + 5 + 9V(x)$

$V(X) = E(x^2) - [E(X)]^2$

$3/4 + 9/4 = E(x^2) \qquad E(x^2) = 3.$

$E(x^2) = 12/4$

$V(y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$

$= 11 + \frac{27}{4}$

$= \frac{44 + 27}{4}$

$= 71/4$

Question 9.

$E(x) = 3$, $V(x) = 4$

$E(y) = 4$, $V(y) = 1$.

Correlation coeff $= -0.3\sqrt{4} \Rightarrow cov(x, y)$

$= -0.6$

$z = x + y$

i) $cov(x, x+y) = cov(x, x) + cov(x, y)$

$= V(x) + cov(x, y)$

$= 4 + (-0.6)$

$= 3.4$

ii $Var(z) = cov(x+y, x+y)$

$= cov(x, x) + 2\,cov(x, y) + cov(y, y)$

$= V(x) + 2\,cov(x, y) + V(y)$

$= 4 + 2(-0.6) + 1$

$= 5 - 1.2$

$= 3.8$

Question 10

$f(x,y) = k x^{-\alpha} e^{-y/\beta}$ $\quad 1 < x < \infty, \; 1 < y < \infty$

$k \int_1^\infty \int_1^\infty x^{-\alpha} e^{-y/\beta} \, dx \, dy = 1$ $\quad \alpha > 1, \; \beta > 0$

$$k \int_1^\infty e^{-y/\beta} \left[\frac{x^{-(\alpha-1)}}{-\alpha+1} \right]^\infty = 1$$

$$\frac{k}{1-\alpha} \int_1^\infty e^{-y/\beta} \left[-1 \right] dy = 1$$

$$-\frac{k}{1-\alpha} \int e^{-y/\beta} \, dy = 1$$

$$\frac{k}{\alpha-1} \left[\frac{e^{-y/\beta}}{-1/\beta} \right]_1^\infty = 1.$$

$$\frac{-k}{\alpha-1} (\beta) \left(e^{-1/\beta} \right) = 1$$

$$\boxed{k = \frac{\alpha-1}{\beta \left(e^{-1/\beta} \right)}}$$