Assignment - Numeric Methods of Algebra

1) ~~Area~~ Measured length = 12.65 cm
Actual length = 12.5 cm

Area using measured length, $A_M = \pi \times (12.65)^2$

$\Rightarrow 502.725$

Area using actual length, $A_A = \pi \times (12.5)^2$

$\Rightarrow 490.874$

Absolute error $= |A_M - A_A| = |502.725 - 490.874|$

$\Rightarrow 11.852$

Relative error $= \dfrac{\text{Absolute error}}{A_A} = 0.0241$

Percentage error = Relative error × 100 = 2.41 %.

2.

x	$f(x) = \log(x)$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6	0.7781513

Using $x=4$ & $x=6$

$x_k = 5$, $x_0 = 4$, $x_1 = 6$

$\cancel{\log(x_k)}\ f(5) = \dfrac{(x_k - x_1)}{(x_0 - x_1)} \cdot f(x_0) + \dfrac{(x_k - x_0)}{(x_1 - x_0)} \cdot f(x_1)$

$\Rightarrow \dfrac{(-1)}{(-2)} \cdot \log(4) + \frac{1}{2} \log 6$

$\Rightarrow 0.30103 + 0.38907565$

$\Rightarrow 0.69010565$

Using $x=4.5$ & $x=5.5$,

$x_k = 5$, $x_0 = 4.5$, $x_1 = 5.5$

$f(5) = \dfrac{(x_k - x_1)}{(x_0 - x_1)} f(x_0) + \dfrac{(x_k - x_0)}{(x_1 - x_0)} f(x_1)$

$\Rightarrow \dfrac{(-0.5)}{(-1)} \log(4.5) + \dfrac{0.5}{1} \log(5.5)$

$\Rightarrow 0.32660625 + 0.37018835$

$\Rightarrow 0.6967876$

3 $x^4 - x - 10 = 0$

So, $f(x) = x^4 - x - 10$

~~Further~~

$\cancel{x_8 = \frac{x_7+x_5}{2} = 1.859875}$

x	y
1	-10
1.5	-6.437
2	4

$x_1 = 1.5$

$x_2 = 2$

$x_3 = \frac{x_1+x_2}{2} = 1.75$

$y_3 = -2.3711$

Further,

$x_4 = \frac{x_3+x_2}{2} = 1.875$

$y_4 = 0.48462$

Further,

$x_5 = \frac{x_4+x_3}{2} = 1.8125$

$y_5 = -1.02025$

Further,

$x_6 = \frac{x_5+x_4}{2} = \cancel{3.687} = 1.843\cancel{8}75$

$y_6 = -0.287734$

Further,

$x_7 = \frac{x_6+x_4}{2} = 1.859375$

4. $x^3 - x - 1 = f(x)$

$\Rightarrow f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow 1 - \frac{(-1)}{2}$$

$$\Rightarrow 1.5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 1.5 - \frac{0.875}{5.75} = \cancel{0.847882}\ 1.35$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow \cancel{0.84782}\ 1.35 - \left(\frac{\cancel{-1.2384}\ 0.110375}{\cancel{1.156}}\right) = \cancel{1.4921}\ 1.325$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.3248$$

So, root of $x^3 - x - 1 = 1.3248$

5. $A^2 = A$

$$7A - (1+A)^3 \Rightarrow 7A - 1^3 - A^3 - 3 \cdot 1A^2 - 3A1^2$$

$$\Rightarrow 7A - 1^3 - A^2 \cdot A - \cancel{B}\,3A^2 - 3A$$

$$\Rightarrow 7A - 1 - A^2 - 3A - 3A$$

$$\Rightarrow 7A - 1 - A - 3A - 3A$$

$$\Rightarrow 7A - 7A - 1$$

$$\Rightarrow -1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

6. $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix} = A$

$T_{11} = -6 \qquad T_{12} = 4 \qquad T_{13} = 4$

$T_{21} = 1 \qquad T_{22} = -1 \qquad T_{23} = -1$

$T_{31} = -6 \qquad T_{32} = 2 \qquad T_{33} = 4$

$|A| = -1(6-8) - 1(+4) \Rightarrow 2 - 4 \Rightarrow -2$

$A^{-1} = \frac{1}{5}\begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$

7. $A = 8i - 9j$
$B = 7i - 9j$

$A_x = 8 \; , \; A_y = -9$
$B_x = 7 \; , \; B_y = -9$

Scalar Product,
$A \cdot B = AB\cos\theta$

or,
$A \cdot B = A_x B_x + A_y B_y$

So,
$A_x B_x + A_y B_y = AB\cos\theta$ —— (1)

$A_x B_x + A_y B_y \Rightarrow (8)(7) + (-9)(-9) \Rightarrow 137$

$A = \sqrt{8^2 + (-9)^2} \Rightarrow \sqrt{64+81} \Rightarrow \sqrt{145}$

$B = \sqrt{7^2 + (-9)^2} \Rightarrow \sqrt{49+81} \Rightarrow \sqrt{130}$

Substituting the values in (1), we get,

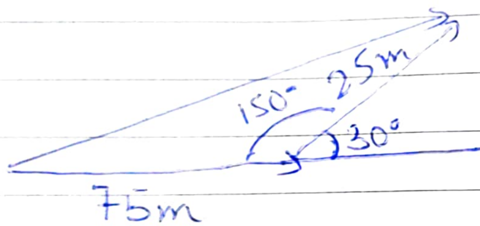
$137 = \sqrt{145} \cdot \sqrt{130} \cos\theta$

$\Rightarrow \cos\theta = \dfrac{137}{\sqrt{145} \cdot \sqrt{130}}$

$\Rightarrow \cos\theta = 0.997$

$\Rightarrow \theta = 4.44^\circ$

8.



$a = 75\,m$

$b = 25\,m$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$\Rightarrow 75 \cdot 25 \cos \cancel{30^\circ}\ 150$

$\Rightarrow$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow \sqrt{75^2 + 25^2 + 2(75)(25)\cos 30^\circ}$

$\Rightarrow 97.46\ m$

10. $a = 101$

$t_n = 998$

$d = 3$

$t_n = a + (n-1)d \Rightarrow 998 = 101 + (n-1)3$

$\Rightarrow \dfrac{897}{3} = n - 1$

$\Rightarrow n = 299 + 1 \Rightarrow n = 300$

$$S_{300} = na + \frac{n(n-1)d}{2}$$

$$\Rightarrow 300 \times 101 + \frac{300(299) \times 3}{2}$$

8

$$\Rightarrow 164850$$

9

10. $T_6 = 32 \Rightarrow ar^5 = 32$

10 $T_8 = 128 \Rightarrow ar^7 = 128$

11 Dividing T_6 by T_8,

Noon $\frac{T_8}{T_6} = \frac{128}{32}$

01

$$\Rightarrow \frac{ar^7}{ar^5} = 4$$

02

$$\Rightarrow r^2 = 4$$

03 $\Rightarrow r = 2$

04 11. Binomial Theorem says,

05 $(a+b)^n = a^n + {}^nC_1 a^{n-1}.b + {}^nC_2 a^{n-2}.b^2 + {}^nC_3.a^{n-3}.b^3 + \dots + b^n$

06

$$(4+3x)^5 = 4^5 + 5.4^4.3x + 10.4^3.9x^2 + 10.4^2.27x^3 + 5.4.81x^4 + 243x^5$$

$$\Rightarrow 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

8

$|A - \lambda I| = 0$

9

$\Rightarrow \left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$

10

11

$\Rightarrow \left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$

Noon

01

$\Rightarrow (3-\lambda)\left[(2-\lambda)(-5-\lambda) - (-3)(2)\right] = 0$

02

$\Rightarrow (3-\lambda)\left[-10 + 5\lambda - 2\lambda + \lambda^2 + 6\right] = 0$

03

$\Rightarrow (3-\lambda)(\lambda^2 + 3\lambda - 4) = 0$

Assuming, $\lambda^2 + 3\lambda - 4 = 0$,

04

$\lambda^2 + 3\lambda - 4 = 0$

05

$\Rightarrow \lambda^2 - 4\lambda + \lambda - 4 = 0$

$\Rightarrow \lambda(\lambda - 4) + 1(\lambda - 4) = 0$

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06

$\Rightarrow (\lambda - 4)(\lambda + 1) = 0$

$\therefore \lambda = 3, 4, -1$

13 $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$f'(x) = 3x^2 - 14x + 8$

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{(-13)}{13} = 6$

8 $\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{f(6)}{f'(6)} = 6 - \frac{9}{32} = 5.71875$

9

10 14. $(1-x+x^2)^4$

$\Rightarrow [(1-x)+x^2]^4$

11 $\Rightarrow (1-x)^4 + 4(1-x)^3x^2 + 16(1-x)^2x^4 + 4(1-x)^4x^6 + \cancel{4}\ 4x^8$

Noon $\Rightarrow 1-4x+16x^2-4x^3+x^4+[4-12x+12x^2-4x^3]x^2$
$+[16-32x+16x^2]x^4 + [4-4x]x^6 + 4x^8$

01

$\Rightarrow 1-4x+16x^2-4x^3+x^4+4x^2-12x^3+12x^4$

02 $-4x^5+16x^4-32x^5+16x^6+4x^6-4x^7+4x^8$

03 $\Rightarrow 1-4x+20x^2-16x^3+29x^4-36x^5+20x^6-4x^7$
$+4x^8$

04 15. $e^{-x} = 3\log(x)$

05 $\Rightarrow$ $e^{-x} - 3\log(x) = 0$

06 $f(x) = e^{-x} - 3\log(x)$

x	$f(x)$
1.1	0.2086
1.2	0.0636
1.3	-0.069
1.4	-0.191

$x_1 = 1.2$

$x_2 = 1.3$

$x_3 = \frac{1.2+1.3}{2}$

$\Rightarrow \frac{2.5}{2} = 1.25$

$y_3 = -0.0042$

$$x_4 = \frac{x_2 + x_3}{2} \Rightarrow \frac{1.2 + 1.25}{2} \Rightarrow 1.225$$

$$y_4 = 0.0293$$

8

$$x_5 = \frac{x_4 + x_3}{2} \Rightarrow \frac{1.225 + 1.25}{2} \Rightarrow 1.2375$$

9

$$y_5 = 0.0125$$

10

$$x_6 = \frac{x_5 + x_3}{2} \Rightarrow 1.24375$$

11

$$y_6 = 0.00841$$

Noon

01

02

03