1. Find the value of $\sqrt{7+\sqrt{7+\sqrt{7+\cdots}}}$

2. Find the value of $\frac{x}{y}$ in the following equation:

$$7x^2 - 6y^2 = -11x^2 - 4y^2$$

3. What ordered pairs satisfy the system

$$x + 3y = 4$$
$$x^2 + y^2 - 4y = 12$$

3. Values that Satisfy :-

$x + 3y = 4$ ①

$x^2 + y^2 - 4y = 12$ ②

$\therefore x = 4 - 3y$ — Putting this in ②

$\therefore (4-3y)^2 + y^2 - 4y = 12$

$16 - 24y + 9y^2 + y^2 - 4y = 12$

$\therefore 10y^2 - 28y + 16 - 12 = 0$

$10y^2 - 28y + 4 = 0$ (divide entirely by 2)

$\therefore 5y^2 - 14y + 2 = 0$

using $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \dfrac{14 + \sqrt{(-14)^2 - 4(10)}}{2(5)}$, $\dfrac{14 - \sqrt{(-14)^2 - 4(10)}}{2(5)}$

$\therefore y = 2.649$, $y = 0.151$

From ①:-

$x + 3y = 4$ | $x + 3y = 4$

$\therefore x + 3(2.649) = 4$ | $x + 3(0.151) = 4$

$\therefore x = -3.947$ | $\therefore x = 3.547$

$\Rightarrow$ The ordered pairs are $[3.947, 2.649]$ & $[3.547, 0.151]$

2. Given:-

$7x^2 - 6y^2 = -11x^2 - 4y^2$

$\therefore 7x^2 + 11x^2 = -4y^2 + 6y^2$

$\therefore 18x^2 = 2y^2$

$\therefore \dfrac{x^2}{y^2} = \dfrac{2}{18}$ [Taking the Square Root on both sides]

$\therefore \boxed{\dfrac{x}{y} = \sqrt{\dfrac{1}{9}}}$

1. Given:-

$\sqrt{7+\sqrt{7+\sqrt{7}}}\ldots$

$\therefore$ Let $x = \sqrt{7+\sqrt{7+\sqrt{7}}}\ldots$

$\therefore x = \sqrt{7+x}$ — ① [$\because x = \sqrt{7+\sqrt{7}}\ldots$]

[On squaring both sides we get]

$x^2 = 7 + x$

$\therefore x^2 - x - 7 = 0$

[By using quadratic formula]

$\dfrac{+1 + \sqrt{(-1)^2 - 4(-7)}}{2}$ or $\dfrac{+1 - \sqrt{(-1)^2 - 4(-7)}}{2}$

$x = 3.1926$ or $x = -2.1926$

$\therefore$ Tallying back both values by putting x values in ①.

when $x = 3.1926$, $x = -2.1926$

$(3.1926) = \sqrt{7 + 3.1926}$ | $(-2.1926) = \sqrt{7 - 2.1926}$

$\therefore 3.1926 = 3.19256$ ✓ | $(-)2.1926 = 2.1926$ ✗

$\Rightarrow x = 3.19256$ is the acceptable value.

4. The quadratic function defined by the equation $d = 2r^2 - 16r + 34$ gives the density of smoke, d, in millions of particles per cubic foot for a certain type of diesel engine. The input variable, r, represents the speed of the engine in hundreds of revolutions per minute
   a. Determine the density of smoke when r = 3.5 (350 revolutions per minute).
   b. Determine the number of revolutions per minute for minimum smoke. What is the minimum output?
   c. If the density of smoke is determined to be 100 million particles per cubic foot, determine the speed of the engine.

5. You contact two car rental companies and obtained the following information for the 1-day cost of renting a car.
   Company 1: Rs 1500 per day plus Rs 25 per km
   Company 2: Rs 2100 per day plus Rs 22 per km
   Let n represent the total number of km driven in 1 day.
   a. Write an expression to determine the total cost, C, of renting a car for 1 day from company 1.
   b. Write an expression to determine the total cost, C, of renting a car for 1 day from company 2.
   c. Use the expressions in parts a and b to write an inequality that can be used to determine for what number of km it is less expensive to rent the car from company 2.
   d. Solve the inequality.

6. Use the Bisection method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval
   0.01
   a. [0, 1]
   b. [1, 3.2]
   c. [3.2, 4]

4. Given:-

$d = 2r^2 - 16r + 34$

density — speed in terms of revolutions.

a) $d \big|_{r=3.5}$

$\therefore d = 2(3.5)^2 - 16(3.5) + 34$

$= 2.5$

$\therefore$ Density $= 2.5$ millions of particles per cubic when there are 350 revolutions/per min.

b) For minimum revolutions r:

$d = 2r^2 - 16r + 34$

$\therefore \dfrac{d(d)}{dr} = 4r - 16$

$\therefore 4r - 16 = 0$

$4r = 16$

$\boxed{r = 4}$

$\therefore \dfrac{d^2d}{dr^2} = 4$

$\therefore \dfrac{d^2d}{dr^2}\bigg|_{r=4}$ $4 > 0$

$\Rightarrow$ 400 revolutions/per min are required for minimum smoke.

The min. smoke is $\Rightarrow d = 2r^2 - 16r + 34$

$= 2(4)^2 - 16(4) + 34$

$= 2$

$\Rightarrow$ 2 million particles per cubic

c) Given :-

Density = 100 million particles per cubic foot.

Speed $(r) = ?$

$\because$ Density $= 2r^2 - 16r + 34$

$\therefore 2r^2 - 16r + 34 = 100$ [$\because d = 100$]

$\therefore r^2 - 8r + 17 = 50$ [divided entirely by 2]

[Since direct factoring isn't possible, using the quadratic formula]

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \underbrace{\boxed{1}}_{a}r^2 \underbrace{\boxed{-8}}_{b}r \underbrace{\boxed{-33}}_{c} = 0$

$-(-8) + \sqrt{(-8)^2 - 4(1 \times -33)}$ or $-(-8) - \sqrt{(-8)^2 - 4(-33}$

5.

ⓐ COMPANY 1:

$C = ₹1500 + ₹25n$

ⓑ COMPANY 2:

$C = ₹2100 + ₹22n$

ⓒ From a & b:

$1500 + 25n = 2100 + 22n$

ⓓ

$25n - 22n = 2100 - 1500$

$3n = 600$

$n = 200$ km

$\because$ We know Company 2 is cheaper than Company 1.

$\therefore 25(200) - 22(200)$

$= 5000 - 4400$

$= ₹600$ [$\because n = 200$]

$\therefore \dfrac{600}{200} = \boxed{3}$

$\Rightarrow$ Company 2 is $\boxed{3km}$ less expensive than Company 1.

$\therefore \boxed{1}x^2 \boxed{-8}x \boxed{-33} = 0$ ($a = 1$, $b = -8$, $c = -33$)

$$\frac{-(-8)+\sqrt{(-8)^2-4(1\times -33)}}{2} \quad \text{or} \quad \frac{-(-8)-\sqrt{(-8)^2-4(-33)}}{2}$$

$= 11$ or -3

$\therefore$ $\boxed{\text{Speed i.e revolutions are 11000/per min.}}$

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6. Given Equation $\rightarrow$ $x^3 - 7x^2 + 14x - 6 = 0$ close to 0.01 value.

a] $[0, 1]$
b] $[1, 3.2]$
c] $[3.2, 4]$

a] $[0, 1]$ ($a = 0$, $b = 1$)

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
0	1	0.5	−6	2	−0.625
0.5	1	0.75	−0.625	2	0.9844
0.5	0.75	0.625	−0.625	0.9844	0.2598
0.5	0.625	0.5625	−0.625	0.2598	−0.1619
0.5625	0.625	0.5938	−0.1619	0.2598	0.0544
0.5625	0.5938	0.5782	−0.1619	0.0544	−0.05211
0.5782	0.5938	0.586	−0.05211	0.0544	$\boxed{0.00146}$

Since precision is reached.

b] $[1, 3.2]$ ($a = 1$, $b = 3.2$) $x^3 - 7x^2 + 14x - 6 = 0$

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1	3.2	2.1	2	−0.112	1.791
2.1	3.2	2.65	1.791	−0.112	0.55213
2.65	3.2	2.925	0.55213	−0.112	0.08583
3	3.01	3.005	0	−0.009799	$\boxed{-0.00495}$

$\underbrace{3 \quad 3.01}_{\text{Found these values using the table } f^n \text{ of Calci.}}$

$\Rightarrow$ The real root of the fn lies at $\boxed{3}$.

The level of precision is achieved.

c] $[3.2, 4]$ $x^3 - 7x^2 + 14x - 6 = 0$

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
3.2	4	3.6	−0.112	2	0.336
3.4	3.5	3.45	−0.016	0.125	0.04613
3.4	3.45	3.425	−0.016	0.04613	0.013016
3.4	3.425	$\boxed{3.4125}$	−0.016	0.013016	−0.001998

$\underbrace{3.4 \quad 3.5}_{\text{finding from table } f^n \text{ of calculator}}$

$\Rightarrow$ 3.4125, since the 10^{-2} i.e 0.01 level of precision (accuracy is achieved.

7. The fourth-degree polynomial

$$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$$

has two real zeros, one in [−1, 0] and the other in [0, 1]. Attempt to approximate these zeros to within 10^{-4} using the Newton's method.

8. You are enrolled in an algebra course at your college. You achieved grades of 70, 86, 81, and 83 on the first four exams. The final exam counts the same as an exam given during the semester.
   a. If x represents the grade on the final exam, write an expression that represents your course average (arithmetic mean).
   b. If your average is greater than or equal to 80 and less than 90, you will earn a B in the course. Using the expression from part a for your course average, write a compound inequality that must be satisfied to earn a B.
   c. Solve the inequality.

9. The NPV for a project at a discount rate of 4% is 8.54. The NPV at a discount rate of 20% is -11.81. Using the above data points, find the IRR using linear interpolation.

10. The makers of a new food delivery app estimate that with x thousand orders their monthly revenue and cost (in lakhs of Rs) are given by the following:

$$R(X) = 32x - 0.21x^2$$
$$C(x) = 195 + 12x$$

Determine the number of orders needed for the app to remain profitable.

8.

a] Avg $= \frac{70 + 86 + 81 + 83 + x}{5}$

$= \boxed{\frac{320 + x}{5}}$

7. Given:- $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$\because$ We know that $x_i - \frac{f(x)}{f'(x)} = x_{i+1}$

a]

i	x_i	f(x)	f'(x)	x_{i+1}
1	−1	156.1398	−1105	−0.60814
2	−0.6081	43.9940	−418.898	−0.2354
3	−0.23541	1.537	−234.246	−0.04759
4	−0.04759	0.0004	−221.704	−0.04666
5	−0.04066	~0	−221.70443	−0.04066

$\therefore \boxed{x = -0.04060}$

b]

i	x_i	f(x)	f'(x)	x_{i+1}
1	0.9	−36.585	509.62	0.97179

a] $\text{Avg} = \frac{70 + 86 + 81 + 83 + x}{5}$

$= \boxed{\frac{320 + x}{5}}$

b] Average = B

$\therefore \quad 80 < \frac{320 + x}{5} < 90$

c] $\therefore$ From (b)

$80 < \frac{320 + x}{5} < 90$

$\therefore \; 400 < 320 + x < 450$

$80 < x < 130$

$\boxed{\therefore 80 \leq x < 130}$

9. We are given;

$x_1 = 0.04, \quad x_2 = 0.2, \quad x = 0$

$f(x_1) = 8.54, \quad f(x_2) = -11.81, \quad f(x) = ?$

$\because \; x = x_1 + \frac{f(x) - f(x_1)}{f(x_2) - f(x_1)} (x_2 - x_1)$

$\therefore \; 0 = 0.04 + \frac{f(x) - 8.54}{-20.35} (0.16)$

$\therefore \; f(x) = 13.6275$

b]

i	x_i	$f(x)$	$f'(x)$	x_{i+1}
1	0.9	-36.585	509.62	0.97179
2	0.97179	6.37668	691.80	0.9625
3	0.9625	0.11523	666.87	0.9624
4	0.9624	0.3267	666.411	0.9624

$\boxed{\therefore x = 0.9624}$

10. Given:-

$R(x) = 32x - 0.21x^2$

$C(x) = 195 + 12x$

$\therefore$ For Profit:- $R(x) > C(x)$

$32x - 0.21x^2 > 195 + 12x$

$20x - 195 - 0.21x^2 > 0$

$0.21x^2 - 20x + 195 < 0$

$0.21x^2 - 20x + 195 = 0$

$\therefore \; x = \frac{20 \pm \sqrt{212.8}}{0.42}$

$\therefore \; x = 82.38, \; 12.88$

$\therefore \; (x - 82.38)(x - 12.88) < 0$

$\therefore \; \boxed{x \in (12.88, 82.38)}$