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Calculus assignment - 2

1. $I = \int_{1/50}^{2} \frac{e^{2/w} dw}{w^2}$

→ let $t = e^{2/w}$

$dt = e^{2/w} \cdot \frac{d}{dw}\left(\frac{2}{w}\right)$

$= e^{2/w} \cdot 2\left(1/w^2\right)$

$dt = -\frac{2e^{2/w} dw}{w^2}$

$$\boxed{\frac{dt}{-2} = \frac{e^{2/w}}{w^2} dw}$$

$I = -\frac{1}{2} \int_{e^{100}}^{e} dt$

$= -\frac{1}{2} \boxed{[t]}_{e^{100}}^{e}$

$= -\frac{1}{2} e^{100} + \frac{1}{2} e$

$= \frac{e}{2} - \frac{e^{100}}{2}$

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(2) $I = \int \frac{2+3-1}{(t^4+2t)^3}\,dt$

let $y = t^4 + 2t$

$dy = (4t^3+2)\,dt$

$= 2(2t+3+1)\,dt \rightarrow (1)$

Putting that in I

$I = \frac{1}{2}\int \frac{dy}{y^3}$

$= \frac{1}{2}\, y^{\frac{-3+1}{-2}} = -\frac{1}{4}\cdot\frac{1}{y^2}$

$= \frac{-1}{4y^2}$

$= \frac{-1}{4(t^4+2t)^2}$

(3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)\,dx$

$= \int (x^4+3x-9)\,dx$

$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$ //

(4) $f(x) \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & x \le 1 \end{cases}$

$\rightarrow \int_{-2}^{3} f(x)\, dx$

$\int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

⇒ So, as per the question

⇒ $I = \int_{-2}^{1} 3x^2 dx + \int_{1}^{3} 6 dx$

$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$

$= 1 - (-8) + 6(3) - 6(1)$

$= 1 + 8 + 18 - 6$
$= 21$

(5) $I = \int 3(8y-1) e^{4y^2-y} dy$

$= 3\int (8y-1) e^{4y^2-y} dy$

= let $t = 4y^2 - y$
$dt = e^{4y^2-y}(8y-1) dy$

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$= I = 3\int dt = 3t + C$

$= 3(e^{4y^2} - y) + C.$

6.) $f''(x) = 15\sqrt{x} + 5x^3 + 6,$

$f(1) = -5/4$ and $f(4) = 404$

$\int f''(x) = f'(x) = \int 15x^{1/2} + 5x^3 + 6\,dx$

$= \frac{2}{\cancel{3}} \cdot \overset{5}{\cancel{15}} x^{3/2} + \frac{5x^4}{4} + 6x$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C$

$= f(x) = \frac{10x^{5/2}}{5/2} + \frac{5}{4}\frac{x^5}{5} + \frac{\overset{3}{\cancel{6}}x^2}{\cancel{2}} + Cx + C'$

$\frac{-5}{4} = 4 + \frac{1}{4} + 3 + C + C'$

$\frac{-3}{2} = 7 + C + C'$

$C + C' = \frac{-17}{2}$ //

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$= f(4) = 4^{7/2} + 4^4 + 48 + 4c + c'$

$404 = 128 + 256 + 48 + 4c + c'$

$-28 = 4c + c'$

$c' = -4c - 28$

$c + c - 4c - 28 = -8.5$

$-3c = 28 - 8.5$

$\underline{\underline{c = -6.5}}$

$\underline{\underline{c' = -2.5}}$

$$\boxed{f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13}{2}x - \frac{5}{2}}$$

(8.) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $r(1) = 2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

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$$\frac{r^{-2+1}}{-1} = \log\theta + C$$

$$-r^{-1} = \log\theta + C$$

$$-\frac{1}{r} = \log\theta + C$$

$$r(1) = 2$$

$$-\frac{1}{r} = \log(1) + C$$

$$-\frac{1}{2} = 0 + C$$

$$\boxed{C = -\tfrac{1}{2}}$$

$$-\frac{1}{r} = \log\theta - \frac{1}{2}$$

$$\boxed{\log\theta = \frac{1}{2} - \frac{1}{r}}$$

9. $$\frac{dv}{dt} = 9.8 - 0.196\,V$$

$$\frac{dv}{9.8 - 0.196V} = dt$$

$$\frac{dv}{\frac{98}{10} - \frac{196}{1000} v} = dt$$

$$= \int \frac{-dv\,(100)}{9800 - 196V} = \int dt$$

$$= \frac{100 \log (9800 - 196)}{-196 - 98} = t + c$$

$$= -\frac{50}{98} \log (9604) = t + C$$

(10.) $ty' + 2y = t^2 - t + 1 \quad y(1) = 1/2$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$t\frac{dy}{dt} \cancel{+2}y = t^2 - t + 1 - 2y$$

$$x = t^2 - t + 1 - 2y$$

$$\frac{dy}{dx} = 2t - 1 - 2\frac{dy}{dt}$$

$$\frac{2dy}{dt} = 2t - 1 - \frac{dx}{dt}$$

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$$\frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}$$

$$\boxed{\frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}}$$

$= \quad y = \frac{1}{2}$ at $t = 1$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$t\left(t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}\right) = x$$

$$t^2 - t/_2 - t/_2 \frac{dx}{dt} = x$$

$$t^2 dt - \frac{t}{2} dt - \frac{1}{2} dx = x dt$$

$$\frac{t^3}{3} - \frac{t^2}{4} - \frac{tx}{2} = xt + C$$

$$\frac{t^3}{3} - \frac{t^2}{4} = \frac{3xt}{2} + C$$

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$$\frac{t^3}{3} - \frac{t^2}{4} - \frac{3t^3}{2} + \frac{3t^2}{2} - \frac{3t}{2} + 3yt =$$

$$-\frac{7t^3}{6} + \frac{10t^2}{8} - \frac{3t}{2} + 3yt = 0c$$

$$-\frac{7}{6} + \frac{10}{8}$$

$$c = \frac{2}{24}$$

$$\boxed{c = 1/12}$$

(11) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$

$\rightarrow$ $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$= \int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}} dx$

$\frac{y^{-2}}{-2} = \cancel{\frac{1}{2}} \frac{t^{1/2}}{1/2} + C$

$\frac{-1}{2y^2} = t^{1/2} + C$

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$\frac{-1}{2(-1)} = C$

$\boxed{C = \frac{1}{2}}$

$1+x^2 > 0$ for all $x \in R$

The function is valid for all no's.

(12.) $f(x) = x^3 - 10x^2 + 6$ for $x=3$

$f'(x) = 3x^2 - 20x$
$f''(x) = 6x - 20$
$f'''(x) = 6$
$f''''(x) = 0$

$f'(3) = 3(3)^2 - (20)3$
$= 27 - 60$
$= -33$

$f''(3) = 18 - 20 = -2$
$f'''(3) = 6$
$f''''(3) = 0$

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$f(3) = 27 - 90 + 6 = -57.$

$\Rightarrow$ Taylor Expansion

$$= -57 + (x-3)(-33) + \frac{18}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3 + 0$$

$$= -57 + 99 - 33x + 9(x-3)^2 + (x-3)^3$$

$$= 42 - 33x + 9(x^2 - 6x + 9) + (x^3 - 27 - 9x(x-3))$$

$$= 42 - 33x - 54x + 81 + x^3 - 27 \cancel{- 9x^2} \cancel{+ 9x^2} + 27x$$

$$= x^3 - 27x - 33x + 42 + 81 - 27$$

$$\Rightarrow \boxed{x^3 - 60x + 96.}$$

15. $f(x) = e^{-6x}$ about $x = -4$

$\rightarrow$ Taylor expansion:-

$$f(-4) + (x-(-4))f'(-4) + \frac{(x-(-4))^2}{2!} f''(-4) + \frac{x-(-4)^3}{3!} f'''(-4)$$

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15. $= f(x) = e^{-6x}$

$f'(x) = \frac{-1}{6} e^{-6x}$

at $x = -4$

$f'(4) = -\frac{1}{6} e^{24}$

$f''(-4) = \frac{1}{36} e^{-6x} = \frac{1}{36} e^{24}$

Taylor Expansion

$= e^{24} + (x+4) \frac{-1}{6} e^{24} + \frac{(x+4)^2}{2!}$

$\frac{1}{36} e^{24}$

$\frac{-1}{216} e^{24} \cdot \frac{(x+4)^3}{3!}$

16 $f(x) = \ln(3+4x)$

Taylor expansion about $x=0$

$= f(0) + (x-0) f'(0) +$
$(x-0)^2 f'(0) + (x-0) f'(0)$

$f(x) = \ln(3+4x) = x = 0$

$\ln(3)$

$$f'(x) = \frac{4}{3+4x} \Rightarrow x = 0 \Rightarrow \frac{4}{3}$$

$$f''(x) = \frac{(4x+3)(0)-(4)(4)}{(3+4x)^2} = \frac{-16}{(3+4x)^2}$$

$$\Rightarrow x = 0$$

$$= \frac{-16}{9}$$

$$f'''(x) = -16\left[\frac{(3+4x)^2(0)-(2)(4x+3)}{(3+4x)^4}\right]$$

$$= \frac{32}{(3+4x)^3} = \frac{32}{27}$$

$$\therefore = \ln(3) + \frac{4x}{3} + (x-0)^2\left(\frac{-16}{9(2!)}\right)$$

$$+ \frac{32}{27(3!)}(x-0)^3 + \ldots \frac{-64}{(81)(4!)}(x-0)^4 \ldots$$

$$= \ln 3 + \frac{4x}{3} + \frac{-16}{9(2!)}x^2 + \frac{32x^3}{27(3!)}$$

$$- \frac{64}{81}\frac{x^4}{(4!)} \ldots\ldots$$

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(3) $f(x) = (1+x)^{u}$

$f(0) = 1^{u} = 1$

$f'(0) = u(1+x)^{u-1}$

$= u(1+0)^{u-1}$

$= u(1) = u$

$= f''(0) = u(u-1)(1+0)^{u-1-1}$

$= u^2 - u(1)$

$= u^2 - u$

$f'''(0) = u(u-1)(u-2)(1+0)^{u-3}$

$= u(u-1)(u-2)1$

$= u(u-1)(u-2)$

→ Maclaurin series

$= f(0) + f'(0)x + \frac{f''(0)x^2}{2!}$

$+ \frac{f'''(0)x^3}{3!} + \cdots$

$= 1 + ux + \frac{(u^2-u)x^2}{2!} + \frac{u(u-1)(u-2)x^3}{3!}$

$$+ \frac{4(4-1)(4-2)(4-3)x^4}{4!} \ldots\ldots$$

(14.) $f(x) = \sqrt{1+x}$

$\rightarrow$ $f(0) = 1$

$= f'(0) = \frac{1}{2}(1+x)^{-1/2} = \frac{1}{2}(1) = \frac{1}{2}$

$= f''(0) = \frac{-1}{2}\frac{1}{2}(1+x)^{-3/2}$

$= \frac{-1}{4}(\cancel{1}) = -\frac{1}{4}$

= Maclaurin series →

$$1 + \frac{1}{2}(x) - \frac{1}{4}\frac{x^2}{(2!)} + \frac{1}{8}\frac{(x^3)}{3!} - \frac{1}{16}\frac{(x^4)}{4!}$$

$$\ldots\ldots \frac{-1}{2^n}\frac{x^n}{n!}$$

(7) let $z = f(x+iy) + g(x-iy)$

differentiating w.r.t ~~y~~ x & y

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we got.

$p = z' = f'(x+iy) + g'(x+iy)$

$m = z' = f'(x+iy)(i) + g'(x-iy)(-i)$

$q = z'' = f''(x+iy) + g'(x-iy)$ ——— (1)

$n = z'' = f''(x+iy)(i^2) + g'(x-iy)(-i)^2$

$n = f''(x+iy)(i^2) + g'(x-iy)(i^2)$

$n = i^{(2)}(q)$

$n = i^{(2)} q$

$n = i^2 q$ ——— from (1)