Assignment - 2

Answers

1) ① APR is the interest rate for the equation of value.

$\therefore \quad 20{,}000 = 12 \times 427.90\, a^{(12)}_{\overline{5}|}$

$a^{(12)}_{\overline{5}|} = 3.89499$

Since.

APR is usually double the flat rate of interest

$\therefore$ Flat Rate $= \dfrac{\text{total interest}}{\text{loan} \times \text{total years}}$

$= \dfrac{5 \times 12 \times 427.90 - 20{,}000}{20000 \times 5}$

$= 5.67\%$

Thus, APR is likely to be around 11%

At $i = 11\%$, $a^{(12)}_{\overline{5}|} = 3.87872$

At $i = 10\%$, $a^{(12)}_{\overline{5}|} = 3.96154$

Interpolating, APR, I is

$$\frac{i - 10\%}{3.89499 - 3.96154} = \frac{11\% - 10\%}{3.87872 - 3.96154}$$

$i = 10.8\%$

At $i = 10.8\%$, $12 \times 427.90\, a^{(12)}_{\overline{5}|} = 20{,}000.2$

At $i = 10.9\%$, $12 \times 427.90\, a^{(20)}_{\overline{5}|} = 19958.3$

$i = 10.8\%$ is more close. Hence, ~~APR~~ APR is 10.8%

(ii) APR = 10.8%

After 1 year, amount left to pay $= 12 \times 427.90\, a^{(12)}_{\overline{4}|}$ @ 10.8%

$= 16775.98$

Let t denote the term of the new loan, thus

$$16775.98 = 12 \times 274.49\, a^{(12)}_{\overline{t}|}$$

$$16775.98 = 12 \times 274.49 \left[\frac{1 - 1.108^{-t}}{12 \times (1.108^{(1/12)} - 1)} \right]$$

$$1.108^{t} = 0.4754$$

$$t = 7.25 \text{ years}$$

The original loan had 4 years of payments remaining, thus

Total interest $= 4 \times 12 \times 427.90 - 16775.98 = \underline{\underline{3763.22}}$

New loan has term of 7.25 years, hence

Total interest $= 7.25 \times 12 \times 274.49 - 16775.98$

$= 7104.65$

Thus, Mr. & Mrs. Jones will pay $(7104.65 - 3763.22) = \underline{3341.43}$ more interest under the restructed loan.

2) (i)

a) The discounted payback period is the smallest time t for which the present value or accumulated value of the returns up to time t exceeds the present or accumulated value of the costs up to time t.

b) The payback period is the same as the discounted payback period, except that the present value calculation (or accumulation) is carried out using an interest rate of 0. In other words, it is the earliest time for which the monetorry value of the returns exceeds the monetory value of the costs.

(ii) Unlike the NPV, neither the DPP nor the PP give any indication of how profitable a project is, as they ignore cash flows after the accumulated value of zero is reached.
There may ~~be~~ not be one unique time when the balance in the investor's account changes from negative to positive. However, the NPV can always be calculated.
The PP can give misleading results, as it dose not take into account the time value of money.
Hence, NPV is a superior as compared to DPP or PP.

(iii)

a) Internal rate of return (IRR)

The IRR for project A is the rate of interest that satisfies the equation of value :

$-170000 - 20000v - 10000v^2 + 20{,}000v + 20\,000v^2 + 20000v^3 = 0$

Simplifying

$10v^2 + 200v^3 = 170$

By trial and error, we find that :-

$i = 7\%$; $10v^2 + 200v^3 = 171.99$

$i = 8\%$; $10v^2 + 200v^3 = 167.34$

Using interpolation :-

$i = 7 + \frac{170 - 171.99}{167.34 - 171.99} * (8-7) = 7.4\%$

For Project B, the income (paid at the end of each of the first 6 years) represents 7% of the initial capital, which is returned at the end of the 6 years.

So we can see immediately that the IRR for Project B is exactly 7%.

Alternatively :-

$14\, a_{\overline{6}|} + 200v^6 = 200$

b) Net Present Value

The net present values are found by discounting the payments at 6%.

Project A : $NPV = -170{,}000 + 10{,}000v^2 + 200{,}000v^3 = £6{,}832.82$

Project B : $NPV = -200{,}000 + 14{,}000\, a_{\overline{6}|} + 200{,}000v^6 = £9{,}834.65$

c) Maximum Interest Rate beyond which the projects are unprofitable.

Each project will be profitable if the rate of interest at which funds can be borrowed is less than the internal rate of return. So, to be profitable, Project A requires a borrowing rate less ~~that~~ than 7.4% and Project B requires a rate lower than 7%.

Other factors to be considered include:

- Project A dose nat provide any net income during the first year.
- The lower internal rate of return for Project B applies for a longer period.
- The rates of interest available in years 4 to 6 (i.e. after Project A has finished) will affect the comparison between the accumulated profits at the end of year 6 (i.e. when Project B finishes).
- The risk associated with the receipt of income. If Project A involves greater uncertainty or risk, it may nat be accepted even though it has a higher IRR.

3) Value at 1st November 1985 is

Annuity 1 $= 200 \times v^{(1/4)} \times \ddot{a}_{\overline{22}|}$ @ 8%

$= 200 \times 0.980944 \times 11.01676$

$= 2161.363$

Annuity 2 $= 320 \times (1+i)^{(1/12)} \times a^{(4)}_{\overline{16.25}|}$ @ 8%

$= 320 \times 1.006434 \times 9.184257$

$= 2957.872$

Annuity 3 $= 180 \times a^{(12)}_{\overline{18.75}|}$

$= 180 \times 9.892583$

$= 1780.665$

Total value of annuities $= 6{,}899.90$

Let the revised annuity be A

value of the revised annity is $= A \times (1+i)^{1/4} \times a^{(2)}_{\overline{21.5}|}$

$$A = \frac{6899.90}{1.019427 \times 10.30886}$$

$= 656.56$

4) From the first principles,

$(I\ddot{a})_n = 1 + 2v + 3v^2 + 4v^3 + 5v^4 + \cdots + nv^{n-1}$ @ I

Multiplying both the sides by 'v'

$v(I\ddot{a})_n = v + 2v^2 + 3v^3 + 4v^4 + 5v^5 + \cdots + nv^n$

Subtracting,

$(1-v)(I\ddot{a})_n = 1 + v + v^2 + v^3 + v^4 + \cdots + v^{n-1} - nv^n$

$= (1-v^n)/(1-v) - nv^n$

$d(I\ddot{a})_n = \ddot{a}_n - nv^n$

$(I\ddot{a})_n = \dfrac{\ddot{a}_n - nv^n}{d}$ Hence proved

5) (i) The time weighted rate of return for property fund

$\frac{16.4}{12.4} - 1 = 0.3226$ or 32.26%

The time weighted rate for equity fund

$\frac{15.5}{12.1} - 1 = 0.2810$ or 28.10%

(ii)

(a) Assuming that the investor bought 100 units per quarter in property fund the equation of value is given by:-

$1240(1+i) + 1310(1+i)^{(3/4)} + 1480(1+i)^{(1/2)} + 1580(1+i)^{(1/4)}$

$= 4 \times 1640 = 6560$

Where 'i' is yeild per annum $= 0.2932$ or 29.32%

(b) Assuming that the investor purchases INR 100 worth of units each quarter

$400 \times \ddot{s}_1^{(4)} = 100 \times 16.4 \left(\frac{1}{12.4} \times \frac{1}{13.1} \times \frac{1}{14.8} \times \frac{1}{15.8} \right)$

$\ddot{s}_1^{(4)} = 1.1801$

this gives $i = 0.2979$ or 29.8%

5) (iii)

(a) Assuming investors bought 100 units per quarter in equity fund the equation ~~of~~ of value is given by:-

$1210\,(1+i) + 920\,(1+i)^{(3/4)} + 1030\,(1+i)^{(1/2)} + 1310\,(1+i)^{(1/4)}$

$= 4 \times 1550$

Where i is yeild per annum = 0.6731 or 67.31%

(b) Assume that the Investor purchase INR 100 worth of units each ~~qtr~~ quarter

$400 \times \ddot{s}_{\overline{1}|}^{(4)} = 100 \times 15.5 \left(\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1} \right)$

$\ddot{s}_{\overline{1}|}^{(4)} = 1.4135$ this gives $i = 0.7088$ or 70.88%

5) (i) Let X be the initial annual repayment

$160000 = X a_{10}$ @ 8%

$X = \frac{160000}{a_{10}} = \frac{160000}{6.701} = 23,844.65$

(ii) The loan outstanding immediately after the fourth payment is made, is:

$23,844.65\, a_6$ ~~at~~ @ 8% $= 23,844.65 \times 4.6229$

$= 110,231.442$

Let Y be the revised annual installment. The equation of value is

$Y a_6 = 110,231.442$ @ 10%

$Y = \frac{110231.442}{a_6} = \frac{110231.442}{4.3553} = 25,309.72$

(iii) The loan outstanding immediately after the seventh payment is made is $25,309.27\, a_3$ @ 10%

$= 25309.72 \times 2.4869 = 62942.74$

Let Z be the revised annual installment

$Z a_3 = 62,942.74$ @ 9%

$Z = \frac{62942.74}{a_3} = \frac{62942.74}{2.5313} =$ ~~[illegible]~~ 24865.77

The yeild can be found as:-

$160000 = 23844.65\, a_4 + v^4 \times 250309.72\, a_3 + v^7 \times 24865.77\, a_3$

By interpolation betn 8% and 9%, the yeild is 8.4%

8% $= -4310.73$

9% $= 2930.935$

8) Working unit of Rs 1 crores and assuming i as the rate of interest:

(i) The present value of outlay is :-

$$2.7 + 0.2\, a_{\overline{3}|} \; @ \; i$$

The expected present value of the income is:

$$0.1 v^3 + \bar{a}_{\overline{10}|} (0.25 v + 0.2 v^2 + 0.1 v^3) \; @ \; i$$

Equating the present value of the outlay to the present value of the income and solving for :

$$2.7 + 0.2\, a_{\overline{3}|} = 0.1 v^3 + \bar{a}_{\overline{10}|} (0.25 v + 0.2 v^2 + 0.1 v^3) \; @ \; i$$

This gives $i = 9.3\%$

(ii) Based on the information, the present value of the company's income is:

$$0.1 v^3 + 0.85\, \bar{a}_{\overline{10}|}\, v^3$$

$$0.1 v^3 + 0.85\, \bar{a}_{\overline{10}|}\, v^3 - \cancel{3}2.7 - 0.2\, a_{\overline{3}|} \; @ \; 10\%$$

$$0.1 \times 0.7513 + 0.85 \times 0.7513 \times 6.4469 - 2.70 - 0.2 \times 2.4868$$

$$= 4.19 - 3.19$$

$$= 1.0089$$

Since NPV is positive it is worth to invest in the project

9) (i) TWRR

$$(1+i)^3 = \frac{\cancel{38}}{30.50} \times \frac{(41.05-4.5)}{(38+6)} \times \frac{45.6}{41.05} \times \frac{47}{(45.6-2.50)}$$

$$(1+i)^3 = 1.081967 \times 0.937179 \times 1.11084 \times 1.090487$$

$$(1+i)^3 = 1.228313$$

$$(1+i) = 1.070951$$

TWRR = 7.0951 %

MWRR

$$30.5 \times (1+i)^3 + 6 \times (1+i)^{2.25} + 4.5 \times (1+i)^{1.5} - 2.5 \times (1+i)^{0.5} = 47$$

At $i = 7\%$, LHS = 46.76

At $i = 7.5\%$, LHS = 47.37

By interpolation,

MWRR = 7.206 %

(ii) Linked rate of return

Year 2011 - equation of value

$$30.5(1+i) + 6(1+i)^{0.25} = 38.50$$

$i = 6\%$, LHS = 38.42

$i = 6.5\%$, LHS = 38.57

By interpolation, $i = 6.266\%$

Year 2012 - equation of value

$$38.50(1+i) + 4.50(1+i)^{0.50} = 45$$

$i = 4.5\%$, LHS = 44.83

$i = 5\%$, LHS = 45.03

By interpolation, $i = 4.92\%$

Year 2013 - equation of value

$45(1+i) - 2.50(1+i)^{0.5} = 47$

$i = 10\%$, LHS $= 46.88$

$i = 10.5\%$ LHS $= 47.097$

By interpolation, $i = 10.276\%$

(iii) TWRR eliminates the effects of the cashflow amount and timing, and therefore gives a fairer basis for assessing the investment performance for the fund. MWRR is sensitive to the amount and timing of the net cashflows.

10) (i) Let the loan amount be ₹L

$L = 180000 \times a_{\overline{15}|} + 20000 \times (Ia)_{\overline{15}|}$ @ 12%

$L = 180000 \times 6.8109 + 20000 \times (\ddot{a}_{\overline{15}|} - 15 \times v^{15}) / i$

$L = 12,25,962 + 20,000 \times (7.6282 - 2.74044)/0.12$

$L = 12,25,962 + 20,000 \times 40.7313 = 20,40,588$

Amount of Loan from the bank = ₹ 20,40,588

Total cost of house $= \dfrac{2040588}{0.8} = 25,50,735$

(ii) The Loan outstanding at the beginning of the 9th year:-

$L = (180000 v + 180000 v^2 + \dots + 180000 v^7) + (9 \times 20000 v + 10 \times 20000 v^2 + \dots + 15 \times 20000 v^7)$

$L(9) = 3,40,000 \times a_{\overline{7}|} + 20000 \times (Ia)_{\overline{7}|}$ @ 12%

$= 340000 \times 4.5638 + 20000 \times (5.1114 - 7 \times 0.45235)/0.12$

$= 1551672 + 324158.33 = ₹\ 18,75,830.33$

Loan schedule :-

Year	Loan o/s	Loan installment	Interest	Cap. repay
9	18,75,850.33	3,60,000	2,25,102.04	1,34,897.96
10	17,40,952.37	3,80,000	2,08,914.28	1,71,085.72
11	15,69,866.65			

10) (iii) Let the new installment be ₹ P

$15,69,866.65 = P \times a_{\overline{5}|} @ 10\%$

$15,69,866.65 = P \times 3.790786$

$P = 4,14,126.87$

Extra payment by Mr. Sam if the loan continued with the first bank:

Total installment less loan outstanding at the end of 10^{th} year

$(400000 + 420000 + 440000 + 460000 + 480000) - 15,69,866.65$

$= 630,133.35$

Extra payment made by Mr. Sam if the loan transferred to second bank :-

Total installments + 1% processing fees − Loan outstanding at the end of 10^{th} year

$= 5 \times 414126.90 + 0.01 \times 15,69,866.65 - 15,69,866.65$

$= 20,70,634.337 + 15,698.665 - 15,69,866.65$

$= 5,16,466.35$

Since the extra payment (interest + processing fees) to second bank is lower than the payments to the first bank, it is advisable to Mr. Sam to transfer his loan to the second bank.

11) (i) Time weighted ~~return~~ rat of return is given by

$$(1+i)^2 = \frac{500}{400} \times \frac{(550+50)}{(500+40)} \times \frac{X}{550}$$

$(1+i)^2 = 1.44$

$X = (1.44 \times 550)/(1.086957 \times 1.1111)$

$X = 655.78$

(ii) Money weighted rate of return is given by

$460(1+i)^2 + 40(1+i)^{7/4} - 50(1+i) = 655.78$

For $i = 20\%$ LHS $= 657.4335$

For $i = 19\%$ LHS $= 646.1394$

So, MWRR $= 19.85\%$ p.a.

(iii) The effective rate of return for the year 2015 is given by solving the eq^n of value :-

The effective rate of return for the year 2016

$$(1+i) = \frac{655.78}{550} = 1.1923$$

So the linked rate of return is :-

$(1+i)^2 = 1.1923 \times 1.2044$

Hence LIRR $= 19.83\%$

(iv) When the sub-intervals chosen for calculating the linked internal rate of return concide with the intervals between net cash flows being received then the linked internal rate of return will be identical to the time weighted rate of return.

11) (vi) • Time weighted rate of return is a better measure fund manager's performance.

• Money-weighted rate of return is sensitive to the amounts and timings of the net cash flows, but fund manager dose ~~cant~~ not have any control on these

• Time-weighted rate of return is calculated using the 'growth factors' reflecting the change in fund value between the times of consecutive cash flows. Thus TWRR is independent of amount and timing of cash flows.

• Hence Time weighted rate of return is a better measure of fund manager's performance.

12) (i) The discounted payback period of an investment project is the first time when the net present value of the cash flows from the project is positive.

(ii) (a) Let us work in units of 1000 to find the DPP

$-800(1+i)^{t} - 50(1+i)^{t-1} + 100 s_{\overline{t-2}|}$ @ 7% = 0

At t = 17 the net present value is -75

At t = 18 the net present value 22.85

Hence By interpolation t = 17.767

(b) The accumulated profit after 22 years is found by accumulating the income after DPP has elapsed at 6% p.a.

$100 s_{\overline{4.233}|}$ @ 6% = 480

The accumulated amount is ₹ 480,000

(iii) The businessman may accumulate his rental income for each year at 6%

$100 s_{\overline{1}|}$ = 102.971 at the end of the year

The DPP is thus the smallest Integer t such that

$-800(1+i)^{t} - 50(1+i)^{t-1} + 102.971 s_{\overline{t-2}|}$ @ 7% ≥ 0

By trial and error t = 18 years

The balance in the businessman's account just after the transaction at time 18 years is

$-800(1+i)^{18} - 50(1+i)^{17} + 102.971 s_{\overline{16}|}$ @ 7% = 9.7714

Hence the accumulated amount after 22 years is

$9.7714(1+i)^{4} + 100 s_{\overline{4}|}$ at 6%

= 462.79

= ₹462,790

13) (i) Let $x/12$ be the monthly repayment. Solving the equation

$$X a_{25}^{(12)} = 9880 \text{ @ } 7\%$$

$$X = 821.76$$

$$x/12 = 821.76/12 = 68.48$$

Hence the monthly repayment is ₹ 68.48

(ii) The loan outstanding immediately after the repayment on 10^{th} march ~~2019~~ 2029

$$X a_{3\,4/12}^{(12)} = 648,600 \text{ @ } 7\%$$

(iii) The loan outstanding just after the repayment on 10^{th} sept. 2026 is made

$$x\, a_{8\,3/6}^{(12)} \text{ @ } 7\%$$

The loan outstanding just after the repayment on 10^{th} oct. 2026 is made

$$x\, a_{13.75}^{(12)} \text{ @ } 7\%$$

The capital repaid on 10^{th} Oct. 2026 is thus

$$x\left[a_{8\,3/6}^{(12)} - a_{13.75}^{(12)}\right] \text{@} 7\%$$

$$= x\left[\frac{v^{13.75} - v^{8\,3/6}}{i^{(12)}}\right] \text{@ } 7\% = 26.86$$

(iv) (i) The capital repayment contained in these 12 installments is

$$x\left[a_{22/3}^{(12)} - a_{19/3}^{(12)}\right] \text{@ } 7\%$$

$$= 516.20$$

(ii) The total interest in these 12 installments is

$$x - 516.20 = 305.56$$

$$x = 821.76$$