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Calculus

Assignment - 1

1.

Sol $\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$

$= \lim_{h\to 0} \frac{2(9+h^2-6h)-18}{h}$

$= \lim_{h\to 0} \frac{18+2h^2-12h-18}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2h-12)}{\cancel{h}}$

$= \lim_{h\to 0} 2(0)-12$

$= -12$

2.

Sol $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$

$= \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$\lim_{x\to 9} \frac{3x+x\sqrt{x}-27-9\sqrt{x}}{9-x}$

$= \lim_{x\to 9} \frac{-3(9-x)-\sqrt{x}(9-x)}{9-x}$

$= \lim_{x\to 9} \frac{-(3+\sqrt{x})\cancel{(9-x)}}{\cancel{9-x}}$

$= \lim_{x\to 9} -(3+\sqrt{9})$

$= -6$

4. (a) $x = -1$

$f(x) = \frac{4x+5}{9-3x}$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$

$f(x)$ exists. therefore $f(x)$ is continuous at $x = -1$

(b) $x = 0$

$f(x) = \frac{4x+5}{9-3x}$

$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$f(x)$ exists, therefore $f(x)$ is continuous at $x = 0$

(c) $x = 3$

$f(x) = \frac{4x+5}{9-3(x)}$

$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0}$

$f(x)$ does not exist, therefore $f(x)$ is discontinuous at $x=3$

7

Sol $f(t) = (4t^2-t)(t^3-8t^2+12)$

$\frac{d}{dx}f(t) = (t^3-8t^2+12)\frac{d}{dx}(4t^2-t) + (4t^2-t)\frac{d}{dx}(t^3-8t^2+12)$

$= (t^3-8t^2+12)(8t-1) + (4t^2-t)(3t^2-16t)$

$= 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 3t^3 - 64t^3 + 16t^2$

$= 8t^4 + 12t^4 - 64t^3 - t^3 - 3t^3 - 64t^3 - 8t^2 + 16t^2 + 96t - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

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8.

Sol $R(w) = \frac{3w+w^4}{2w^2+1}$

$\frac{d}{dx}R(w) = \frac{(2w^2+1)\frac{d}{dx}(3w+w^4) - (3w+w^4)\frac{d}{dx}(2w^2+1)}{(2w^2+1)^2}$

$= \frac{(2w^2+1)(3+4w^3) - (3w+w^4)(4w)}{(2w^2+1)^2}$

$= \frac{6w^2+8w^5+3+4w^3-12w^2-4w^5}{(2w^2+1)^2}$

$= \frac{4w^5+4w^3-6w^2+3}{(2w^2+1)^2}$

9.

Sol $f(x) = 2e^x - 8^x$

$\frac{d}{dx}f(x) = e^x\frac{d}{dx}2 + 2\frac{d}{dx}e^x - 8^x \ln 8$

$= 2e^x - 8^x \ln(8)$

10

Sol $y = z^5 - e^z \ln(z)$

$\frac{dy}{dz} = \frac{d}{dz}z^5 - \left[\ln(z)\frac{d}{dz}e^z + e^z\frac{d}{dz}\ln(z)\right]$

$= 5z^4 - e^z\ln(z) - \frac{e^z}{z}$

11.

Sol (a) $f(x) = (6x^2+7x)^4$

$\frac{d}{dx}f(x) = \frac{d}{dx}(6x^2+7x)^4 \cdot \frac{d}{dx}6x^2+7x$

$= 4(6x^2+7x)^3 \cdot (12x+7)$

(b) $f(t) = 5 + e^{4t+t^7}$

$$\frac{d}{dt} f(t) = \frac{d}{dt}(5) + \frac{d}{dt}\left(e^{4t+t^7}\right) \cdot \frac{d}{dt}(4t+t^7)$$

$$= 0 + e^{4t+t^7}(4+7t^6)$$

$$= e^{4t+t^7}(4+7t^6)$$

12

Sol a) $z = \ln(7-x^3)$

$$\frac{dz}{dx} = \frac{d}{dx}\ln(7-x^3)$$

$$\frac{dz}{dx} = \frac{-3x^2}{\cancel{\ln}(7-x^3)}$$

$$\frac{d^2z}{dx^2} = \frac{d}{dx}\frac{-3x^2}{7-x^3}$$

$$= \frac{(7-x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{-6x(7-x^3) - 3x^2(3x^2)}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-3x(14+x^3)}{(7-x^3)^2}$$

b) $G(v) = \dfrac{2}{(6+2v-v^2)^4}$

$$\frac{d}{dv} G(v) = \frac{(6+2v-v^2)^4 \frac{d}{dv} 2 - 2\frac{d}{dv}(6+2v-v^2)^4}{\left((6+2v-v^2)^4\right)^2} \cdot \frac{d}{dv}(6+2v+v^2)$$

$$= \frac{-2(6+2v-v^2)^3}{(6+2v-v^2)^8} \cdot 4 \cdot 2(1-v)$$

$$= \frac{-16(1-v)}{(6+2v-v^2)^5}$$

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$$= 16 \left[\frac{(6+2v-v^2)^5 \frac{d}{dv}(1-v) - (1-v)\frac{d}{dv}(6+2v-v^2)^5 \cdot \frac{d}{dv}(1-v)\frac{d(6+2v..)}{dv}}{[(6+2v-v^2)^5]^2} \right]$$

$$\cancel{= -16 \left[\frac{-(6+2v-v^2)^5 - 5(1-v)(6+2v-v^2)^4 \cdot (2)(-2)(1-v)}{[(6+2v-v^2)^5]^2} \right]}$$

$$\cancel{= -16 \frac{(6+2v-v^2)^4 [-6-2v+v^2-5+5v] \cdot (2v-2)}{(6+2v-v^2)^{10}}}$$

$$\cancel{= 16 \frac{[v^2+3v-11](2-2v)}{(6+2v-v^2)^6}}$$

$$\cancel{= 16 [2v^2+6v-22 \cdot 2v^3}$$

$$= -16 \left[\frac{-(6+2v-v^2)^5 - 5(1-v)(6+2v-v^2)^4 \cdot (2-2v)}{[(6+2v-v^2)^5]^2} \right]$$

$$= 16 \frac{\cancel{(6+2v-v^2)^4} [(6+2v-v^2) + 5(2v^2-4v+2)]}{(6+2v-v^2)^{10}}$$

$$= 16 \frac{(6+2v-v^2+10v^2-20v+10)}{(6+2v-v^2)^6}$$

$$\frac{d^2y}{dv^2} = 16 \frac{(9v^2-18v+16)}{(6+2v-v^2)^6}$$

c) $f(t) = \ln(1+t^2)$

$$\frac{d}{dt} f(t) = \frac{2t}{1+t^2}$$

$$\frac{d^2}{dt^2} = \frac{(1+t^2)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{2(1+t^2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)} = \frac{-2t^2+2}{(t^2+1)^2}$$

13.

Sol $x^3 - 3x^2 - 9x + 12 = f(x)$

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

$f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$(x+1)(x-3) = 0$

$x = -1, 3$

$f''(-1) = 6(-1) - 6 = -12 < 0$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

Max. Value = 17

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= \cancel{27} - \cancel{27} - 27 + 12$

Min. Value = -15

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$f'(x) = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$x = 1, 2$

$f''(x) = 24x - 36$

$f''(1) = 24(1) - 36 = -12$

$f''(2) = 48 - 36 = 12$

$f(1) = 4(1)^3 - 18(1)^2 + 24 - 7$

$= 4 - 18 + 24 - 7$

Max. Value = 5

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

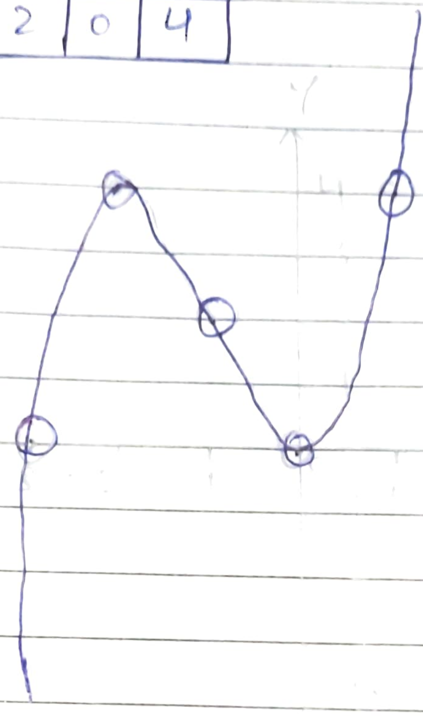
$= 32 - 72 + 48 - 7$

Min. Val. = 1

15. $f(x) = x^2(x+3)$

$y = x^2(x+3)$

x	-3	-2	-1	0	1
y	0	4	2	0	4



16. $f(x) = 3x^2 - 6x$

$y = 3x^2 - 6x$

x	-1	0	1	2	3
y	9	0	-3	0	9