

Probability & Stats

Assignment - II

Qs) 15)

$$\begin{aligned} a) \Sigma x^2 &= 925000 & ; \Sigma x &= 850 \\ \Sigma y^2 &= 372717 & ; \Sigma y &= 1737 \\ \Sigma xy &= 182560 \end{aligned}$$

$$10 \quad S_{xx} = 852750$$

$$S_{yy} = 71000.1$$

$$S_{xy} = 34915$$

$$\alpha \beta = 0.040944$$

$$15 \quad \alpha = 170.219$$

$$\begin{aligned} \therefore Eq^n &= \alpha + \beta x \\ &= 170.219 + 0.040944x \end{aligned}$$

$$b) \therefore \beta = 0.040944$$

i.e β is close to 0

Hence it is not significant.

$$\begin{aligned} 2) 15) \Sigma x^2 &= 12666.58 & ; \Sigma x &= 385.2 \\ \Sigma y^2 &= 119026.9 & ; \Sigma y &= 1162.5 \\ \Sigma xy &= 38191.41 \end{aligned}$$

$$S_{xx} = 301.66$$

$$30 \quad S_{xy} = 875.16$$

$$S_{yy} = 6409.7125$$

$$\beta = 2.9011$$

$$\alpha = \cancel{51.624} \quad 3.7496$$

$$\therefore \hat{y}^n = 3.7496 + 2.9011x$$

$$ii) \sigma^2 = 387.0744$$

$$\therefore \hat{\beta} \sim N(\beta, \sigma^2/S_{xx})$$

$$\therefore 95\% \text{ C.I.}$$

$$= \beta \pm t_{\alpha/2, n-2} \left(\frac{\sigma}{\sqrt{S_{xx}}} \right)$$

$$= 2.9011 \pm \cancel{(0.6955)} \cancel{(-22.2861)} (1.132)$$

$$= \cancel{(-46.752, -22.554)}$$

$$= (0.379, 5.423)$$

$$iii) y \sim N\left(\hat{\alpha} + \beta x, \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right) \sigma^2$$

$$i.e. y \sim N(119.7936 + \dots)$$

$$C.I. = \hat{y}_0 \pm t_{\alpha/2, n-2} \sqrt{\left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right] \sigma^2}$$

$$= 119.7936 + \cancel{2.228} \cancel{(-8.9534)} \quad 22.143$$

$$197.6504 \text{ to } 141.9306$$

Q3) i) In city A & city B, it takes longer while travelling in peak hours while in city C and time diff is similar where as in city D, we might reach early.

10/10

6) $\Sigma x = 39$

$\Sigma x^2 = 207$

$\Sigma y = 1562$

$\Sigma y^2 = 40508$

$\Sigma xy = 2385$

5 $S_{xx} = 54.9$

~~S_{yy}~~ $S_{yy} = 8923.6$

~~S_{xy}~~ $S_{xy} = 661.2$

i) 10

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = \frac{661.2}{\sqrt{54.9 \cdot 8923.6}} = 0.944662$$

ii) 15

$\beta = \frac{12.0437}{12.0437} = 1$

$\alpha = -577.713$

$$\therefore \hat{y} = -577.713 + 12.0437x$$

iii) 20

$SS_{TOT} = SS_{REG} + SS_{RES}$

$SS_{REG} = \frac{S_{xy}^2}{S_{xx}}$

$SS_{RES} = S_{yy} - \frac{S_{xy}^2}{S_{xx}}$

25

$$= \frac{661.2^2}{54.9} = 7963.30$$

$$= 8923.6 - 7963.30 = 960.295$$

30

iv)

$$R^2 = \frac{S_{xy}^2}{S_{xx} \cdot S_{yy}} = \frac{661.2^2}{54.9 \cdot 8923.6} = 0.8928$$

As R^2 is between 0 to 1, it is significant

$$7) \sum x = 174.13, \sum x^2 = 7545.9$$

$$\sum y = 197.84, \sum y^2 = 4747.45$$

$$S_{xx} = 4789.4221; S_{yy} = 1189.2076; S_{xy} = 2176.84$$

$$i) \beta = 2.20017$$

$$\alpha = -16.8432$$

$$\therefore \hat{y} = -16.8432 + 2.20017x$$

$$ii) H_0: \beta = 0$$

$$H_1: \beta \neq 0$$

$$\hat{\sigma}^2 = 199.8121$$

$$t_{cal} = \frac{\hat{\beta} - \beta}{\hat{\sigma}^2 / \sqrt{S_{xx}}} = 10.77176$$

$$t_{tab} = 2.20017$$

$$\therefore t_{cal} > t_{tab}$$

$\therefore$ Reject H_0

$\therefore$ There is a linear relationship between x and y

$$iii) \therefore r = 0.912128$$

$$9) \quad \begin{aligned} \sum x &= 45 & \sum x^2 &= 277.5 \\ \sum y &= 644 & \sum y^2 &= 43956 & \sum xy &= 2602 \end{aligned}$$

$$S_{xx} = 75 \quad ; \quad S_{yy} = 2482.4 \quad ; \quad S_{xy} = -296$$

i) looking at the plot, I can see a negative correlation between x and y

$$ii) \quad r = \frac{S_{xy}}{S_{xx} S_{yy}} = -0.686$$

$\therefore$ There is a strong negative co-relation

$$iii) \quad \tanh^{-1} r \sim N(\tanh^{-1} \rho, \frac{1}{n-3})$$

$$H_0 : \rho = 0$$

$$H_1 : \rho < 0$$

$$\frac{\tanh^{-1} r - \tanh^{-1} \rho}{\sqrt{\frac{1}{n-3}}} = Z_{cal}$$

$$\therefore Z_{cal} = -2.22338$$

$$Z_{tab} = \frac{Z_{0.5}}{Z_{0.5}} = \frac{1.96}{2.5758}$$

at 5% LOS reject H_0

at 1% LOS do not reject H_0

$\therefore$ There is negative co-relation at 5% LOS

$\therefore$ There is no co-relation at 1% LOS