Unit - 1.2

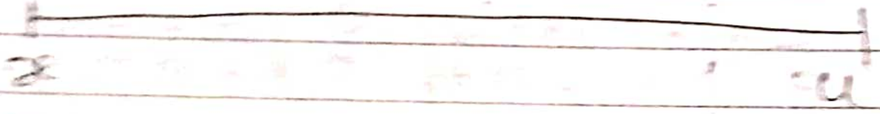
Assignment - 1

1) $T_x = K_x + S_x$

${}_uq_x = u \cdot q_x$ — To show

$0 \le x < u < 1$

${}_uq_x = {}_uq_x \cdot \mu_{x+u}$



So, x = is current age
u = ~~is~~ $0 < u < 1$

${}_uq_x = {}_rq_x \lim_{u \to 1} \mu_{x+u}$

$= \cancel{{}_rq_x}\, u$

$= Pr[K_x = 0 ,\ S_x \le u]$
$= Pr[K_x = 0] \times Pr[S_x \le u]$
as they are independent.

$Pr[S_x \le u] = \int^u 1\, dx = u$
uniformly distributed.

${}_uq_x = u \cdot q_x \quad [\because K_x = 0 \Rightarrow q_x]$

Q3. a) left censoring:- It prevent us from from knowing when entry into the state that we wish to observe took place

b) Right censoring:- cut short the observation in the process of investigation

c) interval censoring:- event of interest fell between some interval of time

d) informative censoring:- when censored data set provide the information is called as informative censoring.

Right censoring ~~cens~~, interval censoring, and informative censoring is present in the data.

Right censoring:- policy maturity date. It state that policy got matured before the death of the individual

Interval censoring:- calender year in which policy is surrendered. It state that ~~one~~ ~~2~~. lack of i

Informative censoring:- date of exist, in case of exist due to any reasons, ~~by this~~ which means, Policyholder might find himself healthy, as compared to the others

Q2) a) Central exposure to Risk:-
exposure to Risk from the start of the investigationd to end of investigation (when person died).

So, exposure to Risk.

1-06-2000 to 25-10-2000

In term of day

~~= 30 + 31 + 30 + 31 + 31~~

= 30 + 31 + 31 + 30 + 25

= 147 days

= 147 / 7 = 21 weeks.

b) Initial exposure to Risk:-
exposure to Risk from the start of investigation to the end of year of age (not the investigation) (till Next birthday).

1-06-2000 to 31-05-2001

= 30 + 31 + 31 + 30 + 30 + 30 + 31 + 30 + 28 + 31 + 30 + 31

= 363 / 7

= 51.85

= 52 weeks

Q.4 Complete expectation of life ; in terms of probability of survival.

$$E(T_x) = \mathring{e}_x = \int_0^{\infty} {}_tP_x = \int_0^{\omega-x} {}_tP_x$$

$$S(x) = P(T_x > t) = {}_tP_x$$

Expectation life at age x. OR
How many ~~live~~ will a life is expected to live given that current age is x.

[ii] curtate expectation of life.

$$e_x = \sum_{K=1}^{\infty} {}_KP_x =$$

~~wt~~

$$\text{So, } {}_KP_x = e^{-\int_0^K \mu \, dk}$$

$$= e^{-0.0325K}$$

$$e_x = \sum_{K=1}^{\infty} e^{-0.0325K} = \frac{e^{-0.0325}}{1 - e^{-0.0325}}$$

$$e_x = 30.27$$

(iii) $_9p_{36} = P(T_{36} > 9)$

$$_9p_{36} = e^{-\int_0^9 \mu \, dt}$$

$$_9p_{36} = e^{-\int_0^9 0.0325 \, dt}$$

$$= e^{-0.0325[t]_0^9}$$

$$_9p_{36} = e^{[illegible]} = 0.7463$$

$$_9p_{36} = 74.63\%$$

(iv) [illegible] the median of the lifetime T [illegible]

[illegible] the lifetime probability

$$_xp_0 = 0.5$$

$$_xp_0 = e^{-\int_0^x 0.0325} = e^{-0.0325x}$$

$$e^{-0.0325x} = 0.5$$

$$-0.0325x = \ln(0.5)$$

$$x = \frac{-\ln(0.5)}{(0.0325)}$$

$$\boxed{x = 21.33}$$

Q5 i) Gompertz law is appropriate model for middle age and older age deaths

ii) probability of survival from age x to $x+t$ is equal to

$${}_tP_x = (g)^{c^x(c^t - 1)}$$

$${}_tP_x = \exp\left[-\int_0^t \mu_{x+s}\, ds\right]$$

$$\mu_x = Bc^x$$

$${}_tP_x = \exp\left[-\int_0^t Bc^{x+s}\, ds\right]$$

$$\int_0^t B c^{x+s}\, ds = \int_0^t B c^x e^{s\log c}\, ds = \frac{Bc^x}{\log c}\left[e^{s\log c}\right]_0^t$$

$$\frac{Bc^x}{\log c}\left[e^{s\log c}\right]_0^t = \frac{Bc^x}{\log c}\left[c^s\right]_0^t = \frac{Bc^x}{\log c}\left[c^t - 1\right]$$

$${}_tP_x = \exp\left(\log g\, c^x\left[c^t - 1\right]\right) = \left(e^{\log g}\right)^{c^x(c^t-1)}$$

$$= g^{c^x(c^t-1)}$$

Q5) I] female smoker

b) Survival function of

male smoker aged 30.

$$h_j(t) = h_0(30) \times \cancel{e^{0.01}} \exp\{(0.01 \times 0 + 0 - 0.05 \times 1)\}$$

$$h_i(30) = h_0(30) \times e^{-0.05}$$

female smoker age 40 at entry

$$h_i(t) = h_0(40) \times \exp\{(0.01(40-30) + 0 - 0.05 \times 0\}$$

$$= h_0(40) \times e^{(0.01 \times (10))}$$

$$h_0(40) \times e^{0.1}$$

so Relatively

$$h_i(t) = \frac{e^{-0.05}}{e^{0.1}} = 0.86070$$

male smoker age 30 has high survival probabilities as compare to female smoker age 40.

But, $S(t) = \exp\left(-\int_0^t h(s)\,ds\right)$ here,

$$S_j(t) = \left(S_i(t)\right)^{0.86070}$$

which implies that

$S_j(t) > S_i(t)$ for all $t > 0$

c] male smoker aged 30.

$$h_j(t) = h_0(t) \times \exp\{0 + 0 - 0.05 \times 1\}$$

$$= e^{-0.05}$$

male smoker aged 40 at entry

$$h_i(t) = h_0(t) \cdot \times \exp\{0.01 \times 10 + 0 - 0.05 \times 1$$

$$= h_0(t) \times e^{(0.1 - 0.05)}$$

$$= h_0(t) \times e^{0.05}$$

$$\frac{h_j(t)}{h_i(t)} = \frac{e^{-0.05}}{e^{0.05}} = 0.9048$$

Q 7] i] The most appriate rate interval to use is the policy year rate interval starting on the policy anniversary where lives are aged x next birthday.

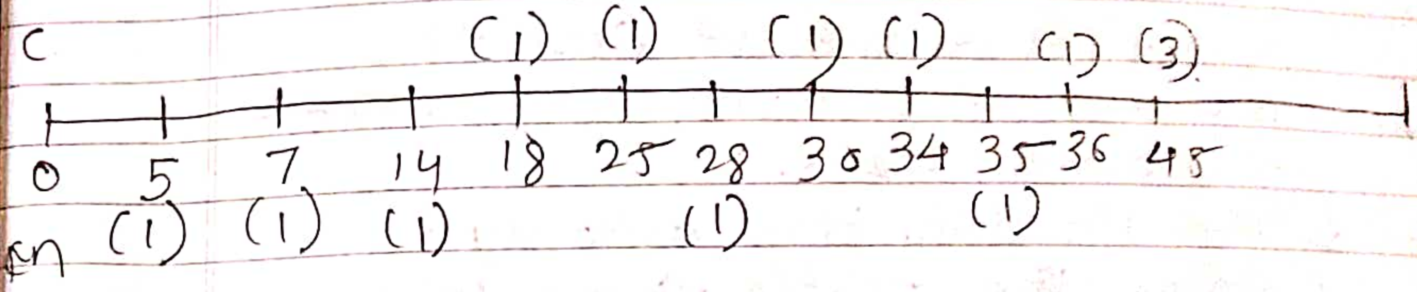
Q.8 i] Informative censoring :- patient those who had taken discharged might be in better health.

Right censoring :- end of observation after 45 days but the patient is still surving.

Type-I censoring :- Study end at predetermined date.

ii) Those patient who were feeling better took the discharge from the Hospital, this is informative censoring.

iii) Kaplan meier estimate of survival fun^n.



j	t_j	d_j	n_j	$\lambda_j = \frac{d_j}{n_j}$	$1-\lambda_j$	$\hat{S}(t) = \prod(1-\lambda_j)$
1	5	1	13	1/13	0.9230	0.9230
2	7	1	12	1/12	0.9167	0.8461141
3	14	1	11	1/11	0.909	0.76912
4	28	1	8	1/8	0.875	0.67298
5	35	1	5	1/5	0.8	0.538384

Value of Survival Function at end of investigation is **0.54**

Assumption:-
1] The censoring happens just after deaths.
2]. Ignore any admission period before the start of the investigation.

iv] Comments]

1] Survival of patient from the infection who given treatmentt is around 54 %.

2] ~~Th~~ Hospital has also ignored ~~to pe~~ the people those who have died in two weeks of observation

3] They assume that patient after the end of investigation will survive

Q9 1] $q_x = 0.3$

Calculate :- m_x (weighted average force of mortality).

a) for UDD :- $\Rightarrow$ ${}_tq_x = t \cdot q_x$ or

${}_tp_x = t \cdot p_x$.

$$\cancel{\int_0^1 {}_tp_x \, dt = \int_0^1 (1 - {}_tq_x) \, dt}$$

$$\cancel{= [t -}$$

$$\int_0^1 t \cdot q_x = \int_0^1 (1 -$$

a) for UDD.

$tP_x = t \cdot p_x$

$\int_0^1 tP_x = \int_0^1 (1 - tq_x) = \int_0^1 (1 - t \cdot q_x)\,dt$

$= \left[t - q_x \frac{t^2}{2}\right]_0^1$

$= \left[1 - q_x \frac{1}{2}\right] = [1 - 0.5q_x]$

$q_x = 0.3$, we have.

$m_x = \frac{q_x}{t_i P_x}$ (weighted average of mortality) $= \frac{d_x}{l_x}$

$\frac{\text{deaths}}{\text{alive}}$

$= \frac{0.3}{1 - 0.5 \times 0.3}$

$= \frac{\cancel{0.3}}{}$

$= 0.3529$

b) Force of mortality.

$tP_x = e^{-\mu}$

$m_x = \mu$

$q_x = 1 - e^{-\mu}$

$$\int_0^1 {}_tp_x \, dt = \int_0^1 e^{-\mu t} \, dt$$

$$= -\frac{1}{\mu}\left[e^{-\mu t}\right]_0^1$$

$$= -\frac{1}{\mu}\left[e^{-\mu} - e^0\right]$$

$$= -\frac{1}{\mu}\left[e^{-\mu} - 1\right]$$

$$= \frac{-1}{-\mu}\left(1 - e^{-\mu}\right)$$

$$= \frac{1}{\mu}\left[1 - e^{-\mu}\right]$$

$$= \frac{q_x}{\mu}$$

$$q_x = 1 - e^{-\mu}$$

$$1 - q_x = e^{-\mu}$$
$$\ln(1 - q_x) = -\mu$$
$$\ln(1 - 0.3) = -\mu$$
$$\mu = -\ln(1 - 0.3)$$
$$\mu = 0.35667$$

(Q1a) i] under cox model each individual's hazard is proportional to the baseline hazard with the constant of proportionality depending on certain measurable quantities called covariates. Hence the model is called proportional hazards model.

ii) baseline hazards refers to annual policy taken through online channel and where premium are paid by direct debit.

iii)

$h_p(t) = h_0(t) \times e^{-\beta_1 x_1 +}$

$e^{(\beta_r + \beta_m)} = 4/3$ — ①

$\exp(\beta_D \times 1) / \exp[(\beta_r \times 1)] = 1$ — ②

$\exp(\beta_m \times 1) / \exp[(\beta_D \times 2)] = 0.75$ — ③

Substituting from ② and ①

$\exp(\beta_D + \beta_m) = 4/3$

$\exp(\beta_D) \times \exp(\beta_m) = 4/3$

$\beta_D = 0.19179$
$\beta_r = 0.19179$
$\beta_m = 0.0919$

(Q15) I) q_x is probability that life aged exactly x will die before reaching exact age $x+1$, and is called the initial rate of mortality.

m_x is called the central rate of mortality & represent the probability that a life alive betn ages x & $x+1$ dies.

$$m_x = \frac{q_x}{\int_0^1 {}_tP_x \, dt}$$

$$m_x = \frac{q_x}{1 - \frac{q_x}{2}}$$

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