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Assingment (Financial Mathematics)

Q1 Nominal rate of discount per anum convertible quarterly is 8%.

1] calculate equivalent force of intrest

$= \delta = \ln(1+i)$

$\delta = \ln(1+0.08)$

$\delta = 0.07696.$

2] calculate effective rate of intrest per anum

$= i = \cancel{\left(1+\frac{i^{(12)}}{12}\right)^{12} - 1}$ $\quad i^{(1)} = 1[(1+i)^{\frac{1}{1}} - 1]$

$= 1[(1+0.08)^{\frac{1}{1}} - 1]$

$= \cancel{\left(1+\frac{0.08^{12}}{12}\right)^{12} - 1} = \boxed{0.79}$

$= \cancel{0.082999}$

3] calculate nominal rate of discount per anum convertible monthly.

$= \cancel{1 \times \left(1 - \frac{d^{(12)}}{12}\right)} = d^{(12)} = 12[1-(1+0.08)^{\frac{1}{12}}]$

$\boxed{= 0.07720836}$

$= \cancel{1 \times \left(1 - \overset{66}{0.08}\right)}$ $\quad \cancel{1 \times (1 - 0.6666)}$

$= \cancel{0.33400}.$

$=$

Q2 The force of intrest $\delta(t) = 0.004t + 0.0002t$ for all t

1] find Present value of sum rs 10,000 payable at the end of 10 years from now

$= \cancel{AV_{10}} = \cancel{1000} \times \cancel{\int_0^5 (0.04t + 0.0002t)\, dt}$

$\cancel{1000 \times e}$

$= \cancel{1000 \times e^{[0.04t + 0.0002t}}$

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$$PV_{10} = 1000 \times e^{\int_0^{10} (0.04t - 0.0002t)\, dt}.$$

b] Find constant annual effective rate of intrest over a 10 year period

$$= e^{\delta} - 1$$

$$= e^{0.04t + 0.0002t} - 1$$

$$= 0.04101$$

3] a] explain the relationship $d = iv$ by genral reasoning where d is the effective annual rate of discount and i is the effective annual rate of intrest

→ Assume that a person borrows 1 at an effective rate of discount d

Then in effect the original principal is $1-d$ & the amount of intrest is d. However from the basic defination of i as the ratio of the amount of intrest to the principal we obtain

$$i = \frac{d}{1-d}$$

$$d = \frac{i}{1+d}$$

ii] Calculate nominal rate of discount per anum convertible half yearly which is equivalent to

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a] effective rate of discount 2.25% per quarter

$= \cancel{(1 - 2.25\%)}$ $d^{(2)} = 2[1-(1+0.0225)^{\frac{1}{2}}]$

$= \cancel{0.9775}$ $d^{(2)} = 1.7000$.

b] Nominal rate of discount of 5% convertible every 2 yearly

$\cancel{d^{(24)} = 24[1-[1+0.05\,d^{(24)}]} = \cancel{24[1-[1+0.05]^{\frac{1}{24}}]}$

$= \cancel{0.051238}$. $d^{(24)} = 24[1-(1+0.05)^{\frac{1}{24}}]$

$= 0.051238$.

iii] A 91 day government bill provides the investor with an annual effective rate of return of 5% calculate the annual simple rate of discount at which the bill is discounted

$$d = \frac{i}{1+i} = d = \frac{0.05}{1+0.05} = 0.0476190$$

4] i] explain the relationship $d = iv$ by genral reasoning where d is the effective annual rate of discount and i is the effective annual rate of intrest

→ Assume that a person borrows 1 at an effective rate of discount d. Then in the effect the original principal is $1-d$ and the amt of intrest is d. However from the basic defination of i as the ratio of the amt of intrest to the principal we obtain

$$i = \frac{d}{1-d} \qquad d = \frac{i}{1+d}$$

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ii) given $i = 0.08$ calculate

a] $d^{(12)}$

$$d = \frac{1}{1+i}$$

$$d^{(P)} = P\left[1 - (1+i)^{-\frac{1}{P}}\right]$$

$$d^{(12)} = 12\left[1 - (1 + 0.08)^{-\frac{1}{12}}\right]$$

$$d^{(12)} = 0.077208361$$

b] $i^{(365)}$

$$i^{(P)} = P\left[(1+i)^{\frac{1}{P}} - 1\right]$$

$$i^{(365)} = 365\left[(1+0.08)^{\frac{1}{365}} - 1\right]$$

$$i^{(365)} = 0.076969155.$$

c] δ

$$= \ln(1+i)$$

$$= \ln(1+0.08)$$

$$= 0.076961041$$

d] $i^{(0.5)}$

$$i^{(P)} = P\left[(1+i)^{\frac{1}{P}} - 1\right]$$

$$i^{(0.5)} = 0.5\left[(1+0.08)^{\frac{1}{0.5}} - 1\right]$$

$$i^{(0.5)} = 0.832000$$

5] The rate of discount per anum convertible quarterly is 6%. calculate

a) The equivalent rate of intrest per anum convertible half yearly.

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$$i^{(2)} = 2\left[(1+i)^{\frac{1}{2}} - 1\right]$$

$$i^{(2)} = 2\left[(1+0.06)^{\frac{1}{2}} - 1\right]$$

$$i^{(2)} = 0.59126028.$$

b] equivalent rate of discount per anum convertible monthly.

$$d^{(12)} : 12\left[1 - (1+i)^{\frac{1}{P}}\right]$$

$$d^{(12)} : 12\left[1 - (1+0.06)^{\frac{1}{12}}\right]$$

$$d^{(12)} : 0.684106.$$

ii] On 15th april 2005 Amit borrowed rs 2,00,000 to be repaid one year later by single payment of 2,20,000 Amit repaid the loan early on 17th July 2005

a] Find the sum paid by Amit to terminate the contract assuming that the intrest is reduced proportinately for early settlement

6] i] The manufacturer of a certain toy sells to retailers on either of the following terms

a) cash payement : 30% below recomended retail price

b] Six months credit 25% below recomended retail price

Find effective annual rate of discount offered by the manufacturer to the retailers who pay cash as opposed to those retailers who accept the credit terms

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$d = \frac{i}{1+i}$ $d = \frac{0.30}{1+0.30}$ $= 0.230769.$

ii] Find

$d^{(12)} = 12 [1 - (1+0.230769)^{\frac{1}{12}}]$

$d^{(12)} = 0.209388.$

$i^{(2)} = 2 [(1+0.230769)^{\frac{1}{12}}]$

$i^{(2)} = 2.034898.$

$\delta = 1 - e^{-1}$

$\delta = 0.6321205.$

7] If an investment fund offers to increase INR 20,000 to INR 26000 in 17 months

a] calculate the following

1] Nominal rate of intrest per anum convertible quarterly

$i^{(3)} = \left(1 + \frac{i^{(3)}}{3}\right)^3$

$i^{(3)} = \left(1 + \frac{2.3^{(3)}}{3}\right)^3$

$i^{(3)} = 1.292216$

2] Nominal rate of discount per anum convertible half yearly

$d^{(2)} = \left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(\frac{1 - 2.3^{(2)}}{2}\right)^2$

$= 2.706025.$

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iii] simple rate of intrest per anum.

$= 20000(\cancel{1+2\cdot 3})\ (1+2\cdot 3\times 17)$

$= 8020.00.$

8] explain with the help of a graph for an effective intrest rate 8% p.a the relation-ship between equivalent nominal rates of intrest convertible pthly ($i^{(p)}$) and p.

effective rate of intrest 8% p.a

$i^{(P)} = P((1+i)^{\frac{1}{P}} - 1)$

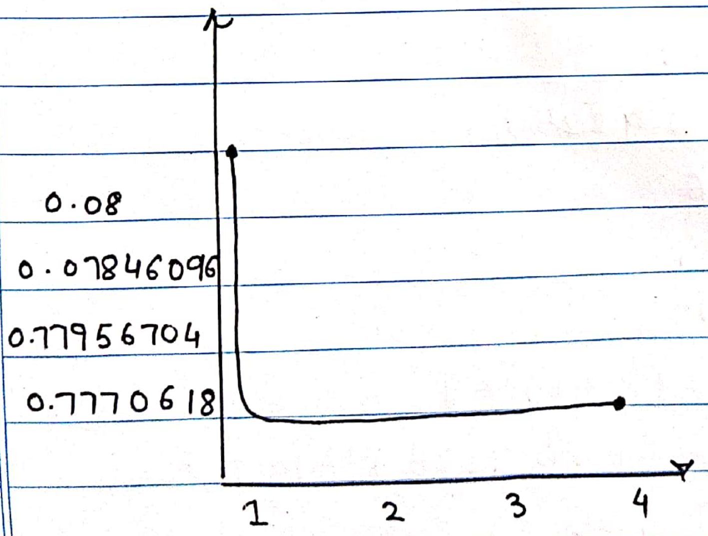
$i^{(1)} = 1((1+0.08)^{\frac{1}{1}} - 1) = 0.08$

$i^{(2)} = 2((1+0.08)^{\frac{1}{2}} - 1) = 0.078460969$

$i^{(3)}\ \ 3((1+0.08)^{\frac{1}{3}} - 1) = 0.077956704.$

$i^{(4)} = 4((1+0.08)^{\frac{1}{4}} - 1) = 0.07770618.$

$$i^{(P)} = P[(1+i)^{\frac{1}{P}} - 1]$$



□ from the above graph we can see as the values of p increases the nominal intrest rate i.e $i^{(P)}$

starts to fall

2] But after a certain value as the corresponding gets more frequent the decrease in $i^{(p)}$ becomes less significant.

9] i] Define force of intrest

→ Measure of interest at individual moments of time is called the force of intrest

ii] If $i^{(2)} = 0.0775$

calculate i

$$0.0775 = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$0.0775 = (1+i)^2$$

$$0.0775 - 1 = i$$

$$i = 0.9225.$$

calculate δ

$$\delta = \ln(1+i)$$

$$\delta = \ln(1+0.9225)$$

$$\delta = 0.653626.$$

calculate $d^{(12)}$

$$d^{(12)} = 12\left[1-(1+i)^{\frac{1}{12}}\right]$$

$$d^{(12)} = 12\left[1-(1+0.9225)^{\frac{1}{12}}\right]$$

$$d^{(12)} = 0.671755$$

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$i^{(1/2)} = 1/2\ ((1+i)^{\frac{1}{1/2}} - 1]$

$i^{(1/2)} = 1/2\ ((1+0.9225)^{\frac{1}{1/2}} - 1]$

$i^{(1/2)} = 0.5\ [(1+0.9225)^{\frac{1}{0.5}} - 1]$

$i^{(1/2)} = 1.348003$.

10] $\delta(t)$ the force of intrest per anum at time t years is defined as

$$\delta(t) = \begin{cases} 0.08 \text{ for } 0 \leq t \leq 5 \\ 0.06 \text{ for } 5 \leq t \leq 10 \\ 0.04 \text{ for } t \geq 10 \end{cases}$$

using $\delta(t)$ derive the expression for $v(t)$ the present value of 1 due at time t

$= PV = c \cdot v(t)$

$= 1 \times \exp\left[-\left[\int_0^5 0.08\,dt + \int_5^{10} 0.06\,dt + \int_{10}^{t} 0.04\,dt\right]\right]$

$= \exp\left[-[0.08t)_0^5 + (0.06t)_5^{10} + (0.04t)_{10}^{t}\right]$

$= \exp\left[-[0.4 + 0.3 + 0.4t - 0.4]\right]$

$= \exp - [0.04t + 0.3]$

$\therefore = v(t) = e^{-0.04t - 0.3}$.

11] A 9 month loan repayable by a single payement of 50,000 is is issued at rate of commercial discount 18%. p.a. What amt was initally lent to borrower ?

PV = fund value × discounting factor

$$= 50{,}000 \left(\frac{1}{1+i}\right)^9$$

$$= 50{,}000 \left(\frac{1}{1+0.18}\right)^9$$

$$PV = 11272.80355.$$

b] If $\delta = 6\%$ find $d(12)$

$$d(12) = 12[1-(1+0.06)^{\frac{1}{12}}]$$

$$d(12) = 0.058410$$

ii] If $d(4) = 6$ find

1] $i(12)$

$$i(12) = 12[(1+0.06)^{\frac{1}{12}}-1]$$

$$i(12) = 0.58470$$

2] $i(4)$

$$i(4) = 4[(1+0.06)^{\frac{1}{4}}-1]$$

$$i(4) = 0.58695385.$$

12] d] explain in words why relationship $d = iv$ holds true

→ Intrest earned on invrestment of 1 paid at begining of the period is d. Intrest earned on an investment of 1 paid at the end of period is i, therefore if we discount i from the end of the period to the begining of the period with the discount factor v we obtain id $\therefore$ $d = iv$.

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12] A deposit of 7049 is accumulated at the following rates of intrest 6% per anum convertible monthly for first two years followed by 1.5% per quarter for next two and a half year followed by rate of discount of 6% per anum convertible monthly for next one and a half year convertible monthly. Calculate accumulated amount after 6 years.

$$AV_6 = 7049\,(0,6)$$

$$AV_6 = 7049\,(1+i)^{0-6}$$

$$AV_6 = 7049\,(1+0.135)^{6}$$

$$AV_6 = \cancel{7049}\ 15069$$