NMA Assignment

Date / /
Page ____

1) $r = 12.65$ cm

$r_{true} = 12.5$ cm

$$\text{Area} = \pi r^2$$
$$= 3.14(12.65)^2$$
$$= 502.47065 \text{ cm}^2$$

$$\text{Area}_{true} = 3.14(12.5)^2$$
$$= 490.625 \text{ cm}^2$$

(i) Absolute error $= \text{Area} - \text{Area}_{true}$

$$= 502.47065 - 490.625$$
$$= 11.84565 \text{ cm}^2$$

(ii) Relative error $= \dfrac{\text{Absolute error}}{\text{Actual value}}$

$$= \frac{11.84565}{490.625}$$

$$= 0.024 \text{ or } 2.4\%$$

(iii) Percentage error $= 2.4\%$

2)

x	4	4.5	5.5	6
$f(x) = \log x$	0.60206	0.6532125	0.7403627	0.7181513

→ (a) Using Langrange's interpolation

$$\log 5 = \frac{(5-4.5)(5-5.5)(5-6)}{(4-4.5)(4-5.5)(4-6)} \times 0.60206 +$$

$$= \frac{(5-4)(5-5.5)(5-6)}{(4.5-4)(4.5-5.5)(4.5-6)} \times 0.6532125 +$$

$$+ \frac{(5-4)(5-4.5)(5-6)}{(5.5-4)(5.5-4.5)(5.5-6)} \times 0.7403627 +$$

$$\frac{(5-4)(5-4.5)(5-5.5)}{(6-4)(6-4.5)(6-5.5)} \times 0.7781513$$

$= \log 5 = -0.100343 + 0.4354475 + 0.493575 - 0.12969$

$= \boxed{\log 5 = 0.699017}$

(b) Using $x = 4.5$ and $x = 5.5$

$$\log 5 = \frac{(5-5.5)}{(4.5-5.5)} \times 0.6532125 + \frac{(5-4.5)}{(5.5-4.5)} \times 0.740366$$

$= 0.326606 + 0.37018$

$= 0.696786$

3) $f(x) = x^4 - x - 10$

$\rightarrow$ $a = 1.5$, $b = 2$

$= f(a) = -6.4375$

$f(b) = 4.$

= Bisection Method,

$$x_1 = \frac{a+b}{2} = \frac{3.5}{2} = 1.75$$

$$f(x_1) = (1.75)^4 - 1.75 - 10$$
$$= -2.3711$$

$$x_2 = \frac{1.75+2}{2} = 1.875$$

$$f(x_2) = (1.875)^4 - 1.875 - 10$$
$$= 0.4846$$

$$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$$

$$f(x_3) = (1.8125)^4 - 1.8125 - 10$$
$$= -1.02625$$

$$x_4 = \frac{1.8125 + 1.875}{2}$$
$$= 1.84375$$

$$f(x_4) = -0.2877$$

$$x_5 = \frac{1.84575 + 1.875}{2} = 1.859375$$

$$f(x_5) = 0.0934$$

$$x = 1.859375$$

Date / /
Page

4) $f(x) = x^3 - x - 1$

x	0	1	2
$f(x)$	-1	-1	5

$= f'(x) = 3x^2 - 1$

$= x_0 = 2$

$= f(x_0) = 5$

$= f'(x_0) = 74$

$= x_1 = 1.93243$

$f(x_1) = 4.28384.$

$f'(x_1) = 10.20286$

$= x_2 = 1/2 - \frac{1.375}{0.25} = 1.512563$

$= f(x_2) = \frac{0.447952}{6}$

$= f'(x_2) = 5.86354.$

$= x_3 = 1.512563 - \frac{0.947952}{5.86354} = 1.350894$

$= f(x_3) = 0.1143727$

$= f'(x_3) = 4.47473$

$= x_4 = 1.350894 - \frac{0.1143727}{4.47473} = 1.31342.$

$= f(x_4) = -0.04767$

Date / /
Page

$= f'(x_4) = 4.175216$

$= x_5 = 1.31342 - \frac{0.04767}{4.175216} = 1.324838$

$= f(x_5) = 0.00051474$
$= f'(x_5) = 4.265587$

$= x_6 = 1.324838 - \frac{0.0001474}{4.265587} = 1.324717$

$= f(x_6) = 0.00000268$
$= f'(x_6) = 4.264625$

= The roots of this equation $x^3 - x - 2$ is $\underline{\underline{1.324717}}$

If $A^2 = A$

$\Rightarrow 7A - (1+A)^3$
$= 7A - \{1 + A^3 + 3A^2 + 3A\}$
$= \cancel{7A} - 1 - A^3 - 3A^2 + 4A$
$= -1 - \cancel{A}^2 - 3\cancel{A}^2 + 4\cancel{A}^2 \quad (\because A^2 = A)$
$= \underline{\underline{-1}} \quad \Rightarrow 1$

5) $A \rightarrow$ Square Matrix
$A^2 = A$
$7A - (I + A)^3 = 7A - (I + A^3 + 3A^2I + 3AI^2)$
$= IA = A$ and $I^3 = I$

$= 7A - \{ I + A^2 + 3IA + 3IA \}$

$= 7A - \{ I + A + 6A \}$

$= \cancel{7A} - I - \cancel{7A}$

$= -I$

$=$

So,

$$7A - \{ I + A \}^3 = -I.$$

6) $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$\rightarrow |A| = 1(0-6) + 1(-4-0) + 2(4-0)$

$= -6 - 4 + 8$

$= -2 \neq 0$ hence

$=$ So, A^{-1} does not exist

$$\text{Adj } A = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$= A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ 2 & -1/2 & 2 \end{bmatrix}$$

7) $\vec{A} = 8\hat{i} - 9\hat{j}$

$\vec{B} = 7\hat{i} - 9\hat{j}$

→ By Scalar product,

$= \vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

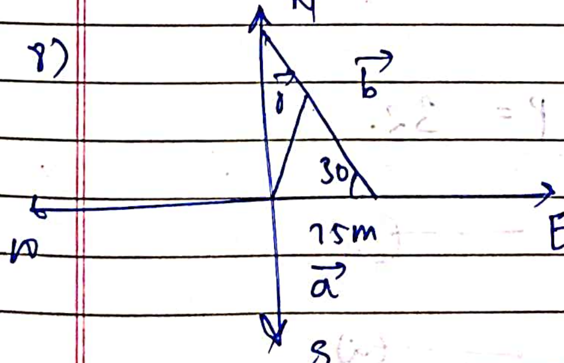
$= (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j}) = \sqrt{8^2 + (9)^2} \cdot \sqrt{7^2 + (-9)^2} \cos\theta$

$= 56 + 81 = \sqrt{145 \times 130} \cos\theta$

$= \cos\theta = 0.997849$

$= \{\theta = 3.7587\} (3.758683119)$

8)



$\vec{R} = \vec{a} + \vec{b}$

$\vec{R}^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$

$= |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos\theta$

$= 75^2 + 25^2 + \cancel{2}(75)(25)\frac{\sqrt{3}}{\cancel{2}}$

$= 9497.57$

$\therefore \vec{R} = 97.46 \text{ m}$

9) The three digit numbers when divided by 3 leaves remainder 2, forms an AP.

→ AP = 101, 104, 107, ... 998

Using ~~an~~ $a_n = a + (n-1)d$

$\Rightarrow 998 = 101 + (n-1)3$

$= \left(\frac{998 - 101}{3}\right) + 1 = n$

$= \boxed{n = 300}$

$= S_n = \frac{n}{2}(a + a_n)$

$= \frac{300}{2}(101 + 998)$

$=$ ~~1044~~ 164850

10) 6th term of G.P = 32.

→ $ar^5 = 32$ —— (1)

$= a_8 = 128$

$ar^7 = 128$ —— (2)

$=$ (1) / (2) → $\frac{\cancel{a}r^5}{\cancel{a}r^7} = \frac{\cancel{32}}{\cancel{128}}$ 4.

$= \frac{1}{r^2} = \frac{1}{4}$

= $r = 2$

11) Binomial Theorem $(4+3x)^5$

→ $5C_0(3x)^5 4^0 + 5C_1(3x)^4(4)^1 + 5C_2(3x)^3(4)^2$

$+ 5C_3(3x)^2 4^3 + 5C_4(3x)4^4 + 5C_5(3x)^0 4^5$

= $(3x)^5 + 5(3x)^4 \cdot 4 + 10(3x)^3 \cdot 4^2 + 10(3x)^2 4^3$
$+ 5(3x)4^4 + 4^5$

= $243x^5 + 1620x^4 + 4320x^3 + 5760x^2 +$
$3840x + 1024$

12) $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

→ $|A - \lambda I| = 0$

= $\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = 0$

= $-(2-\lambda)(5+\lambda)(3-\lambda) + 3(2)(3-\lambda) = 0$

= $(\lambda+2)(\lambda+5)(3-\lambda) + 6(3-\lambda) = 0$

= $(\lambda^2 + 7\lambda + 10)(3-\lambda) + 18 - 6\lambda = 0$

= $3\lambda^2 + 21\lambda + 30 - \lambda^3 - 7\lambda^2 - 10\lambda + 18 - 6\lambda = 0$

= $\cancel{\lambda^3} \cancel{+3\lambda} + 30 - \lambda^3 + 3\lambda^2$

$= \quad -30+18+19\lambda-6\lambda+\lambda^3=0$

$= \quad \lambda^3-13\lambda+12=0$

Using trial and error,

$\lambda=1 \longrightarrow$ eq^n satisfied,

$$
\begin{array}{r|l}
\lambda-1 & \lambda^3-13\lambda+12 \quad (\lambda^2+\lambda-12 \\
 & \lambda^3-\lambda^2 \\
\hline
 & \lambda^2-13\lambda+12 \\
 & \lambda^2-\lambda \\
\hline
 & -12\lambda+12 \\
 & -12\lambda+12 \\
\hline
 & 0
\end{array}
$$

$= \quad \therefore (\lambda-1)(\lambda^2+\lambda-12)=0$

$= \quad (\lambda-1)(\lambda+4)(\lambda-3)=0$

$\lambda=1,-4,+3$

13) $f(x)= x^3-7x^2+8x-3$

$x_0=5$

$= \quad f(x_0)=-13$

$= \quad f'(x_0)=3x^3-14x+8$

$= 13$

= Newton's Method :-

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$= x_1 = 5 - \frac{(-13)}{13}$

$= x_1 = 6$

$= f(x_1) = 9$

$f'(x_1) = 32$

$= x_2 = 6 - 9/32$

$x_2 = 5.71875$

14) To expand = $(1-x+x^2)^4$

$= (1-x+x^2)^4 = [(1-x)+x^2]^4$

$= [1-x]^4 + 4[1-x]^3(x^2) + 6[1-x]^2(x^2)^2 + 4(1-x)(x^6) + x^8$

on solving we get,

$(1-x+x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1$

15) $e^{-x} = 3\log x$

$e^{-x} - 3\log x = 0$

$\rightarrow f(x) = e^{-x} - 3\log x$

x	1	2
$f(x)$	0.3679	-0.7678

= Using bisection Method

= $x_1 = \frac{1+2}{2} = 1.5$

= $f(x_1) = -0.3051$

= $x_2 = \frac{1+1.5}{2} = 1.25 \rightarrow f(x_2) = -0.004225$

= $f(x_2) = \frac{1+1.5}{2} = 1.125$

= $f(x_3) = 0.1712$

$x_4 = 1.125 + 1.25 = 1.18875$

$f(x_4) = 0.0811$

$x = 1.18875$