

# HOMEWORK: 1 & 2

7/2/2022

1.

| x  | y |
|----|---|
| 7  | 3 |
| 8  | 1 |
| 9  | 1 |
| 10 | 9 |

Find the value of y at  $x=9.5$

$$\Rightarrow f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3 +$$

$$\frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1 +$$

$$\frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1 +$$

$$\frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$= \frac{(1.5) \cdot (0.5) \cdot (-0.5)(3)}{-6} + \frac{(2.5)(0.5)(-0.5)}{2} +$$

$$\frac{(2.5)(1.5)(-0.5)}{-2} + \frac{(2.5)(1.5)(0.5)(9)}{6}$$

$$= \frac{-1.125}{-6} + \frac{(-0.625)}{2} + \frac{(-1.875)}{-2} + \frac{16.875}{6}$$

$$= \frac{1.125}{6} - \frac{0.625}{2} + \frac{1.875}{2} + \frac{16.875}{6}$$

$$= \frac{29}{8}$$

$$f(9.5) = 3.625 //$$

|                       |     |     |    |    |
|-----------------------|-----|-----|----|----|
| 2. Dependent variable | 3   | 7   | 9  | 10 |
| Independent variable  | 168 | 120 | 72 | 63 |

~~$f(x) = ?$~~  Find the value of  $f(6)$ .

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120$$

$$+ \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$= \frac{-2016}{-168} + \frac{36(120)}{24} + \frac{12(72)}{12} + \frac{9(63)}{21}$$

$$= \frac{+2016}{+168} + \frac{4320}{24} + \frac{864}{12} + \frac{567}{21}$$

$f(6) = 291$

3.

| x | y |
|---|---|
| 1 | 4 |
| 2 | 5 |
| 7 | 5 |
| 8 | 4 |

Find the value of  $f(6)$ .

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5 +$$

$$\frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5 + \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$= \frac{6(4)}{-42} + \frac{10(5)}{30} + \frac{(-40)(5)}{-30} + \frac{(-20)(4)}{42}$$

$$= \frac{-24}{42} + \frac{50}{30} + \frac{200}{30} - \frac{80}{42}$$

$$= \frac{41}{7}$$

$$= \frac{41}{7}$$

$$f(6) = 5.857142857 //$$

$\therefore$  The value of  $f(6) = 5.857142857$

4.

| x | y   |
|---|-----|
| 1 | 2   |
| 2 | 4   |
| 3 | 8   |
| 4 | 16  |
| 7 | 128 |

Find the value of  $f(5)$ .

$$\begin{aligned}
 f(5) &= \frac{(5-1)(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 + \frac{(5-1)(5-2)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4 \\
 &\quad + \frac{(5-1)(5-2)(5-3)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 + \frac{(5-1)(5-2)(5-3)(5-4)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16 \\
 &\quad + \frac{(5-1)(5-2)(5-3)(5-4)(5-7)}{(7-1)(7-2)(7-3)(7-4)} \times 128
 \end{aligned}$$

$$= \frac{-24}{36} + \frac{(-64)}{-10} + \frac{-192}{8} + \frac{(-768)}{-18} + \frac{3072}{360}$$

$$= \frac{-24}{36} + \frac{64}{10} - \frac{192}{8} + \frac{768}{18} + \frac{3072}{360}$$

$$= \frac{494}{15}$$

$$\therefore f(5) = 32.9333$$

$\therefore$  The value of  $f(5) = 32.9333$ .

5.

| x  | y    |
|----|------|
| 10 | 0.25 |
| 20 | 0.2  |
| 14 | 0.15 |
| 16 | 0.10 |

Find the value of  $f(15)$ .

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25 + \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2$$

$$+ \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15 + \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10$$

$$= \frac{-5.25}{-240} + \frac{(-1)}{240} + \frac{3.75}{48} + \frac{(-2.5)}{-48}$$

$$= \frac{5.25}{240} - \frac{1}{240} + \frac{3.75}{48} + \frac{2.5}{48}$$

$$= \frac{71}{480}$$

$$\approx f(15) = 0.147916667 //$$

The value of  $f(15) = 0.147916667$

Q. Using false position method, find the approximate values of the root of the equation:

1.  $x^3 + x - 1 = 0$  lying between 0.5 and 1.

| $x$ | $y$    |
|-----|--------|
| 0.4 | -0.536 |
| 0.5 | -0.375 |
| 0.6 | -0.184 |
| 0.7 | 0.043  |
| 0.8 | 0.312  |

$$x_1 = 0.6 \quad x_2 = 0.7$$

$$y_1 = -0.184 \quad y_2 = 0.043$$

$$x_n = \frac{(0.6 \times 0.043) - [0.7 \times (-0.184)]}{0.043 - (-0.184)}$$

$$x_n = 0.681058$$

$$y = (0.681058)^3 + (0.681058) - 1 \\ = -0.003040$$

$$x_1 = 0.681058$$

$$x_2 = 0.7$$

$$y_1 = -0.003040$$

$$y_2 = 0.043$$

$$x_n = \frac{(0.681058 \times 0.043) - [0.7 \times (-0.003040)]}{0.043 - (-0.003040)}$$

$$x_n = 0.682309 //$$

$$y_n = -0.000046 //$$

$\therefore$  Root of  $x^3 + x - 1 = 0$  is 0.682309 //

2.  $x^3 - 2x + 0.5 = 0$  lying between 0 and 1.

|   |       |       |        |        |        |        |
|---|-------|-------|--------|--------|--------|--------|
| x | 0.1   | 0.2   | 0.3    | 0.4    | 0.5    | 0.6    |
| y | 0.301 | 0.108 | -0.073 | -0.236 | -0.375 | -0.484 |

$$x_1 = 0.2$$

$$x_2 = 0.3$$

$$y_1 = 0.108$$

$$y_2 = -0.073$$

$$x_n = \frac{[0.2 \times (-0.073)] - (0.3 \times 0.108)}{-0.073 - 0.108}$$

$$x_n = 0.259669$$

$$y_n = (0.259669)^3 - 2(0.259669) + 0.5 \\ = -0.001829$$

$$x_1 = 0.259669$$

$$x_2 = 0.2$$

$$y_1 = -0.001829$$

$$y_2 = 0.108$$

$$x_n = \frac{(0.259669 \times 0.108) - [0.2 \times (-0.001829)]}{0.108 - (-0.001829)}$$

$$x_n = 0.258968 //$$

$$y_n = 0.017793539 //$$

$\therefore$  The root of  $x^3 - 2x + 0.5 = 0$  is 0.258968. //

3.  $x^3 + 2x - 20 = 0$  lying between 2 and 2.5.

| x | 2  | 2.1    | 2.2    | 2.3    | 2.4    | 2.5   |
|---|----|--------|--------|--------|--------|-------|
| y | -8 | -6.539 | -4.952 | -3.233 | -1.376 | 0.625 |

$$x_1 = 2.4$$

$$x_2 = 2.5$$

$$y_1 = -1.376$$

$$y_2 = 0.625$$

$$x_n = \frac{(2.4 \times 0.625) - [(2.5) \times (-1.376)]}{0.625 - (-1.376)}$$

$$x_n = 2.468766$$

$$y_n = (2.468766)^3 + 2(2.468766) - 20$$

$$y_n = -0.015810$$

$$x_1 = 2.468766$$

$$x_2 = 2.5$$

$$y_1 = -0.015810$$

$$y_2 = 0.625$$

$$x_n = 2.469537 //$$

$$y_n = -0.000175562 //$$

$\therefore$  The root of  $x^3 + 2x - 20 = 0$  is 2.469537 //

4.  $3x - \cos x - 1 = 0$  lying between 0 and 1.

| $x$ | $y$     |
|-----|---------|
| 0   | -2      |
| 0.1 | -1.6998 |
| 0.2 | -1.3998 |
| 0.3 | -1.0998 |
| 0.4 | -0.7998 |
| 0.5 | -0.4998 |
| 0.6 | -0.1998 |
| 0.7 | 0.1     |
| 0.8 | 0.4     |
| 0.9 | 0.7     |
| 1   | 1       |

$$x_1 = 0.6 \quad x_2 = 0.7$$

$$y_1 = -0.1998 \quad y_2 = 0.1$$

$$x_n = \frac{(0.6 \times 0.1) - [(0.7) \times (-0.1998)]}{0.1 - (-0.1998)}$$

$$x_n = 0.666645 //$$

$$y_n = 3(0.666645) - \cos(0.666645) - 1$$

$$y_n = 0.0000002688 //$$

$\therefore$  The root of  $3x - \cos x - 1 = 0$  is  $0.666645 //$

5.  $x^3 - 3x - 5$  lying between 2 and 2.5.

| x | 2  | 2.1    | 2.2    | 2.3   | 2.4   | 2.5   |
|---|----|--------|--------|-------|-------|-------|
| y | -3 | -2.039 | -0.952 | 0.267 | 1.624 | 3.125 |

$$x_1 = 2.2$$

$$x_2 = 2.3$$

$$y_1 = -0.952$$

$$y_2 = 0.267$$

$$x_n = \frac{(2.2 \times 0.267) - [(2.3) \times (-0.952)]}{0.267 - (-0.952)}$$

$$x_n = 2.303002$$

$$y_n = 0.305698$$

$$x_1 = 2.303002$$

$$x_2 = 2.2$$

$$y_1 = 0.305698$$

$$y_2 = -0.952$$

$$x_n = 2.277966$$

$$y_n = -0.013238$$

$\therefore$  The root of  $x^3 - 3x - 5 = 0$  is 2.277966

6.  $x \log_{10} x - 1.2 = 0$  lying between 2 and 3.

|     |      |          |          |
|-----|------|----------|----------|
| $x$ | 1    | 2        | 3        |
| $y$ | -1.2 | -0.59794 | 0.231363 |

$$x_n = 2.72101$$

$$y_n = -0.017099$$

$$x_1 = 2.72101 \quad x_2 = 3$$

$$y_1 = -0.017099 \quad y_2 = 0.231363$$

$$x_n = 2.740206_{//}$$

$$y_n = -0.000384_{//}$$

$\therefore$  The root of  $x \log_{10} x - 1.2 = 0$  is 2.740206

7.  $x^3 - 4x + 1 = 0$

|     |    |   |
|-----|----|---|
| $x$ | 1  | 2 |
| $y$ | -2 | 1 |

$$x_n = \frac{5}{3} = 1.666667 \quad x_2 = 2$$

$$y_n = -1.037022 \quad y_2 = 1$$

$$x_n = 1.83636 \quad x_2 = 2$$

$$y_n = -0.1528 \quad y_2 = 1$$

$$x_n = 1.8580 \quad x_2 = 2$$

$$y_n = -0.0175 \quad y_2 = 1$$

$$x_n = 1.860442$$

$$y_n = 0.00124$$

$\therefore$  The root of  $x^3 - 4x + 1 = 0$  is 1.860442

8.  $x^3 + 2x^2 - 8 = 0$

|   |    |   |
|---|----|---|
| x | 1  | 2 |
| y | -5 | 8 |

$$x_n = \frac{18}{13} = 1.384615$$

$$x_2 = 2$$

$$y_n = -1.511156$$

$$y_2 = 8$$

$$x_n = 1.482$$

$$x_2 = 2$$

$$y_n = -0.346$$

$$y_2 = 8$$

$$x_n = 1.5035$$

$$x_2 = 2$$

$$y_n = -0.08$$

$$y_2 = 8$$

$$x_n = 1.5084$$

$$x_2 = 2$$

$$y_n = -0.017$$

$$y_2 = 8$$

$$x_n = 1.5095$$

$$y_n = -0.0032$$

$\therefore$  The root of  $x^3 + 2x^2 - 8 = 0$  is 1.5095

9.  $x^3 - 9x + 1 = 0$

|     |   |    |
|-----|---|----|
| $x$ | 0 | 1  |
| $y$ | 1 | -7 |

$$x_n = 0.125$$

$$x_2 = 1$$

$$y_n = -0.123$$

$$y_2 = -7$$

$$x_n = 0.1093$$

$$y_n = 0.017605$$

∴ The root of  $x^3 - 9x + 1 = 0$  is 0.1093