NUMERICAL METHODS AND ALGEBRA

1] Observed radius = 12.65 cm
Actual Radius = 12.5 cm

Observed area = πr^2

$A_o = \pi \times (12.65)^2$

$= 502.725 \, cm^2$

Actual area = πr^2

$= \pi \times (12.5)^2$

$= 490.874 \, cm^2$

i) Absolute error = Observed Value - Actual value

$= 502.725 - 490.874$

$= 11.851 \, cm^2$

ii) Relative error = $\frac{\text{Absolute error}}{\text{Actual value}}$

$= \frac{11.851}{490.874}$

$= 0.02414$

iii) Percentage error = Relative error $\times 100$

$= 2.414\%$

2]

x	$f(x) = \log x$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6	0.7781513

a)

$x_1 = 4$ $\qquad$ $y_1 = 0.60206$

$x = 5$ $\qquad$ $y = ?$

$x_2 = 6$ $\qquad$ $y_2 = 0.7781513$

$$y = \frac{y_2(x - x_1) + y_1(x_2 - x_1)}{x_2 - x_1}$$

$$= \frac{0.7781513(5-4) + 0.60206(6-5)}{6-4}$$

$$= 0.6901$$

b)

$x_1 = 4.5$ $\qquad$ $y_1 = 0.6532125$

$x = 5$ $\qquad$ $y = ?$

$x_2 = 5.5$ $\qquad$ $y_2 = 0.7403627$

$$y = \frac{y_2(x - x_1) + y_1(x_2 - x)}{x_2 - x_1}$$

$$= \frac{0.7403627(5-4.5) + 0.6532125(5.5-5)}{5.5 - 4.5}$$

$$= 0.6901$$

3] $x^4 - x - 10 = 0$

$a = 1.5, \quad b = 2$

$f(a) = (1.5)^4 - (1.5) - 10$

$= -6.4375$

$f(b) = 2^4 - 2 - 10$

$= 4$

1st iteration:

$x_1 = \frac{2+1.5}{2} = 1.75$

$f(x_1) = 1.75^4 - 1.75 - 10 = -2.3711$

2nd iteration:

$x_2 = \frac{1.75+2}{2} = 1.875$

$f(x_2) = 1.875^4 - 1.875 - 10 = 0.4846$

3rd iteration:

$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$

$f(x_3) = 1.8125^4 - 1.8125 - 10 = -1.0201$

4th iteration

$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$

$f(x_4) = 1.8437^4 - 1.8437 - 10 = -0.8277$

5th iteration

$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$

$f(x_3) = 1.859375^4 - 1.859375 - 10 = 0.093378$

$\therefore$ Root of $x^4 - x - 10 = 0$ is 1.859375
which is approx. 1.86

4] $f(x) = x^3 - x - 1$
$f'(x) = 3x^2 - 1$

x	0	1	2
y	-1	-1	5

$\therefore x_0 = \frac{1+2}{2} = 1.5$

$f(x_0) = 1.5^3 - 1.5 - 1 = 0.875$
$f'(x_0) = 5.75$
$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1.5 - \frac{0.875}{5.75} = 1.3478$

$f(x_1) = 0.10056$ $\quad f'(x_1) = 4.4496$
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.3252$
$f(x_2) = 0.00205$ $\quad f'(x_2) = 4.2684$
$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.32247$
$f(x_3) = 0.000007545$ $\quad f'(x_3) = 4.2645$
$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.3247$
$\therefore$ Root of $x^3 - x - 1$ is 1.3247

Page No.

Date

5] $A^2 = A$

$7A - (I+A)^3$

$= 7A - (I^3 + A^3 + 3A^2 I + 3AI^2]$

$7A - I^3 - A^3 - 3A^2 I + 3AI^2$

$7A - I^3 - A^2 \cdot A - 3A - 3A$

$7A - I - A - 6A$

$7A - 7A - I$

$\therefore \; 7A - (I + A)^3 = -I$

6] $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1(-6) + 1(-4) + 2(4) = -2$

$|A| \neq 0 \; \therefore$ inverse exists

Cofactor matrix $= \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 2 & 4 \end{bmatrix}$

Adjoint Matrix $= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \times$ Adjoint

$= \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$

9. The smallest 3 digit no. that leave remainder 2 is 101 and the largest is 998

$a = 101, \quad l = 998$

$d = 3$

$t_n = a + (n-1) d$

$998 = 101 + (n-1) 3$

$n - 1 = \frac{998 - 101}{3}$

$n = 300$

$S_n = \frac{n}{2}(a + l) = \frac{300}{2}(101 + 998)$

$= 164850$

10. $t_6 = 32$

$t_8 = 128$

$t_6 = ar^5 \qquad t_8 = ar^7$

$32 = ar^7 \qquad 128 = ar^7$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$\cancel{[r = 2]} \qquad r = 2$

Page No
Date

11. Expand $(4+3x)^5 = (3x+4)^5$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + b^n$$

$$\therefore (3x+4)^5 = (3x)^5 + 5(3x)^4 + \frac{5\times 4(3x)^3}{2} + \frac{5(4)(3)(3x)^2 (4^3}{3\times 2} + \frac{5(4)(3)(2)(3x)4^4}{4\times 3\times 2} + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\lambda^3 - 0\lambda^2 + (-13)\lambda - (30+18+0) = 0$$
$$\lambda^3 - 13\lambda + 12 = 0$$
$\therefore \lambda = 1$

1	1	0	-13	12
	↓	1	1	-12
	1	1	-13	$\boxed{0}$

$$\therefore \lambda^2 + \lambda - 12 = 0$$
$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$
$$(\lambda+4)(\lambda-3) = 0$$
$\lambda = -4$ or $\lambda = 3$

13. $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$f(x) = x^3 - 7x^2 + 8x - 3$

$\cancel{f(5) = -13}$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) = -13 \qquad f'(5) = 13$

$x_1 = 5 - \left(\frac{-13}{13}\right)$

$= 6$

$f(6) = 9 \qquad f'(6) = 32$

$x_2 = 6 - \frac{9}{32}$

$x_2 = 5.71875$

14. $(1 - x + x^2)^4$

$[x^2 + (1-x)]^4$

$x + 1 - x = y$

$\therefore [x^2 + y]^4$

using binomial theorum

$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} \ldots + b^n$

$[x^2+y]^4 = (x^2)^4 + 4(x^2)^3 y + \frac{4(3)(x^2)^3 y^2}{2} + \frac{4(3)(2)(x^2)y^3}{3 \times 2} + y^4$

$= x^8 + 4x^6 y + 6x^4 y^2 + 4x^2 y^3 + y^4$

$\therefore [x^2 + (1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2 + 4x^2(1-x)^3 + (1-x)^4$

15. $e^{-x} = 3\log(x)$

$f(x) = 3\log x - e^{-x}$

$a = 1 \quad b = 2$

$f(a) = 0.367879$

$f(b) = 1.944100$

$x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$

$f(x_1) = 0.99326$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = 0.382925$

$x_3 = \frac{1+1.5}{2} = 1.125$

$f(x_3) = 0.028696$

$x_4 = \frac{1+1.125}{2} = 1.0625$

$f(x_4) = -0.16372$