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Calculus
Assignment - 2

Q.1 Solve using substitution:-

$$I = \int_{\frac{1}{50}}^{2} \frac{e^{2/\omega}}{\omega^2} d\omega$$

$$= \int_{1/50}^{2} \frac{\log e^{2/\omega}}{\log \omega^2} d\omega$$

$$= \int_{1/50}^{2} \frac{2\omega^{-1}}{\log\omega^2} d\omega \qquad [\because \log_e e = 1]$$

Put $\log \omega^2 = t$

$$\frac{2\cancel{\omega}}{\omega^{\cancel{2}}} d\omega = dt$$

$$2\omega^{-1} d\omega = dt$$

$$I = \int_{1/50}^{2} \frac{dt}{t}$$

$$I = [\log t]_{1/50}^{2} + c$$

$$I = \log 2 - \log \frac{1}{50} + c$$

$$I = \log \frac{2}{1/50} + c$$

$$I = \log 100 + c$$

$$\boxed{I = 2 + c}$$

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Q. 2 Solve using Substitution:

$$I = \int \frac{2t^3 + 1}{(t^4+2t)^3} dt$$

Put $t^4+2t = x$

$(4t^3 + 2) dt = dx$

$$I = \frac{1}{2}\int \frac{2(2t^3+1)}{(t^4+2t)^3} dt$$ [Multiplying & dividing 2]

$$I = \frac{1}{2}\int \frac{4t^3+2}{(t^4+2t)^3} dt$$

$$I = \frac{1}{2}\int \frac{1}{x^3} dx$$

$$I = \frac{1}{2}\left[\frac{x^{-2}}{-2}\right]$$

$$I = \frac{-x^{-2}}{4}$$

Resubstituting,

$$I = \frac{-(t^4+2t)^{-2}}{4}$$

$$\boxed{I = \frac{-1}{(t^4+2t)^2}}$$

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Q. 3. $f'(x) = x^4 + 3x - 9$

$$f(x) = \int x^4 + 3x - 9 \quad dx$$

$$= \int x^4 \, dx + \int 3x \, dx - 9 \int dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x$$

$$\therefore f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x$$

Q.4. $f(x) = \begin{cases} 6 & , \text{if } x > 1 \\ 3x^2 & , \text{if } x \leq 1 \end{cases}$

Let $I = \int_{-2}^{3} f(x) \, dx$

$$= \int_{-2}^{1} 6 \, dx + \int_{1}^{3} 3x^2 \, dx.$$

$$= 6 \left[x \right]_{-2}^{1} + 3 \left[\frac{x^3}{3} \right]_{1}^{3} + C$$

$9 - \frac{1}{3}$
$\frac{27 - 1}{3}$
$\frac{26}{3}$

$$= 6 \left[1 - (-2) \right] + 3 \left[\frac{\cancel{3}^{\cancel{3}\,2}}{\cancel{3}} - \frac{1}{3} \right] + C$$

$$= 18 + 26$$
$$= 44$$

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Q.5. Solve.

→ Let $I = \int 3(8y-1)\, e^{4y^2-y}\, dy$

$= \cancel{\int (24y-1)\,(e^{4y^2-y})\, dy}$

$= 3\int (8y-1)\,(e^{4y^2-y})\, dy$

Put $e^{4y^2-y} = t$

differentiate w.r.t y

$e^{4y^2-y} \cdot \frac{d}{dy}(4y^2-y) = dt$

$(e^{4y^2-y})(8y-1)\, dy = dt$

$\therefore I = 3\int dt$

$= 3[t] + c$

{ Resubstituting,

$\boxed{I = 3e^{4y^2-y} + c}$

Q.6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f(1) = \frac{-5}{4}$

$f(4) = 404.$

→

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$$f(x) = \int 15\sqrt{x} + 5x^3 + 6 \quad dx$$

$$= 15\int \sqrt{x}\,dx + 5\int x^3\,dx + 6\int dx$$

$$= \frac{\cancel{15}^{5} \times 2x^{3/2}}{\cancel{3}} + \frac{5 \times x^4}{4} + 6x + C$$

$$= 10x^{3/2} + \frac{5x^4}{4} + 6x + C.$$

$$f(x) = 10\int x^{3/2}dx + \frac{5}{4}\int x^4\,dx + \int 6x\,dx + \int C\,dx$$

$$= \frac{2}{\cancel{5}} \times \overset{2}{\cancel{10}}\, x^{5/2} + \frac{\cancel{5}}{4} \times \frac{x^5}{\cancel{5}} + 6\,\frac{x^2}{2} + Cx + D$$

$$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + D$$

$$f(1) = 4(1)^{5/2} + (1)^5/4 + 3(1)^2 + C(1) + D$$

$$\frac{-5}{4} = 4 + \frac{1}{4} + 3 + C + D$$

$$\therefore C + D = \frac{-34}{4} = \underline{\underline{\frac{-17}{2}}}$$

$$f(4) = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + 4C + D$$

$$404 = 4(816) + 4C + D$$

$$4C + D = -1948$$

Solving simultaneously equation 1 & 2,

$$C = -1293/2.$$

$$D = 638$$

$$\boxed{\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{1293}{2}x + 638.}$$

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Q.7. Formulate the PDE from $f(x+iy)+g(x-iy)$

Sol^n - Let $z = f(x+iy) + g(x-iy)$ — (1)

Differentiating partially w.r.t. x

$$\frac{\partial z}{\partial x} = f'(x+iy) + g'(x-iy)$$

Differentiating again partially w.r.t. x.

$$\frac{\partial^2 z}{\partial x^2} = f''(x+iy) + g''(x-iy) \quad (2)$$

Differentiating eq^n (1) w.r.t. t.

$$\frac{\partial z}{\partial t} = f'(x+iy)(i) + g'(x-iy)(-i)$$

Differentiating again partially w.r.t. t

$$\frac{\partial^2 z}{\partial t^2} = f''(x+iy)(i^2) + g''(x-iy)(i^2)$$

$$= i^2\left[f''(x+iy) + g''(x-iy)\right]$$

$$\boxed{\frac{\partial^2 z}{\partial t^2} = i^2\left[\frac{\partial^2 z}{\partial x^2}\right]} \quad \text{...(from eq}^n\text{ (2))}$$

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Q.8. Solve the following:

$$\frac{dr}{d\theta} = \frac{r^2}{\theta} \quad r(1) = 2.$$

→

$$\therefore \frac{1}{r^2} dr = \frac{1}{\theta} d\theta$$

Integrating on both sides, we get,

$$\int \frac{1}{r^2} dr = \int \frac{1}{\theta} d\theta$$

$$-\frac{1}{r} = \log|\theta| + c$$

$$\boxed{r(\theta) = \frac{-1}{\log|\theta| + c}}$$

Given that $r(1) = 2$

$$\therefore r(1) = \frac{-1}{\log 1 + c}$$

$$2 = \frac{-1}{c}$$

$$\boxed{\therefore c = \frac{-1}{2}}$$

$$\therefore r(\theta) = \frac{-1}{\log\theta - 1/2}$$

Q.9. $\dfrac{dv}{dt} = 9.8 - 0.196v$

$\rightarrow$ ~~DOR~~ =

$$\frac{dv}{dt} + 0.196v = 9.8$$

$$I.F = e^{\int 0.196\,dt}$$

$$= e^{0.196t}$$

Solution $\rightarrow$

$$y \times I.F = \int Q \times I.F \; dt$$

$$y \cdot e^{\int 0.196t} = \int (9.8) \cdot (e^{0.196t})\,dt$$

$$y \cdot e^{0.196t} = (1.9208)(e^{0.196t}) + C.$$

$$\boxed{\therefore\; y \cdot e^{0.196t} = (1.9208) \cdot (e^{0.196t}) + C.}$$

Q.10. $ty' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$

$\rightarrow$ $y' + \frac{2}{t}y = t^2 - t + 1$

Dividing throughout by t,

$$\frac{dy}{dt} + \frac{2}{t}y = t - 1 + \frac{1}{t}$$

$$I.F = e^{\int \frac{2}{t}dt}$$

$$= e^{2\log t}$$

$$= e^{\log_e t^2}$$

$$= t^2$$

$$y \times I.F = \int Q \times I.F \; dt$$

$$y \times t^2 = \int \left(t - 1 + \frac{1}{t}\right)(t^2)\, dt$$

$$y \cdot t^2 = \int (t^3 - t^2 + t)\, dt$$

$$y \cdot t^2 = \frac{1}{4}t^4 - \frac{1}{3}t^3 + \frac{1}{2}t^2 + c$$

$$y = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{c}{t^2}$$

We have, $y(1) = 2.$

$$2 = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + c$$

$$\boxed{\therefore \quad c = \frac{19}{12}}$$

$$\boxed{\therefore \quad y = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{19}{12t^2}}$$

$$\frac{dy}{dx} + Py = Q$$

Q.11. $\quad y' = \dfrac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1.$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\int \frac{dy}{y^3} = \frac{x\,dx}{\sqrt{1+x^2}} \longrightarrow$$

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$$\cancel{y' - \frac{x}{\sqrt{1+x^2}} y^3 = 0}$$

$$\cancel{I \cdot f = e^{\int}}$$

$$\cancel{y^{-3}\, dy = \frac{x}{\sqrt{1+x^2}}\, dx}$$

$$y' - \frac{x}{\sqrt{1+x^2}} y^3 = 0.$$

$$I \cdot f = e^{\int \frac{x}{\sqrt{1+x^2}} dx}$$

$$= e^{\sqrt{1+x^2}}$$

NOW,

$$y \times I \cdot f = \int Q \times I \cdot f \, dx$$

$$\frac{dy}{dx} - \frac{x}{\sqrt{1+x^2}} y^3 = 0$$

$$\sqrt{1+x^2} \cdot \frac{dy}{dx} - \sqrt{1+x^2} \cdot \frac{x}{\sqrt{1+x^2}} y^3 = 0$$

$$\frac{d}{dx}\left(\sqrt{1+x^2} \cdot y\right) - x \cdot y^3 = 0$$

$$\int \frac{d}{dx}\left(\sqrt{1+x^2} \cdot y\right) - x \cdot y^3 = \int 0 \, dx$$

$$\sqrt{1+x^2} \cdot y = C$$

$$y = \frac{1}{\sqrt{1+x^2}} \, c$$

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We know, $y(0) = -1$

$$\therefore -1 = \frac{1}{\sqrt{1+0^2}} c$$

$$\boxed{\therefore c = -1}$$

$$\boxed{y = \frac{-1}{\sqrt{1+x^2}}}$$

Q.12 $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f(x) = x^3 - 10x^2 + 6$ $x_0 = 3$

$f(x) = -57$

$f'(x) = 3x^2 - 20x$ $f'(x) = -33$

$f''(x) = 6x - 20$ $f''(x) = -2$

$f'''(x) = 6$ $f'''(x) = 6$

$\therefore$ Using Taylor series expansion,

$$f(x) = f(x) - f'(\cancel{x})(x - \cancel{x}) + \frac{f''(x)}{2!}(x-x)^2 \ldots$$

$$\sum_{n=0}^{\infty} = \frac{f^n(a)}{n!}(x-a)^n$$

$$\therefore f(x) = \cancel{x^0} \frac{-57}{0!}(x-3)^0 + \frac{(-33)}{1!}(x-3)^1$$

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$$+ \frac{(-2)}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3$$

Hence, Taylor series for $f(x) = x^3 - 10x^2 + 6$ is,

$$\boxed{f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3}$$

Q.13 Find the Maclaurin Series for $(1+x)^{\mu}$

Let $f(x) = (1+x)^{\mu}$

$\cancel{(1+x)^{\mu} = 0+}$

$f(x) = (1+x)^{\mu}$
$f(0) = 1$
$f'(x) = \mu(1+x)^{\mu-1}$
$f'(0) = \mu^{\mu-1}$
$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$
$f''(0) = \mu(\mu-1)(1)^{\mu-2}$
$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$
$f'''(0) = \mu(\mu-1)(\mu-2)(1)^{\mu-3}$
$f^{n}(x) = \mu(\mu-1)(\mu-2)\ldots$
$\ldots(\mu-n+1)(1+x)^{\mu-n}$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \ldots$$
$$+ \frac{f^{n}(0)}{n!}x^{n}$$

$$\therefore f(x) = 1 - x\mu^{\mu-1} + \frac{\mu(\mu-1)}{2!} + \frac{\mu(\mu-1)(\mu-2)}{3!} +$$
$$\frac{(\mu-n+1)(1)^{\mu-n}}{n!}$$

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Q.14. Determine the Maclaurin series for $f(x) = \sqrt{1+x}$

$$f(x) = \sqrt{1+x} \qquad f(0) = 1.$$

$$f'(x) = \frac{1}{2\sqrt{1+x}}, \qquad f'(0) = \frac{1}{2}.$$

$$f''(x) = -\frac{1}{4}(1+x)^{-3/2}, \qquad f''(0) = -\frac{1}{4}$$

$$f'''(x) = \frac{3}{8}(1+x)^{-5/2}, \qquad f'''(0) = \frac{3}{8}.$$

Maclaurin Series,

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}(x^2) + \frac{f'''(0)}{3!}x^3 + \frac{f^n(0)}{n!}x^n + \cdots$$

$$f(x) = 1. + \frac{1}{2}x - \frac{1/4}{2!}x^2 + \frac{\frac{3}{8}}{6}x^3 - \frac{15/16}{24}x^4 +$$

$$\boxed{f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots}$$

Q.15. Find the Taylor Series for $f(x) = e^{-6x}$ about $x = -4$

→ $f(x) = e^{-6x}$

$x = -4$

$f(x) = e^{24}$

$f'(x) = -6e^{-6x}$ $\qquad$ $f'(x) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\qquad$ $f''(x) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ $\qquad$ $f'''(x) = -216e^{24}$

$f''''(x) = 1296e^{-6x}$ $\qquad$ $f''''(x) = 1926e^{24}$

Now, By expansion of Taylor Series,

$$\sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$f(x) = \frac{e^{24}}{0!}(x-a)^0 + \frac{(-6e^{-6x})}{1!}(x-a)^1 + \frac{36e^{-6x}}{2!}(x-a)^2$$

$$+ \frac{(-216e^{-6x})}{3!}(x-a)^3 +$$

$$+ \frac{1296e^{-6x}}{4!}(x-a)^4$$

$$\boxed{f(x) = e^{24} - 6e^{-6x}(x+4) + 18e^{-6x}(x+4)^2 - 36e^{-6x}(x+4)^3 + 54e^{-6x}(x+4)^4 - \ldots\ldots}$$

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Q.16. Find the Taylor Series for $f(x) = \ln(3+4x)$ about $x=0$

$f(x) = \ln(3+4x)$ $\qquad x=0$

$f(x) = \ln(3)$

$f'(x) = \frac{4}{3+4x}$ $\qquad f'(x) = \frac{4}{3}$

$f''(x) = -\frac{16}{(3+4x)^2}$ $\qquad f''(x) = -\frac{16}{9}$

$\frac{128}{27}$

$f'''(x) = \frac{128}{(4x+3)^3}$ $\qquad f'''(x) = \frac{128}{7}$

$\frac{27}{81}$

$f''''(x) = -\frac{1024}{(4x+3)^4}$ $\qquad f'''(x) = -\frac{1024}{81}$

By Taylor Series, $\sum_{n=0}^{N} \frac{f^n(a)}{n!}(x-a)^n$

$f(x) = \frac{\ln(3)}{0!}(x-a)^0 + \frac{4}{3}/1!\,(x-a)^1 -$

$\frac{16}{9}/2!\,(x-a)^2 + \frac{128/7}{3!}(x-a)^3 -$

$- \frac{1024}{81}/4!\,(x-a)^4 + \ldots\ldots$

$\frac{16}{64}$ $\frac{128}{256}$ $\frac{1024}{81} \times \frac{1}{4\times3\times2}$

$\frac{64}{16} \frac{1}{2}$

$\frac{64}{128} \times \frac{1}{3\times2}$
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$$\boxed{f(x) = \ln(3) + \frac{4}{3}x - \frac{8}{9}x^2 + \frac{64}{21}x^3 - \frac{16}{3}x^4 + \ldots\ldots}$$