Calculus Assignment 2.

1) $I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw$

$\rightarrow$ let $t = e^{2/w}$ —— (1)

$= e^{2/w} \cdot \frac{d}{dw}(2/w)$

$= e^{2/w} \cdot 2\left(-1/w^2\right)$

$dt = \frac{-2e^{2/w}}{w^2} \, dw$

$\boxed{\frac{dt}{-2} = \frac{e^{2/w}}{w^2} \, dw}$

w	$1/50$	2
t	e^{100}	e

.... from (1)

$I = \frac{-1}{2} \int_{e^{100}}^{e} dt$

$= \frac{-1}{2} \left[(t)\right]_{e^{100}}^{e}$

$= \frac{-1 e^{100}}{2} + \frac{1e}{2}$

$= \frac{e}{2} \quad \frac{-e^{100}}{2}$

2) $I = \int \frac{2+3-1}{(t4+2t)^3} \, dt$

$\rightarrow$ Let $y = t4+2t$

$dy = (4+3+2)\,dt$

$= 2(2+3+1)\,dt$ —— (1)

= Putting in I, we get,

$I = \frac{1}{2} \int \frac{dy}{y^3}$

$$= \frac{1}{2} \frac{y^{-3+1}}{-2} = \frac{-1}{4} \cdot \frac{1}{y^2}$$

$$= \frac{-1}{4y^2}$$

$$= \frac{-1}{4(\pm 4 + 2t)^2}$$

3) $f(x) = x^4 + 3x - 9$

$\rightarrow$ $f(x) = \int f'(x)\, dx$

$$= \int (x^4 + 3x - 9)\, dx$$

$$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$\rightarrow$ $\int_{-2}^{3} f(x)\, dx$

$$= \int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$$

= So, as per the question

$$I = \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$$

$$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= 1 - (-8) + 6(3) - 6(1)$$

$$= 1 + 8 + 18 - 6$$

$$= 21$$

5) $I = \int 3(8y-1)e^{4y^2-y}\,dy$

$\longrightarrow 3\int (8y-1)e^{4y^2-y}\,dy$

$=$ let $t = \cancel{4}\, e^{4y^2-y}$

$dt = e^{4y^2-y}(8y-1)\,dy$

~~let~~ $I = 3\int dt = 3t + c$

$= 3(e^{4y^2-y}) + c$

6) $f''(x) = 15\sqrt{x} + 5\times 3 + 6$, $f(1) = \frac{-5}{4}$ and $f(4) = 404$

$\longrightarrow \int f''(x) = f'(x) = \int 15 \times ½ + 5\times 3 + 6\,dx$

$= \frac{2}{\cancel{3}} \cdot \cancel{15}^{5} x^{3/2} + \frac{5x^4}{4} + 6x$

$= f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C$

$f(x) = \frac{\cancel{10}^{4} x^{5/2}}{\cancel{5/2}} + \frac{5}{4}\frac{x^5}{5} + \frac{6x^2}{2} + Cx + c'$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + c'$

$\frac{-5}{4} = 4 + \frac{1}{4} + 3 + c + c'$

$\frac{-3}{2} = 7 + c + c'$

$\boxed{c + c' = \frac{-17}{2}}$

$= f(4) = 4 \cdot 4^{5/2} + (4)^4 + 3(4)^2 + 4c + c'$

$404 = 4^{7/2} + 4^4 + 48 + 4c + c'$

$404 = 128 + 256 + 48 + 4c + c'$

$$-28 = 4c + c'$$

$$c' = -4c - 28$$

$= \quad c + (-4c) - 28 = -8.5$

$$-3c = 28 - 8.5$$

$$c = -6.5$$

$= \quad c' = -8.5 + 6.5 = -2.5$

$$c' = -2.5$$

$= \quad \left[f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + \frac{-13}{2} \times \frac{-5}{2} \right]$

8) $\frac{dr}{r\theta} = \frac{r^2}{\theta} \qquad r(1) - 2$

$\longrightarrow \quad \frac{dr}{r^2} = \frac{d\theta}{\theta}$

= integrating on both sides

$$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$$

$$\frac{r^{-2+1}}{-1} = \log \theta + c$$

$$-r^{-1} = \log \theta + c$$

$$\frac{-1}{r} = \log \theta + c$$

$= \quad r(1) = 2$

$$\frac{-1}{r} = \log(1) + c$$

$$\frac{-1}{2} = 0 + c$$

$$\boxed{c = -1/2}$$

$$= \frac{-1}{8} = \log \theta \frac{-1}{2}$$

$$= \boxed{\log \theta = \frac{1}{2} - \frac{1}{8}}$$

9) $$\frac{dw}{dt} = 9.8 - 0.196\,v$$

$$\longrightarrow \frac{dv}{9.8 - 0.196v} = dt$$

$$= \frac{dv}{\frac{98}{10} - \frac{196}{1000}v} = dt$$

$$\frac{\cancel{100}^{50} \log (9800 - 196)}{-\cancel{196} - 98} = t + c$$

$$= \boxed{\frac{-50}{98} \log (9604) = t + c}$$

10) $$fy' + 2y = t^2 - t + 1 \qquad y(1) = 1/2$$

$$\longrightarrow t\frac{dy}{dt} + 2y = t2 - t + 1$$

$$t\frac{dy}{dt} = t^2 - t + 1 - 2y$$

$$= x = t2 - t + 1 - 2y$$

$$= \frac{dx}{dt} = 2t - 1 - 2\frac{dy}{dt}$$

$$= 2\frac{dy}{dt} = 2t - 1 - \frac{dx}{dt}$$

$$= \frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}$$

$$= \boxed{\frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dy}{dt}}$$

$= \quad y = \frac{1}{2} \text{ at } t = 1$

$= \quad t\frac{dy}{dt} + 2y = t^2 - t + 1$

$= \quad t\left(t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}\right) = x$

$= \quad t2 = \frac{t}{2} - \frac{t}{2}\frac{dx}{dt} = x$

$= \quad t^2 dt - \frac{t}{2}dt - \frac{t}{2}dx = x\,dt$

$= \quad \frac{t^3}{3} - \frac{t^2}{4} - \frac{tx}{2} = xt + c$

$= \quad \frac{t^3}{3} - \frac{t^2}{4} = \frac{3xt}{2} + c$

$= \quad \frac{t^3}{3} - \frac{t^2}{4} = \frac{3x}{2}t\,(t^2 - t + 1 - 2y) + c$

$= \quad \frac{t^3}{3} - \frac{t^2}{4} = \frac{3t}{2}(t^2 - t + 1 - 2y) + c$

$= \quad \frac{t^3}{3} - \frac{t^2}{4} - \frac{3t^3}{2} + \frac{3t^2}{2} - \frac{3t}{2} + 3yt = c$

$-\frac{7t^3}{6} + \frac{10t^2}{8} - \frac{3t}{2} + 3yt = 0 + c$

$-\frac{7}{6} + \frac{10}{8} = c$

$\frac{-28 + 30}{24} = c$

$c = 2/24$

$\underline{c = 1/12}$

11) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$\longrightarrow \dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$

$= \displaystyle\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\, dx$

$= \dfrac{y^{-2}}{-2} = \dfrac{1}{2}\dfrac{t^{1/2}}{1/2} + c$

$= \dfrac{-1}{2y^2} = t^{1/2} + c$

$= \dfrac{-1}{2(-1)} = 0 + c$

$= c = 1/2$

$= 1+x^2 > 0$ for all $x \in R$

$\therefore$ The function is valid for the numbers.

12) $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$\longrightarrow f'(x) = 3x^2 - 20x$

$f''(x) = 6x$

$f'''(x) = 6$

$f''''(x) = 0$

$\therefore f'(3) = 3(3)^2 - 20(3)$

$= 27 - 60$

$= -33$

$f''(3) = 18$

$f'''(3) = 6$

$f''''(3) = 0$

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$f(3) = 27 - 90 + 6 = -57$

$\therefore$ Taylor expansion $= -57 + (x-3)(-33) + \frac{18}{2!}(x-3)^2 + \frac{6}{3!}(x-3$

$+ 0$

$= -57 + 99 - 33x + 9(x-3)^2 + (x-3)^3$

$= 42 - 33x + 9(x^2 - 6x + 9) + (x^3 - 27 - 9x(x-3$

$= 42 - 33x + \cancel{9x^2} - 54x + 81 + x^3 - 27 - \cancel{9x^2} + 27x$

$= x^3 - 27x - 33x + 42 + 81 - 27$

$= x^3 - 60x + 123 - 27$

$= x^3 - 60x + 96.$

15) $f(x) = e^{-6x}$ about $x = -4$

$\longrightarrow$ Taylor Expansion :-

$= f(-4) + (x-(-4))f'(-4) + \frac{(x-(-4))^2}{2!}f''(-4) + \frac{(x-(-4))^3}{3!}f'''(-4) \ldots\ldots$

$= f(x) = e^{-6x}.$

$f'(x) = \frac{-1}{6}e^{-6x}$

$\therefore$ at $x = -4$

$f'(-4) = \frac{-1}{6}e^{24}$

$f''(-4) = \frac{1}{36}e^{-6x} = \frac{1}{36}e^{24}$

$\therefore$ Taylor expansion $=$

$= e^{24} + (x+4)\frac{-1}{6}e^{24} + \frac{(x+4)^2}{2!}\frac{1}{36}e^{24}$

$-\frac{1}{216}\frac{e^{24}(x+4)^3}{3!} \ldots$

7) let $z = f(x+iy) + g(x-iy)$

$\longrightarrow$ differentiating z wrt x & y
we get,

$= p = z' = f'(x+iy) + g'(x+iy)$

$= m = z' = f'(x+iy)(i) + g(x-iy)(-i)$

$= q = z'' = f''(x+iy) + g'(x-iy)$ — (1)

$= n = z'' = f''(x+iy)(i^2) + g'(x-iy)(-i)^2$

$n = f''(x+iy)(i2) + g'(x-iy)(i2)$

$n = i(2)\{q\}$

$n = i(2)q$

$\boxed{n = i^2 q}$

16) $f(x) = \ln(3+4x)$

$\longrightarrow$ Taylor expansion about $x=0$, ..

$= f(0) + (x-0)f'(0) + (x-0)^2 f''(0) + (x-0)^3 f'''(0) \ldots$

$= \because f(x) = \ln(3+4x) = x=0 \Rightarrow \ln(3)$

$f'(x) = \frac{4}{3+4x} \Rightarrow x=0 \Rightarrow \frac{4}{3}$

$= f''(x) = \frac{(4x+3)(0) - (4)(4)}{(3+4x)^2} = \frac{-16}{(3+4x)^2} \Rightarrow x=0$

$= \frac{-16}{9}$

$$= f'''(x) = -16\frac{[(3+4x)^2(10)-1(2)(4x+3)]}{(3+4x)^4}$$

$$= \frac{32}{(3+4x)^3} = \frac{32}{24}$$

$$= \ln(3) + \frac{4x}{3} + (x-0)^2\frac{(-16)}{9(2!)} + \frac{32}{27(3!)}(x-0)^3 + \cdots - \frac{-64}{81(4!)}(x-0)^4 \cdots$$

$$= \ln 3 + \frac{4x}{3} + \frac{-16}{9}\frac{x^2}{(2!)} + \frac{32}{27}\frac{x^3}{(3!)} \; \frac{-64}{81}\frac{x^4}{(4!)} \cdots$$

13) $f(x) = (1+x)^{\mu}$

$\longrightarrow$ $f(0) = 1^{\mu} = 1$

$= f'(0) = \mu(1+x)^{\mu-1}$

$= \mu(1+0)^{\mu-1}$

$= \mu(1) = \mu$

$= f''(0) = \mu(\mu-1)(1+0)^{\mu-1-1}$

$= \mu^2 - \mu(1)$

$= \mu^2 - \mu)$

$= f'''(0) = \mu(\mu-1)(\mu-2)(1+0)^{\mu-3}$

$= \mu(\mu-1)(\mu-2)1$

$= \mu(\mu-1)(\mu-2)$

Mclaurin Series $\longrightarrow$

$$= f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x3}{3!} + \cdots$$

$$= 1 + \mu x + \frac{(\mu^2-\mu)x^2}{2!} + \frac{\mu(\mu-1)(\mu-2)x3}{3!}$$

$$+ \frac{\mu(\mu-1)(\mu-2)(\mu-3)x^4}{4!} + \cdots$$

14) $f(x) = \sqrt{1+x}$

$\longrightarrow$ $f'(0) = 1$

$= f'(0) = \frac{1}{2}(1+x)^{-1/2} = \frac{1}{2}(1) = \frac{1}{2}$

$= f''(0) = \frac{-1}{2}\frac{1}{2}(1+x)^{-3/2} = \frac{-1}{4}(1) = \frac{-1}{4}$

Malanrian Series $\longrightarrow$

$$1 + \frac{1}{2}(x) - \frac{1}{4}\frac{(x^2)}{2!} + \frac{1}{8}\frac{(x^3)}{3!} - \frac{1}{16}\frac{(x^4)}{4!} \cdots$$

$$\cdots \frac{-1}{2^n}\frac{x^2}{n!} \cdots$$