

Calculus Assignment 2

$$Q1 \int_{1/50}^2 \frac{e^{2\omega}}{\omega^2} d\omega$$

$$\begin{aligned} \omega t &= e^{2\omega} \\ \frac{d}{d\omega} e^{2\omega} &= 2e^{2\omega} \\ \frac{d}{d\omega} e^{2\omega} &= 2e^{2\omega} \end{aligned}$$

$$= \int_{1/50}^2 \frac{t \omega^2 dt}{-2e^{2\omega} \omega^2}$$

$$-\frac{1}{2} \int_{1/50}^2 dt$$

$$-\frac{1}{2} (\omega^2)_{1/50}^2$$

$$= -\frac{1}{2} (2 - 0.02)$$

$$= -0.99$$

$$Q2 \int \frac{2t^3 + 1}{(t^4 + 2t + 5)^3} dt$$

$$\int \frac{2t^3 + 1}{u^3} \frac{du}{dt} dt$$

$$\text{let } u = (t^4 + 2t + 5) \\ \text{diff w.r.t } t \\ \frac{du}{dt} = 3(t^4 + 2t + 5)^2$$

$$= \frac{1}{3} \int \frac{u^3 du}{u^3}$$

$$= \frac{1}{3} (t^4 + 2t + 5)^{-2} + C$$

$$Q3 f'(x) = x^4 + 3x - 9$$

$$f(x) = \int (x^4 + 3x - 9) dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

Q4 $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

$\int_{-2}^3 f(x) dx = \int_{-2}^1 f(x) dx + \int_1^3 f(x) dx$

$\int_{-2}^1 3x^2 dx + \int_1^3 6 dx$

$(x^3)_{-2}^1 + (6x)_1^3$

$1^3 - (-2)^3 + 18 - 6 = 4$

Q5 $\int 3(8y-1)e^{4y^2-y} dy$

$\int \frac{3(8y-1)e^{4y^2-y}}{(e^{4y^2-y})'(8y-1)} dy$

$u = e^{4y^2-y}$

$\frac{du}{dy} = (e^{4y^2-y})'(8y-1)$

$$3n + C$$

$$3(e^{4y^2-5}) + C$$

Q1 $f''(n) = 1\sqrt{n} + 5n^3 + 6$

$$f'(n) = \int 1\sqrt{n} + 5n^3 + 6 \, dn$$

$$= \frac{1n^{3/2}}{3/2} + \frac{5n^4}{4} + 6n + k$$

$$f(n) = \int \frac{1n^{3/2}}{3/2} + \frac{5n^4}{4} + 6n + k \, dn$$

$$f(n) = \frac{4n^{5/2}}{5} + \frac{n^4}{4} + 3n^2 + k + C$$

20 $f(1) = \frac{4}{5} = \frac{4}{5} + \frac{1}{4} + 3 + k + C$

$$k + C = -8.5$$

25 $f(n) = 404 = \frac{4(n)^{5/2}}{5} + \frac{(n)^4}{4} + 3(n)^2 + k + C$

$$k + C = 164$$

$$4h + c = 104$$

$$-k + c = -8.5$$

$$3k = 112.5$$

$$k = 37.5$$

$$c = -66$$

$$f(x) = 4x^{\frac{5}{2}} + x^4 + 3x^2 + 37.5x - 66$$

$$Q9 \frac{dv}{dn} = 9.8 - 0.196v$$

$$\log(9.8 - 0.196v)$$

$$= 0.196t + C$$

Q12 Taylor series for $f(x) = x^3 + \log x^2$

$$f(x) = x^3 + \log x^2$$

$$f'(x) = 3x^2 + \frac{2}{x}$$

$$f''(x) = 6x - \frac{2}{x^2}$$

$$f'''(x) = 6 + \frac{4}{x^3}$$

$$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2}f''(3) + \frac{(x-3)^3}{6}f'''(3) + \dots$$

$$= 27 + (x-3)(13) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{6}(10) + \dots$$

Q13 MacLaurin series

$$f(x) = \frac{1}{(1+x)^4}$$

$$f'(x) = -4(1+x)^{-5}$$

$$f''(x) = 20(1+x)^{-6}$$

$$f'''(x) = -120(1+x)^{-7}$$

$$f(x) \approx f(0) + \frac{x^1}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$$(1+x)^{-4} = 1 + x + \frac{(4)(3)(2)}{2!} x^2 + \frac{(4)(3)(2)(1)}{3!} x^3 + \dots$$

Q14 $f(x) = \sqrt{1+x}$

$$f'(x) = \frac{1}{2}(1+x)^{-1/2}$$

$$f'(0) = \frac{1}{2}$$

$$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$$

$$f''(0) = -\frac{1}{4}$$

$$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$$

$$f'''(0) = \frac{3}{8}$$

$$\sqrt{1+x} \approx \sqrt{1+x} + \frac{x^1}{1!} \left(\frac{1}{2}\right) + \frac{x^2}{2!} \left(-\frac{1}{4}\right) + \frac{x^3}{3!} \left(\frac{3}{8}\right)$$