DATE

CALCULUS
ASSIGNMENT - 1

1. $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$\lim_{h \to 0} \dfrac{2(9+h^2-6h)-18}{h}$

$\lim_{h \to 0} \dfrac{18 + 2h^2 - 12h - 18}{h}$

$\lim_{h \to 0} \dfrac{h(2h-12)}{h} = \lim_{h \to 0} 2h - 12$

$= 2(0) - 12$

$= -12$

2. $g(x) = \begin{cases} 2x & x < 6 \\ x - 1 & x \geq 6 \end{cases}$

For $g(x) = x - 1$

$\lim_{x \to 6^-} 2x = 12$

$\lim_{x \to 6^+} x - 1 = 5$

$\because$ LHS $\neq$ RHS

It is discontinuous

DATE

3. $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$\lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{(9-x)}$

$\lim_{x \to 9} \frac{-(9-x)(3+\sqrt{x})}{(9-\cancel{x})}$

$\lim_{x \to 9} -(3+\sqrt{x})$

$-(3+\sqrt{9}) = -6$

4. a) $x = -1$

$f(x) = \frac{4x+5}{9-3x}$

$f(-1) = \frac{4(-1)+5}{9-3(-1)}$

$= \frac{1}{12}$

∵ Since limit exists, the function ~~has a removable discontinuity~~ is continuous.

b) $x = 0$

$f(x) = \frac{4x+5}{9-3x}$

$f(0) = \frac{4(0)+5}{9-3(0)}$

$= \frac{5}{9}$

Since Limit exists, the function ~~has a removable disconti~~ is continuous.

DATE

c) $x = 3$

$f(x) = \dfrac{4x + 5}{9 - 3x}$

$f(3) = \dfrac{4(3) + 15}{9 - 3(3)}$

$= \dfrac{17}{0}$

Since limit doesnot exist, the function is discontinuous

5.

6. $g(y) = \begin{cases} y^2 + 5 & y < -2 \\ 1 - 3y & y \geq -2 \end{cases}$

b) $\lim_{y \to -2} g(y)$

$\lim_{y \to -2^-} y^2 + 5 = 4$

$\lim_{y \to -2^+} 1 - 3y = 7$

7. $j(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$j'(t) = (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - t)$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= (12t^4 - 64t^3 - 3t^3 + 16t^2) + (8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12)$

$= 12t^4 - 67t^3 + 16t^2 + 8t^4 - 65t^3 + \cancel{104t - 12} + 8t^2 + 96t - 12$

$= 20t^4 - 132t^3 \cancel{-80t} \cancel{-12} + 24t^2 + 96t - 12$

Ans] $20t^4 - 132t^3 + 24t^2 + 96t - 12$

8. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

Diff. w.r.t w

$$R'(w) = \frac{(2w^2 + 1)\frac{d}{dw}(3w + w^4) - (3w + w^4)\frac{d}{dx}(2w^2 + 1)}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{(2w^2 + 1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{(6w^2 + 8w^5 + 3 + 4w^3) - (12w^2 + 4w^5)}{2w^2 + 1}$$

$$R'(w) = \frac{-6w^2 + 4w^5 + 4w^3 + 3}{(2w^2 + 1)^2}$$

9.] $f(x) = 2e^x - 8^x$

Diff. w.r.t x

$f'(x) = 2e^x - 8^x \log 8$

Ans] $f'(x) = 2e^x - 8^x \log 8$

10.] $y = z^5 - e^z \ln(z)$

Diff. w.r.t z

$\frac{dy}{dz} = 5z^4 - e^z \frac{d}{dz} \ln(z) - \ln(z) \frac{d}{dz} e^z$

$\frac{dy}{dx} = 5z^4 - \frac{e^z}{z} - \ln(z) \cdot e^z$

Ans] $\frac{dy}{dx} = 5z^4 - \frac{e^z}{z} - e^z \cdot \ln(z)$

11.] a) $f(x) = (6x^2 + 7x)^4$

Diff w.r.t x

$f'(x) = 4(6x^2 + 7x) \cdot \frac{d}{dx}(6x^2 + 7x)$

$f'(x) = 4(6x^2 + 7x)(12x + 7)$

Ans] $f'(x) = 4(6x^2 + 7x)(12x + 7)$

b) $f(t) = 5 + e^{4t + t^7}$

Diff. w.r.t t

$f'(t) = e^{4t + t^7} \cdot \frac{d}{dx}(4t + t^7)$

$f'(t) = e^{4t + t^7} \cdot (4 + 7t^6)$