Numerical Methods and Algebra
Assignment - 1

Name: Shagun Alag
Class : F.Y.B
Roll no. : 97

1) Area of circular plate with radius 12.65 cm = 502.726
Area of circular plate with radius 12.5 cm = 490.874

i) Absolute error = |490.874 − 502.726| = |Actual value − Error value|
= 11.852

ii) Relative error = $\frac{\text{Absolute error}}{\text{Actual value}}$

$= \frac{11.852}{490.874}$

$= 0.024$

iii) Percentage error = Relative error × 100
= 0.024 × 100
= 2.4 %

2) a)

x	y (f(x))	Δy (Δf(x))
4	0.60206	~~0.1760913~~
6	0.7781573	

$h = |x_1 - x_0| = |4 - 6| = 2$

$U = \frac{x - x_0}{h} = \frac{5 - 4}{2} = \frac{-1}{2}$

$f(x) = 1 \cdot f(x_0) + U \Delta f(x_0)$

$\log(5) = (0.60206) + (-1/2)(0.1760913)$

$\log(5) = 0.60206 \div 0.08805$

$\log(5) = 0.57401$

Scanned with CamScanner

b)

x	y (f(x))	Δy (Δf(x))
4.5	0.6532125	0.0871502
5.5	0.7403627	

$h = |4.5 - 5.5| = 1$

$u = \frac{5 - 4.5}{1} = -0.5$

$\log(5) = (0.6532125) + (-0.5)(0.0871502)$

$\log(5) = 0.6532125 - 0.0435751$

$\log(5) = \underline{\underline{0.6096874}}$

3)

$\cancel{f(x)}\, y = x^4 - x - 10 = 0$

$a = 1.5, \quad b = 2$

x	y
1.5	−6.43875
2	4

$x_1 = 1.5, \quad y_1 = -6.4375$

$x_2 = 2, \quad y_2 = 4$

$x_3 = \frac{x_1 + x_2}{2} = \frac{1.5 + 2}{2} = \frac{2.5}{2} = 1.75$

$\cancel{y_3 = (2.5)^4 - (2.5) - 10 = 25.5625}$

$\cancel{y_3 = (1.75)^4 - (1.75) - 10 = -8.8}$

$\cancel{x_4 = \frac{x_1 + x_3}{2} = \frac{1.5 + 1.25}{2} = 1.375}$

$\cancel{y_4 = (1.375)^4 - (1.375) - 10 = -7.8}$

$y_3 = (1.75)^4 - (1.75) - 10 = -2.371$

$x_4 = \frac{x_3 + x_2}{2} = \frac{1.75 + 2}{2} = 1.875$

$y_4 = (1.875)^4 - (1.875) - 10 = 0.4846$

$x_5 = \frac{x_3 + x_4}{2} = \frac{1.75 + 1.875}{2} = 1.8125$

$y_5 = (1.8125)^4 - (1.8125) - 10 = -1.020$

$x_6 = \frac{x_4 + x_5}{2} = \frac{1.875 + 1.8125}{2} = 1.84375$

$y_6 = (1.84375)^4 - (1.84375) - 10 = -0.1972$

$x_7 = \frac{x_4 + x_6}{2} = \frac{1.875 + 1.84375}{2} = 1.859375$

$y_7 = (1.859375)^4 - (1.859375) - 10 = 0.0934$

Root of $x^4 - x - 10 = 0$ is $\cancel{0}$. $\underline{x = 1.859375}$

4) $f(x) = x^3 - x - 1$

$x_N = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f'(x) = 3x^2 - 1$

x	0	1	2
$f(x)$	-1	-1	5

$x_0 = 1,\ f(x_0) = -1,\ f'(x_0) = \cancel{2}\ 11$

$x_1 = 1 - \frac{(-1)}{\cancel{2}11} = \cancel{1.5}\ 1.09091$

$\cancel{f(x_1) = 3(1.5)^3 - (1.5) - 1 = 0.87}$

$f'(x_1) = 3(1.5)^2 - 1 = 5.75$

$x_2 = 1 - \frac{(0.87)}{5.75} = 0.84$

$f(x_2) = (0.84)^3 - (0.84) - 1 = -1.247\cancel{226}$

$f'(x_2) = 3(0.84)^2 - 1 = 1.1168$

$x_3 = 1 - \frac{(-1.247)}{1.1168} = 2.11658$

$f(x_3) = (2.11658)^3 - (2.11658) - 1 =$

4) $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	0	1	2
$f(x)$	-1	-1	5

$$x_N = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 1,\ f(x_0) = -1,\ f'(x_0) = 2$

$$x_1 = 1 - \left(\frac{-1}{2}\right) = 1.5$$

$f(x_1) = (1.5)^3 - (1.5) - 1 = 0.87$

$f'(x_1) = 3(1.5)^2 - 1 = 5.75$

$$x_2 = 1.5 - \left(\frac{0.87}{5.75}\right) = 1.3487$$

$f(x_2) = (1.3487)^3 - (1.3487) - 1 = 0.1046$

$f'(x_2) = 3(1.3487)^2 - 1 = 4.457$

$$x_3 = 1.3487 - \left(\frac{0.1046}{4.457}\right) = 0.9765$$

$f(x_3) = (0.9765)^3 - (0.9765) - 1 = -1.0454$

$f'(x_3) = 3(0.9765)^2 - 1 = 1.8607$

$$x_4 = 0.9765 - \left(\frac{-1.0454}{1.8607}\right) = 1.5383$$

$f(x_4) = (1.5383)^3 - (1.5383) - 1 = 1.1019$

$f'(x_4) = 3(1.5383)^2 - 1 = 6.0991$

$$x_5 = (1.5383) - \left(\frac{1.1019}{6.0991}\right) = 1.3576$$

$$f(x_5) = (1.3576)^3 - (1.3576) - 1 = 0.1446$$
$$f'(x_5) = 3(1.3576)^2 - 1 = 4.5292$$

$$x_6 = 1.3576 - \left(\frac{0.1446}{4.5292}\right) = 1.3257$$

$$f(x_6) = (1.3257)^3 - (1.3257) - 1 = 0.00419$$
$$f'(x_6) = 3(1.3257)^2 - 1 = 4.2724$$

$$x_7 = 1.3257 - \left(\frac{0.00419}{4.2724}\right) = 1.3247$$

$$f(x_7) = (1.3247)^3 - (1.3247) - 1 = -0.0000766$$
$$f'(x_7) = 3(1.3247)^2 - 1 = 4.2645$$

Root of the equation $f(x) = x^3 - x - 1$, is $x = 1.3247$.

5) $A^2 = A$,

$$7A - (I+A)^3 = 7A - (I^3 + 3I^2A + 3IA^2 + A^3)$$
$$= 7A - I^3 - 3A - 3A^2 - A^2 \cdot A$$
$$(A^2 = A)$$
$$= 7A - I - 3A - 3A - A \cdot A$$
$$= A - I - A^2$$
$$= \cancel{A} - I - \cancel{A}$$
$$7A - (I+A)^3 = \underline{\underline{-I}}$$

6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$$A^{-1} = \frac{1}{D} C^T$$

$M_{11} = -6$, $M_{12} = -4$, $M_{13} = 4$

$M_{21} = -1$, $M_{22} = -1$, $M_{23} = 1$

$M_{31} = -6$, $M_{32} = -2$, $M_{33} = 4$

$$C = \begin{bmatrix} -6 & 4 & 4 \\ -1 & -1 & -1 \\ -6 & -2 & 4 \end{bmatrix} \Rightarrow C^T = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$D \cancel{|A|} = 1(-6) - (-1)(-4) + 2(4)$$

$$= -6 - 4 + 8$$

$$D = -2$$

$$A^{-1} = \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

7) $A = 8i - 9j \Rightarrow |A| = \left|\sqrt{(8)^2 + (-9)^2}\right| = 12.042$

$B = 7i - 9j \Rightarrow |B| = \left|\sqrt{(7)^2 + (-9)^2}\right| = 11.402$

tal =

$A \cdot B = (8i - 9j) \cdot (7i - 9j)$

$= (8)(7) + (-9)(-9)$

$= 56 + 81$

$A \cdot B = 137$

$\cos\theta = \dfrac{A \cdot B}{|A||B|}$

$\cos\theta = \dfrac{137}{(12.042)(11.402)}$

$\cos\theta = 0.998$

$\theta = \cos^{-1}(0.998)$

$\theta = 3.62^\circ$

8) Magnitude $= \sqrt{(75)^2 + (25)^2}$

$= \sqrt{5625 + 625}$

$= \sqrt{6250}$

$= 79.057$

9) ~~5, 8, 11, 14,~~ .....

$\cancel{S_n = \frac{n}{2}[a_1 + (n-1)d] + a_n}$

4) 5, 8, 11, ...

$S_n = \frac{1}{2} n [2a + (n-1) d]$

$S_n = \frac{n}{2} [2(5) + (n-1)(3)]$

$S_n = \frac{n}{2} (10 + 3n - 3)$

$S_n = \frac{n}{2} (7 + 3n)$

$S_n =$

9) 101, 104, ..., 998.

$a = 101, \quad d = 3, \quad a_n = 998.$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$3(n-1) = 998 - 101$

$n - 1 = \frac{897}{3}$

$n - 1 = 299$

$n = 299 + 1$

$n = \cancel{298}\ 300$

$S_n = \frac{n}{2} [2a + (n-1)d]$

$S_{300} = \frac{300}{2} [2(101) + (300-1)(3)]$

$= 150 [202 + 3(299)]$

$= 150 (202 + 897)$

$= 150(1099)$

$\therefore S_{200} = 164850$

10) $T_6 = 32$, $T_8 = 128$

$T_n = a \cdot (r^{n-1})$

$T_6 = ar^5$

$32 = ar^5$

$T_8 = ar^7$

$128 = ar^7$

$\frac{T_8}{T_6} = \frac{\cancel{a}r^{7\,2}}{\cancel{ar^5}}$

$\frac{128}{32} = r^2$

$r^2 = 4$

$r = 2$

11) $(4+3x)^5 = {}^5C_0(4)^5(3x)^0 + {}^5C_1(4)^4(3x)^1 + {}^5C_2(4)^3(3x)^2 + {}^5C_3(4)^2(3x)^3 + {}^5C_4(4)^1(3x)^4 + {}^5C_5(4)^0(3x)^5$

$= 1024 + 5(256)(3x) + 10(64)(9x^2) + 10(16)(27x^3) + 5(4)(81x^4) + 243x^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

$(4+3x)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024.$

12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

~~Equation~~ $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = \cancel{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \; 0$$

$$(2-\lambda)\begin{vmatrix} -5-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} - (-3)\begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} + 0\begin{vmatrix} 2 & -5-\lambda \\ 0 & 0 \end{vmatrix} = 0$$

$$(2-\lambda)(-5-\lambda)(3-\lambda) + 3(2)(3-\lambda) = 0$$

$$\cancel{(2-\lambda)(-15+5\lambda-3\lambda+\lambda^2)+6(3-\lambda)=0}$$

$$(3-\lambda)[(-10-2\lambda+5\lambda+\lambda^2)+6] = 0$$

$$(3-\lambda)(\lambda^2+3\lambda-4) = 0$$

$$(3-\lambda)(\lambda^2+4\lambda-\lambda-4) = 0$$

$$(3-\lambda)(\lambda+4)(\lambda-1) = 0$$

$$\lambda = -4, 1, 3$$

Eigen values of A are $-4, 1, 3$.

Eigen vectors of A when $\lambda = -4$,

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

At $\lambda = -4$

$$\begin{bmatrix} 2+4 & -3 & 0 \\ 2 & -5+4 & 0 \\ 0 & 0 & 3+4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_1 \rightarrow R_1 - 2(R_2)$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2x_1 - x_2 \\ 7x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 - x_2 = 0$

$2x_1 = x_2 \rightarrow (1)$

$7x_3 = 0$

$x_3 = 0 \rightarrow (2)$

So, the eigen vectors for $\lambda = -4$ are $\begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix}$

When $\lambda = 1$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

~~When λ =~~

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 3x_2 \\ 0 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 3x_2 = 0$

$x_1 = 3x_2 \to (1)$

$2x_3 = 0$

$x_3 = 0 \to (2)$

So, the eigen vectors when $\lambda = 1$ are $\begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix}$

When $\lambda = 3$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-x_1 - 3x_2 = 0$

$x_1 = -3x_2 \to (1)$

$2x_1 - 8x_2 = 0$

$2(-3x_2) - 8x_2 = 0$

$-6x_2 - 8x_2 = 0$

$-14x_2 = 0$

$x_2 = 0 \rightarrow (2)$

Substitute (2) in (1)

$x_1 = -3x_2$

$x_1 = -3(0)$

$x_1 = 0 \rightarrow (3)$

So, the eigen vectors when $\lambda = 3$ are $\begin{bmatrix} 0 \\ 0 \\ \text{Undefined} \end{bmatrix}$

13) $f(x) = x^3 - 7x^2 + 8x - 3$, $x_0 = 5$, $x_2 = ?$

$f'(x) = 3x^2 - 14x + 8$

$f(x_0) = f(5) = (5)^3 - 7(5)^2 + 8(5) - 3 = -13$

$f'(x_0) = f'(5) = 3(5)^2 - 14(5) + 8 = 13$

$$x_N = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 5 - \left(\frac{-13}{13}\right) = 6$$

$f(x_1) = f(6) = (6)^3 - 7(6)^2 + 8(6) - 3 = 9$

$f'(x_1) = f'(6) = 3(6)^2 - 14(6) + 8 = 32$

$$x_2 = 6 - \left(\frac{9}{32}\right) = 5.718$$

$\therefore x_2 = 5.719$

14) $(1-x+x^2)^4 = [(x^2-x)+1]^4$

Using binomial theorem

$[(x^2-x)+1]^4 = {}^4C_0(x^2-x)^4(1)^0 + {}^4C_1(x^2-x)^3(1)^1 + {}^4C_2(x^2-x)^2(1)^2 + {}^4C_3(x^2-x)^1(1)^3$
$+ {}^4C_4(x^2-x)^0(1)^4$

$= (x^2-x)^4 + 4(x^2-x)^3 + 6(x^2-x)^2 + 4(x^2-x) + 1$

$= [{}^4C_0(x^2)^4(-x)^0 + {}^4C_1(x^2)^3(-x)^1 + {}^4C_2(x^2)^2(-x)^2 + {}^4C_3(x^2)^1(-x)^3$
$+ {}^4C_4(x^2)^0(-x)^4] + 4[{}^3C_0(x^2)^3(-x)^0 + {}^3C_1(x^2)^2(-x)^1 + {}^3C_2(x^2)^1(-x)^2$
$+ {}^3C_3(x^2)^0(-x)^3] + 6(x^4-2x^3+x^2) + 4x^2 - 4$

$= x^8 - 4x^7 + 6x^6 - 4x^5 + x^4 + 4(x^6 - 3x^5 + 3x^4 - x^3)$
$+ 6x^4 - 12x^3 + 6x^2 + 4x^2 - 4$

$= x^8 - 4x^7 + 6x^6 - 4x^5 + x^4 + 4x^6 - 12x^5 + 12x^4 - 4x^3$
$+ 6x^4 - 12x^3 + 6x^2 + 4x^2 - 4$

$(1-x+x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4$

15) $e^{-x} = 3\log(x)$

$e^{-x} - 3\log(x) = 0$

x	1	2
y	0.368	-0.768

$x_1 = 1, \; y_1 = 0.368$

$x_2 = 2, \; y_2 = -0.768$

$x_3 = \frac{x_1+x_2}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$y_3 = e^{-1.5} - 3\log(1.5) = -0.305$

$x_4 = \frac{x_1+x_3}{2} = \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$

$y_4 = e^{-1.25} - 3\log(1.25) = -0.004$

$\therefore x = x_4 = \underline{1.25}$