

FINANCIAL MATHEMATICS

ASSIGNMENT 2

Q.1. (i) Amt - 20,000.
Period - 12 years.

$$20,000 = 12 \times 427.90 a_{\overline{12}|i}$$

$$\therefore a_{\overline{12}|i} = 3.89499.$$

$$\text{Flat rate} = \frac{(5 \times 12 \times 427.90) - 20,000}{20,000 \times 5}.$$

$$= 5.67 \%$$

Approx APR = 11 %.

$$\text{At } i = 11 \%, a_{\overline{12}|i} = 3.87812$$

$$\text{At } i = 10 \%, a_{\overline{12}|i} = 3.96154.$$

By interpolation,

$$\frac{i - 10\%}{3.89499 - 3.96154} = \frac{11\% - 10\%}{3.87812 - 3.96154}$$

$$i = 10.8\%.$$

$$\boxed{\text{APR} = 10.8\%}$$

(ii) APR = 10.8 %.

$$\text{After 1 year capital left to pay} = 17 \times 427.90 a_{\overline{11}|10.8\%}$$

$$= 16,775.98$$

Let x denote the term of new loan.

$$\therefore 16,775.98 = 12 \times 274.49 a_{\overline{x}|10.8\%}$$

$$16,775.98 = 3293.88 \left(\frac{1 - (1.08)^{-x}}{12 \times (1.08)^{x-1}} \right)$$

$$1.108^{-x} = 0.4754$$

$$x = 7.25$$

$\therefore$ Term of new loan = 7.25 years

(ii) Interest for original loan = $4 \times 12 \times 427.40 - 16778.98$

$$= 3763.22.$$

Interest for new loan = $7.25 \times 12 \times 274.49 - 16778.98$

$$= 7104.65$$

Now $7104.65 - 3763.22 = 3341.43.$

$\therefore$ Mr. & Mrs Jones will pay INR 3341.43 more interest under the new loan.

Q.2. (i) a) Discounted payback Period -

It is a capital budgeting procedure used to determine the profitability of a project.

b) Payback Period -

This refers to the time required to recoup the funds expended in an investment.

(ii)

Q. 3. Value at 1/11/1983

$$\text{Annuity 1} = 2000 \times v^{\frac{1}{4}} \times \ddot{a}_{\overline{22}|} @ 8\%$$

$$= 2000 \times 0.980944 \times 11.01676$$

$$= \text{INR } 2161.363$$

$$\text{Annuity 2} = 320 \times (1+i)^{\frac{1}{4}} \times a_{\overline{16}|}^{(4)}$$

$$= 320 \times 1.0064 \times 9.1842$$

$$= \text{INR } 2957.872$$

$$\text{Annuity 3} = 180 \times a_{\overline{18}|}^{(12)}$$

$$= 180 \times 9.842583$$

$$= 1780.665$$

$$\text{Total} = 2161.363 + 2957.872 + 1780.665$$

$$= \text{INR } 61899.96$$

$$\text{Revised Annuity} = x \times (1+i)^8 a + a \frac{(1+i)^8 - 1}{i}$$

$$x = 6899.90$$

$$1.019427 \times 10.30886$$

$$x = 656.56$$

$$\boxed{\text{Revised annuity} = 656.56}$$

Q.4. $(I\ddot{a})_{\overline{n}|} = 1 + 2v + 3v^2 + 4v^3 + 5v^4 + \dots + nv^{n-1}$

Multiplying v on both sides,

$$v(I\ddot{a})_{\overline{n}|} = v + 2v^2 + 3v^3 + 4v^4 + \dots + nv^n$$

Subtraction

$$(1-v)(I\ddot{a})_{\overline{n}|} = 1 + v + v^2 + v^3 + \dots + v^{n-1}$$

$$(1-v)(I\ddot{a})_{\overline{n}|} = \frac{(1-v^n)}{(1-v) - nv^n}$$

$$d(I\ddot{a})_{\overline{n}|} = \ddot{a} - nv^n$$

$$(I\ddot{a})_{\overline{n}|} = \frac{a_n - nv^n}{d}$$

Hence Proved

Q.5. i) TWRR for property fund.

$$= \frac{16.4}{12.4} - 1 = 0.3226$$

$$= 32.26\%$$

TwRR for equity fund

$$= \frac{18.5}{12.1} - 1 = 0.5372$$

$$= 53.72\%$$

(ii) a) Assumption = 100 units.

$$= 1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} + 1586(1+i)^{1/4}$$

$$= 4.0640 \quad 4.1640$$

By interpolation,

$$i = 0.02932$$

$$i = 29.32\%$$

b) Assumed purchased at INR 100 made each ~~at~~ each quarter.

$$400 \times \ddot{s}_{\overline{4}|i}^{(4)} = 100 \times 16.4 \left(\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.8} + \frac{1}{15.1} \right)$$

$$\ddot{s}_{\overline{4}|i}^{(4)} = 1.180$$

$$i = 0.2979$$

$$i = 29.82\%$$

(iii) a) $1210(1+i) + 920(1+i)^{3/4} + 1630(1+i)^{1/2} + 1310(1+i)^{1/4} = 4 \times 1556$

$$i = 0.673$$

$$i = 67.31\%$$

b) $400 \times \ddot{s}_{\overline{4}|i}^{(4)} = 100 \times 15.5 \left(\frac{1}{12.2} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{12.4} \right)$

$$\ddot{s}_{\overline{4}|i}^{(4)} = 0.4735$$

$$\therefore i = 0.7088$$

$$= 70.88\%$$

Q.6. (i) $1,60,000 = X a_{\overline{10}|8\%}$

$$X = \frac{16,000}{a_{\overline{10}|8\%}}$$

$$= \frac{16,000}{6.7101} = 23,844.65$$

(ii) $23,844.65 \times a_{\overline{6}|8\%}$

$$= 23,844.65 \times 4.6229$$

$$= 110,231.422$$

$Y = \text{new installment}$

$$Y \cdot a_{\overline{6}|6\%} = 110,231.442 \text{ at } 6\%$$

$$Y = \frac{110,231.442}{a_{\overline{6}|6\%}}$$

$$= \frac{110,231.442}{4.3553}$$

$$= 25,309.72$$

(iii) $25,309.77 a_{\overline{3}|10\%}$

$$= 25,309.77 \times 2.4869$$

$$= 62,942.74$$

$Z = \text{new installment}$

$$Z a_{\overline{3}|9\%} = 62,942.74 \text{ @ } 9\%$$

$$= \frac{62,942.74}{2.5313}$$

$$= 24,865.77$$

Q.7. (i) 1) There are large buying and selling experience

- 2) The return on property may be lower, expect in short term.
- 3) The maintenance cost at the property is high.

(ii) a) Government raise money, by floating a loan on stock exchange. The terms of issue are set out by tender - The investors are invited to nominate the price that they are prepared to pay the loan is issued to highest bidder.

b) Advantages to Government is that they know the price that investor pay therefore it will know the cost of borrowing

Disadvantage to Government is that they may not meet its finance requirement.

c) Name of 3 securities are

- 1) Equity - Dividend varies according to companies performance
- 2) Property - Rent payments are subject to present
- 3) Index - Linked Bond - final redemption price can increase.

Q.8 i) $2.7 + 0.2 a_{\overline{3}|i}$ @ i $\rightarrow$ Present Value

$$0.1 v^3 + a_{\overline{10}|i} (0.25 v + 0.2 v^2 + 0.11) @ i$$

$\rightarrow$ Expected value

Equating both equation sides,

$$2.7 + 0.2 a_{\overline{3}|i} = 0.1 v^3 + a_{\overline{10}|i} (0.25 + 0.2v + 0.11) @ i$$

$$\therefore i = 9.31\%$$

$$\begin{aligned} \text{ii)} \quad & 0.1 v^3 + 0.85 a_{\overline{10}|i} v^3 \\ & = 0.1 v^3 + 0.85 a_{\overline{10}|i} v^3 - 2.7 a_{\overline{2}|i} @ i = 10\% \\ & = 0.1 \times 0.7513 \times 0.85 \times 0.7513 \times 6.4463 - 2.70 \times \\ & \quad 0.2 \times 2.4868. \end{aligned}$$

$$= \frac{2.70 \times 4.19}{4193.19}$$

$$= 1.0089$$

As NPV is positive it is worth investing.

Q.9 i) TWRR

$$(1+i)^3 = \frac{33}{30.56} \times \frac{4105.45}{33+6} \times \frac{45.6}{41.65} \times \frac{47}{(45.62.50)}$$

$$(1+i)^3 = 1.061967 \times 0.93717 \times 1.11084 \times 1.9$$

$$(1+i)^3 = 1.228315$$

$$1+i = \sqrt[3]{1.22831}$$

$$1+i = 1.70951$$

$$i = 1.70951 - 1$$

$$i = 7.0951\%$$

$$MKRR = 30.5 \times (1+i)^3 + 6 \times (1+i)^{2.23} + 4.5 \times (1+i)^{1.15} - 2.5 \times (1+i)^{6.5}$$

$$= 47$$

$$= 7\%$$

$$\therefore MKRR = 7.20\%$$

(iii) TWRR eliminates the effect of the cashflow amount & timing and therefore gives a fair price basis for assessing investment performance.