Sasithar 42

Page:
Date:

NMA Assignment

Q1] Actual length = 12.5 cm

Actual area $= \pi r^2 = \pi \times 12.5^2 = 490.87385\ cm^2$

Approx radius = 12.65 cm

Approx radius $= \pi r^2 = 502.72551\ cm^2$

i) Absolute error = Approx area error − Actual area

$= 502.72551 - 490.87385$

$= 11.85166\ cm^2$

ii) Relative error $= \dfrac{\text{Absolute error}}{\text{Actual area}}$

$= \dfrac{11.85166}{490.87385}$

$= 0.024144$

iii) % error = (Relative error) × 100

$= 2.4144\%$

Q2] a) Using linear interpolation

$x = 5$ $\quad x_1 = 4 \quad y_1 = 0.60206$ $\quad y = ?$

$x_2 = 6 \quad y_2 = 0.7781513$

~~(6-4) = (0.7781513 − 0.60206)~~

~~2 = 0.1760913~~

$$y = \frac{0.7781513\,(5-4) + 0.60206\,(6-5)}{6-4}$$

$= 0.6901$

b) $x_1 = 4.5$ $\quad$ $y_1 = 0.6532125$

$x = 5$ $\quad$ $y = ?$

$x_2 = 5.5$ $\quad$ $y_2 = 0.7403627$

$$y = \frac{0.7403627(5-4.5) + 0.6532125(5.5-5)}{5.5 - 4.5}$$

$= 0.6901$

Q3] $x^4 - x - 10 = 0$

	x	y
x_0	1.5	-6.4375
x_1	2	4

1) $x_2 = \dfrac{x_0 + x_1}{2}$ $\quad$ $y_2 = -2.37109$

$= 1.775$

2) $x_3 = \dfrac{x_2 + x_1}{2}$ $\quad$ $y_3 = 0.48462$

$= 1.875$

3) $x_4 = \dfrac{x_2 + x_3}{2}$ $\quad$ $y_4 = -1.0202$

$= 1.8125$

4) $x_5 = \dfrac{x_4 + x_3}{2}$ $\quad$ $y_5 = -0.28773$

$= 1.84375$

5) $x_6 = \dfrac{x_5 + x_3}{2} = 1.859375$ $\quad$ $y_6 = 0.0933$

Q4]

$x^3 - x - 1 = 0$

$f'(x) = 3x - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1 \qquad f(x_0) = -1 \qquad f'(x_0) = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \left(-\frac{1}{2}\right)$$

$$= 1.5$$

$y_1 = 0.875$

$f(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \left(\frac{0.875}{5.75}\right)$$

$$= 1.34783$$

$f(x_2) = 0.100699$

$f'(x_2) = 4.44994$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.3252$$

$y_3 = 0.00205$

$f'(y_3) = 4.26846$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.32472$$

$y_4 = 0.000008712$

Q5] $A^2 = A$

$$7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3A I^2)$$
$$= 7A - (I + A^2(A) + 3A + 3A)$$
$$= 7A - (I + A^2 + 6A)$$
$$= -I$$

Q6] $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

Using 3rd row,

$$|A| = 0 - 1(6-8) + (-1)(0+4)$$
$$= 2 - 4$$
$$= -2$$

$$\text{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{adj}(A)$$

$$= \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

Q7] $\vec{a} = 8\hat{i} - 9\hat{j}$

$\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a}\cdot\vec{b} = ab\cos\theta$

$\vec{a}\cdot\vec{b} = (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$

$= 56 + 81$

$= 137$

$a = \sqrt{64+81}$ $\quad$ $b = \sqrt{49+81}$

$= \sqrt{145}$ $\quad$ $= \sqrt{130}$

$\vec{a}\cdot\vec{b} = ab\cos\theta$

$\cos\theta = \dfrac{137}{\sqrt{130} \times \sqrt{145}}$

$\cos\theta = 0.9978849146$

$\theta = 3.78586^\circ$

Q8] Displacement vector $= \vec{R}$

$\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 30^\circ$

$R^2 = 75^2 + 25^2 + 2(75)(25)\cos 30^\circ$

$R^2 = 9497.5953$

$R = 97.456$

Thus displacement $= 97.456$ m

Q9] Common diff, $d = 3$

$A.P = 101, 104, 107, \ldots\ldots, 998$

$a = 101$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$n = 300$

$\therefore S_n = n/2\,(a + a_n)$

$= \frac{300}{2}(101 + 998)$

$= 150 \times 1099$

$S_n = 1{,}64{,}850$

Q10] $a_6 = 32$

$ar^5 = 32$

$a_8 = 128$

$ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = 2$ or -2

Rejecting as ~~give~~ ratio can't be negative

$r = 2$

Q11] $(3x+4)^5$

Using Pascal's Δ

				1	1	1					
			1		2		1				a
		1		3		3		1			$(a+b)$
	1		4		6		4		1		$(a+b)^3$
1		5		10		10		5		1	$(a+b)^4$
											$(a+b)^5$

$$(3x+4)^5 = 3x^5 + (5 \times 3x^4 \times 4) + (10 \times 3x^3 \times 4^2) + (10 \times 3x^2 \times 4^3) + (5 \times 3x \times 4^4) + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

Q12] $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$|A - \lambda I|$

Expanding using 3^{rd} row

$$|A - \lambda I| = 0 - 0 + (3-\lambda)[(2-\lambda)(-5-\lambda)+6]$$

$$= (3-\lambda)[-10-2\lambda+5\lambda+\lambda^2+6]$$

$$= (3-\lambda)\left[\lambda^2 + 3\lambda - 4\right]$$

$$= \cancel{3\lambda^2} + 9\lambda - 12 - \lambda^3 - \cancel{3\lambda^2} + 4\lambda$$

$$|A - \lambda I| = -\lambda^3 + 13\lambda - 12$$

$$-\lambda^3 + 13\lambda - 12 = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

Comparing coefficients,

$$a + b + c + d = 1 - 13 + 12 = 0$$

Thus, one root = 1

$$\begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ & 1 & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$$\lambda^2 + \lambda - 12 = 0$$

$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$

$$\lambda(\lambda + 4) - 3(\lambda + 4) = 0$$

$$(\lambda - 3)(\lambda + 4) = 0$$

$$\lambda = 3 \text{ or } -4$$

Values = -4, 1 and 3.

Q.13] $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) = -13$

$f'(5) = 13$

Page :
Date :

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \left(\frac{13}{-13}\right)$$

$$= 6$$

$$f(x_1) = 9$$

$$f'(x_1) = 32.$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 6 - \frac{9}{32}$$

$$x_2 = 5.71875$$

$$f(x_2) = 0.84787$$

Q14] $(x^2 - x + 1)^4 = [(x-1)^2 + x]^4$

Using Pascal's triangle.

1	a
1 1	$(a+b)$
1 2 1	$(a+b)^2$
1 3 3 1	$(a+b)^3$
1 4 6 4 1	$(a+b)^4$

$$[(x-1)^2 + x]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$$

$$(x^2 - x + 1)^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4$$

Page :
Date :

Q. 15] $e^{-x} = 3\log(x)$

$3\log(x) + e^{-x} = 0$

x	y
0.1	-2.09516
0.2	-1.27818
0.3	-0.82782
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	0.03188

$x_0 = 0.6$, $y_0 = -0.11673$

$x_1 = 0.7$, $y_1 = 0.03188.$

1. $x_2 = \frac{x_0 + x_1}{2}$

$= 0.65$

$y_2 = -0.03921$

2. $x_3 = \frac{x_2 + x_1}{2}$

$= 0.675$

$y_3 = -0.00293$

3. $x_4 = \frac{x_3 + x_1}{2}$

$= 0.6875$

$y_4 = 0.01465$

4. $x_5 = \frac{x_4 + x_3}{2}$

$= 0.68125$

$y_5 = 0.0059$