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NUMERICAL METHODS ALGEBRA

Assignment

1. Measured radius = 12.65 cm
   actual radius = 12.5 cm

(i) Absolute error

Area of circular plate = πr^2 sq. units

$$A(r) = \pi r^2$$

$$\frac{dA(r)}{dr} = 2\pi r$$

$$dA(r) = 2\pi r \, dr$$

$$dr = 12.65 - 12.5 = 0.15 \text{ cm}$$

$$\Rightarrow dA(r) = 2\pi(12.5)(0.15)$$

$$dA(r) = 3.75\pi \text{ cm}^2$$

Exact calculation for change in area:

$$A(12.65) - A(12.5)$$

$$= \pi(12.65)^2 - \pi(12.5)^2$$

$$= 160.0225\pi - 156.25\pi$$

$$= 3.7725\pi \text{ cm}^2$$

Absolute error $= 3.7725\pi - 3.75\pi$

$$\boxed{\text{Absolute error} = 0.0225\pi \text{ cm}^2}$$

(ii) Relative error = $\frac{\text{Actual change} - \text{approx. change}}{\text{actual change}}$

$$= \frac{0.0225\pi}{3.7725\pi}$$

$$\boxed{\text{relative error} = 0.006 \ cm^2}$$

(iii) percentage error = relative error × 100

$= 0.006 \times 100$

$$\boxed{\text{percentage error} = 0.6\%}$$

2. (a) when $\underset{x_1}{x} = 4$ $f(x_1) = \underset{y_1}{0.60206}$

$\underset{x_2}{x} = 6$ $f(x_2) = \underset{y_2}{0.7781513}$

interpolation : $y = y_1 + (x - x_1) \frac{(y_2 - y_1)}{(x_2 - x_1)}$

let $y = \log(5)$ and $x = 5$

$\log(5) = 0.60206 + (5-4) \frac{(0.1760913)}{2}$

$= 0.60206 + (1)(0.08804565)$

$$\boxed{\log(5) = 0.6901}$$

(b) $x_1 = 4.5$ $\quad y_1 = 0.6532125$

$x_2 = 5.5$ $\quad y_2 = 0.7403627$

$$\log(5) = 0.6532125 + \frac{(5-4.5)(0.0871502)}{(5.5-4.5)}$$

$$= 0.6532125 + \frac{(0.5)(0.0871502)}{(1)}$$

$$= 0.6532125 + 0.0435751$$

$$\boxed{\log(5) = 0.6968}$$

3. $x^4 - x - 10 = 0$ Bisection method

$f(x) = x^4 - x - 10$

$a = 1.5 \quad b = 2$

$f(a) = f(1.5) = (1.5)^4 - 1.5 - 10$

$= -6.4375 \rightarrow -ve$

$f(b) = (2)^4 - 2 - 10$

$= 4 \rightarrow +ve$

first iteration :

$x_1 = \frac{2+1.5}{2} = 1.75$

$f(x_1) = f(1.75) = -2.37109375 \rightarrow -ve$

second iteration :

$x_2 = \frac{1.75+2}{2} = 1.875$

$f(x_2) = f(1.875) = 0.48462 \rightarrow +ve$

third iteration : $x_3 = \dfrac{1.875 - 1.75}{2}$

$x_3 = 1.8125$

$f(x_3) = f(1.8125) = -1.02025 \rightarrow -ve$

fourth iteration : $x_4 = \dfrac{1.875 + 1.8125}{2}$

$x_4 = 1.84375$

$f(x_4) = f(1.84375) = -0.28774 \rightarrow -ve$

fifth iteration : $x_5 = \dfrac{1.875 + 1.84375}{2}$

$= 1.859375 \approx 1.86$

$f(x_5) = f(1.86) = 0.09378$

$\therefore$ root of $f(x)$ is $\boxed{1.86}$

4. $x^3 - x - 10 = 0$

$f(x) = x^3 - x - 10 = 0$

$f'(x) = 3x^2 - 1$

x	$f(x)$
0	-10
1	
2	

4. $f(x) = x^3 - x - 1$

Newton Raphson Method

$f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
y -ve $\leftarrow$ 1	-1
y +ve $\leftarrow$ 2	5

first iteration: $x_0 = \frac{1+2}{2} = 1.5$

$f(1.5) = 0.875$

$f'(1.5) = 5.75$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.5 - \frac{0.875}{5.75}$$

$$x_1 = 1.34783$$

second iteration: $f(x_1) = f(1.34783)$
$= 0.10068$

$f'(x_1) = 4.44991$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.34783 - \frac{0.10068}{4.44991}$$

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$x_2 = 1.3252$

third iteration : $f(x_2) = f(1.3252) = 0.00206$

$f'(x_2) = 4.26347$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.3252 - \frac{0.00206}{4.26347}$$

$x_3 = 1.32472$

fourth iteration : $f(x_3) = f(1.32472) = \underline{\underline{0}}$

$f'(x_3) = 4.26463$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.32472 - \frac{0}{4.26463}$$

$x_4 = 1.32472$

$\therefore$ root of $f(x) = x^3 - x - 1$ is $\boxed{1.32472}$

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5. $A^2 = A$ (given)
find value of $7A - (I + A)^3$

Soln. $\Rightarrow$ $7A - (I + A)^3$

$= 7A - (I^3 + A^3 + 3A^2I + 3I^2A)$

$= 7A - (I + A^2.A + 3A^2 + 3A)$

$= 7A - (I + A.A + 3A + 3A)$ [since $A^2 = A$]

$= 7A - (I + A^2 + 6A)$

$= 7A - (I + A + 6A)$

$= 7A - (I + 7A)$

$= \cancel{7A} - I - \cancel{7A}$

$= \boxed{-I}$

6. let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 0(-6) - 1(6-8) - 1(+4)$

$|A| = 0 + 2 - 4$

$|A| = -2$

finding cofactors:

$a_{11} = -6$ $\quad a_{21} = +1$ $\quad a_{31} = -6$

$a_{12} = +4$ $\quad a_{22} = -1$ $\quad a_{32} = +2$

$a_{13} = 4$ $\quad a_{23} = -1$ $\quad a_{33} = 4$

$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$$\boxed{A^{-1} = \frac{1}{2} \begin{bmatrix} 6 & -1 & 6 \\ -4 & 1 & -2 \\ -4 & 1 & -4 \end{bmatrix}}$$

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7. $\vec{A} = 8\hat{i} - 9\hat{j}$
$\vec{B} = 7\hat{i} - 9\hat{j}$

scalar product $= \vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ (θ is angle b/w $\vec{A}$ & $\vec{B}$)

$$\theta = \cos^{-1}\left[\frac{(8\hat{i} - 9\hat{j})\cdot(7\hat{i} - 9\hat{j})}{|8\hat{i} - 9\hat{j}|\,|7\hat{i} - 9\hat{j}|}\right]$$

$$\theta = \cos^{-1}\left[\frac{(8)(7) + (-9)(-9)}{\sqrt{8^2 + (-9)^2}\sqrt{7^2 + (-9)^2}}\right]$$

$$\theta = \cos^{-1}\left[\frac{56 + 81}{\sqrt{145}\sqrt{130}}\right]$$

$$\theta = \cos^{-1}\left[\frac{137}{\sqrt{18850}}\right]$$

$\theta =$ ~~$\cos^{-1}(0.7647749)$~~ $\cos^{-1}(0.99785)$

$$\boxed{\theta = 3.76^\circ}$$

8. let : resultant magnitude be R
$A = 75\,m$
$B = 25\,m$
$\theta = 30^\circ$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{75^2 + 25^2 + 2(75)(25)\cos 30^\circ}$$

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$$= \sqrt{5625 + 625 + 3750\left(\frac{\sqrt{3}}{2}\right)}$$

$$\boxed{R = 97.4556 \text{ m}}$$

9. ~~given~~ $a = 101$ $a_n = 998$
$d = 3$

$a_n = a + (n-1)d$
$998 = 101 + (n-1)3$
$897 = 3n - 3$
$900 = 3n$
$\boxed{n = 300}$

$S_n = \frac{n}{2}(a + a_n)$

$S = \frac{300}{2}(101 + 998)$

$\boxed{S = 1,64,850}$

∴ sum of all 3 digit numbers that leave a remainder of 2 when divided by 3 is 164850.

10. given : $a_6 = ar^5 = 32$ — ①
$a_8 = ar^7 = 128$ — ②
$r = ?$
② ÷ ①

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$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$\boxed{r = 2}$$

$\therefore$ common ratio of the GP is 2.

11. $(4 + 3x)^5 = (3x + 4)^5$

formula for binomial theorem :

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots + b^n$$

so,

$$(3x+4)^5 = (3x)^5 + 5(3x)^4(4) + \frac{5(4)}{2!}(3x)^3 4^2 + \frac{(5)(4)(3)}{3!}(3x)^2(4)^3 + \frac{(5)(4)(3)(2)(3x)(4)^4}{4!} + 4^5$$

$$\boxed{= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024}$$

(12) **

$$A = \begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

So, $|A - \lambda I| = 0 \rightarrow$ characteristic equ.

$$\begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$0 + 0 + 3-\lambda\left((2-\lambda)(-5-\lambda) - (-3)(2)\right) = 0$$

$$= 3-\lambda\left[-10 - 2\lambda + 5\lambda + \lambda^2 + 6\right] = 0$$

$$= (3-\lambda)(\lambda^2 + 3\lambda - 4) = 0$$

$$= \cancel{3\lambda^2} + 9\lambda - 12 - \lambda^3 - \cancel{3\lambda^2} + 4\lambda = 0$$

$$= -\lambda^3 + 13\lambda - 12 = 0$$

$$= \lambda^3 - 13\lambda + 12 = 0$$

$$= \lambda(\lambda^2 - 13) + 12 = 0$$

$$\cancel{\lambda(\lambda^2 - 13) = -12}$$

$$\lambda = -4, 1, 3.$$

Eigen values $\rightarrow -4, 1, 3$

for Eigen vectors :

$$\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

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at $\lambda = -4$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$6x - 3y = 0$ — ①

$2x - y = 0$ — ②

$7z = 0$ — ③

from ③, $z = 0$

solving ① & ②

$y = 2$

$x = 1$

$v_1 = (1, 2, 0)$

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at $\lambda = 3$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\rightarrow$

$$-x - 3y = 0 \quad -(1)$$
$$2x - 8y = 0 \quad -(2)$$
$$0$$

solving (1) & (2)

$$-x - 3y = 0$$
$$x = -3y$$

subsituting in (2):

$$2(-3y) - 8y = 0$$
$$-6y - 8y = 0$$
$$-14y = 0$$
$$y = 0$$

13. $f(x) = x^3 - 7x^2 + 8x - 3$

Newton's method

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

first iteration: $f(x_0) = f(5) = -13$

$f'(x_0) = f'(5) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\boxed{x_1 = 5 - \frac{(-13)}{13} = 6}$$

second iteration: $f(x_1) = f(6) = 9$

$f'(x_1) = f'(6) = 32$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 6 - \frac{9}{32} = 5.71875$$

Hence, $\boxed{x_2 = 5.71875}$

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14. $(1-x+x^2)^4 = \cancel{(1+(x^2-x))^4}$

$= ((1-x)+x^2)^4$

$= (y+x^2)^4$ [let $1-x=y$]

$= y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$

substitute value of y back:

$= (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

$= \boxed{1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8}$.

15. $e^{-x} = 3\log(x)$

Bisection method

$f(x) = 3\log(x) - e^{-x}$

$a = 0.5 \quad b = 1.5$

$f(a) = f(0.5) = -1.5092620647$ → -ve

$f(b) = f(1.5) = 0.30514361 7$ → +ve

first iteration: $x_1 = \frac{0.5+1.5}{2} = 1$

$f(x_1) = f(1) = -0.3678794412$ → -ve

second iteration : $x_2 = \dfrac{1 + 1.5}{2} = 1.25$

$f(1.25) = 4.225242164 \times 10^{-3} \to +ve$

third iteration: $x_3 = \dfrac{1 + 1.25}{2} = 1.125$

$f(1.125) = -0.1711949 \to -ve$

fourth iteration : $\dfrac{1.125 + 1.25}{2} = 1.187$

$f(1.187) = -0.08179$

approximated value $\to$ $\boxed{x = 1.187}$