Probability & Statistics
Assignment - 1

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Q.1)
$$\frac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} \sim N(0,1)$$

$$\hat{\lambda} = \frac{\Sigma x_i}{n}$$

$$\hat{\lambda} = 200$$

$$P(Z_{2.5\%} < Z < Z_{97.5\%}) = 0.95$$

$$P(-1.96 < Z < 1.96) = 0.95$$

$$P\left(-1.96 < \frac{\hat{\lambda} - \lambda}{\sqrt{\hat{\lambda}/n}} < 1.96\right) = 0.95$$

$$P\left(-1.96\sqrt{\frac{\hat{\lambda}}{n}} - \hat{\lambda} < \lambda < 1.96\sqrt{\frac{\hat{\lambda}}{n}} - \hat{\lambda}\right) = 0.95$$

$$P\left(\hat{\lambda} - 1.96\sqrt{\frac{\hat{\lambda}}{n}} < \lambda < \hat{\lambda} + 1.96\sqrt{\frac{\hat{\lambda}}{n}}\right) = 0.95$$

95% CI for $\lambda = \left(200 - 1.96\sqrt{200}, 200 + 1.96\sqrt{200}\right)$

$$= (172.28141, 227.7185)$$

Q.2) X_i, $i = 1, 2, \ldots\ldots\ldots, n$ are iid

Mean $= \mu$.
Variance $= \sigma^2$.

$$\bar{X} = \frac{\Sigma X_i}{n}$$

$$S^2 = \frac{1}{n-1}\left(\Sigma X_i^2 - n\bar{X}^2\right)$$

(i) $= E(\bar{X}) = E\left(\frac{\Sigma X_i}{n}\right)$

$$= \frac{\Sigma E(X_i)}{n}.$$

$$= \frac{n\mu}{n}$$

$$= \mu$$

(ii) $V(\bar{X}) = V\left(\frac{\Sigma X_i}{n}\right)$

$$= \frac{1}{n^2} V(\Sigma X_i)$$

$$= \frac{\sigma^2}{n}$$

$$E(s^2) = \sigma^2.$$

$$E(s^2) = E\left[\frac{1}{n-1}\left(\Sigma X_i^2 - n\bar{X}^2\right)\right]$$

$$= \frac{1}{n-1}\left[E(\Sigma X_i^2) - nE(\bar{X}^2)\right]$$

$$= \frac{1}{n-1}\left[\Sigma E(X_i^2) - n E(\bar{X}^2)\right].$$

$$V(X) = E(X_i^2) - \left[E(X)\right]^2.$$

$$E(X_i^2) = \mu^2 + \sigma^2.$$

$$V(\bar{X}) = E(\bar{X}^2) - \left[E(\bar{X})\right]^2.$$

$$E(\bar{X}^2) = \mu^2 + \frac{\sigma^2}{n}$$

$$E(s^2) = \frac{1}{n-1}\left[\Sigma(\mu^2+\sigma^2) - n\left(\mu^2 + \frac{\sigma^2}{n}\right)\right]$$

$$= \frac{1}{n-1}\left(n\mu^2 + n\sigma^2 - n\mu^2 - \sigma^2\right).$$

$$= \frac{1}{n-1}\sigma^2(n-1)$$

$$E(s^2) = \sigma^2$$

(iii) Now, $X \sim N(\mu, \sigma^2)$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$V\left(\frac{(n-1)s^2}{\sigma^2}\right) = \frac{(n-1)^2}{\sigma^4} V(s^2)$$

$$V(\chi^2_{n-1}) = \frac{(n-1)^2}{\sigma^4} V(s^2)$$

$$2(n-1) = \frac{(n-1)^2}{\sigma^4} V(s^2)$$

$$V(s^2) = \frac{2\sigma^4}{n-1}$$

(iv) $P(0.5\sigma^2 < s^2 < 1.5\sigma^2)$

$= P(s^2 < 1.5\sigma^2) - P(s^2 < 0.5\sigma^2)$

$n = 10$
$\sigma^2 = 100$

$$P(s^2 < 1.5\sigma^2) = P(s^2 < 150)$$

$$= P\left(\frac{9s^2}{\sigma^2} < \frac{150 \times 9}{100}\right)$$

$$= P(\chi^2_9 < 13.5)$$

$$= 0.8587$$

$$P(S^2 < 0.55^2) = P(S^2 < 50)$$

$$= P\left(\frac{9S^2}{\sigma^2} < \frac{50 \times 9}{100}\right)$$

$$= P(\chi_9^2 < 4.5)$$

$$= 0.1245$$

$$P(50 < S^2 < 150) = 0.8587 - 0.1245$$
$$= 0.7342.$$

Q3) Let no. of claims be a random variable X.

$$X \sim Bin(4, p)$$

(i) $L = \prod_{i=1}^{180} f(x_i)$

$$= (P(X=0))^{86} \times (P(X=1))^{75} \times [P(X=2)]^{16} \times [P(X=3)]^2 \times [P(X=4)]^1.$$

$$= \left({}^4C_0 p^0 (1-p)^4\right)^{86} \times \left({}^4C_1 p^1 (1-p)^3\right)^{75} \times \left({}^4C_2 p^2 (1-p)^2\right)^{16} \times \left({}^4C_3 p^3 (1-p)\right)^2 \left({}^4C_4 p^4\right)^1$$

$$= \text{constant} \times p^{117} \times (1-p)^{603}$$

$$\log L = \log(\text{constant}) + 117 \log(p) + 603 \log(1-p)$$

$$\frac{dl}{dp} = \frac{117}{p} - \frac{603}{(1-p)}$$

$$\frac{dl}{dp} = 0$$

$$\frac{117}{p} = \frac{603}{1-p}$$

$$117 - 117p = 603p$$

$$\hat{p} = \frac{117}{720}$$

$$\hat{p} = 0.1625$$

ii) Goodness of fit test

H_0 = No. of claims conforms to the binomial model.

H_1 = No. of claims does not conform to the binomial method

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.542	68.724	19.998	2.574	0.108

$p = 0.1625$

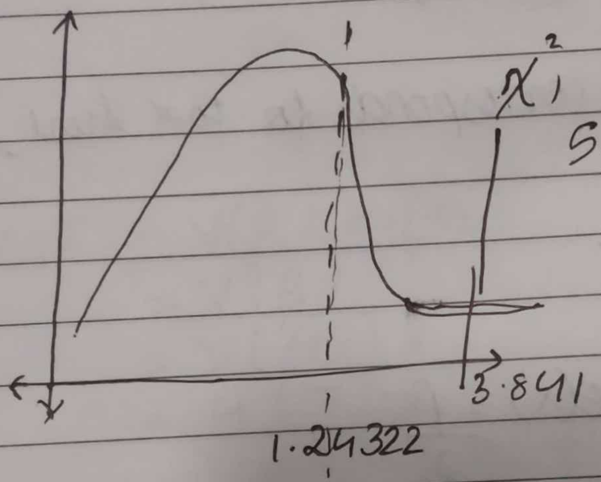
$P(X=0) = 0.4919$
$P(X=1) = 0.3818$
$P(X=2) = 0.1111$
$P(X=3) = 0.0143$

$P(X=4) = 0.0006$.

O_i	86	75	19
E_i	88.542	68.724	22.68

$$T.S = \frac{\Sigma(O_i - E_i)^2}{E_i}$$

$$1.224322 \sim \chi_1^2$$



5% level of significance

We have insufficient to reject H_0 at 5% level of significance. Therefore no. of claims conforms binomial model

4) $f(x) = \frac{c\beta^3}{(x+\beta)^4}$; $x > 0$, $\beta > 0$

(i) Comparing PDF of pareto distribution

For Pareto,

$$\text{PDF} = \frac{\alpha \lambda^{\alpha}}{(\lambda + x)^{\alpha+1}}$$

Thus β corresponds ~~to~~ to α here)

$$E(X) = \frac{\lambda}{\alpha - 1}$$

$$E(X) = \frac{\beta}{2}$$

$$V(X) = \frac{\alpha \lambda^2}{(\alpha-1)^2(\alpha-2)} = \frac{3\beta^2}{4}.$$

$$E(X) = \frac{\beta}{2}$$

$$E(X) = \bar{X}$$

$$\frac{\beta}{2} = \bar{X}$$

$$\hat{\beta} = 2\bar{X}$$

$\Rightarrow$ Bias $= E(\hat{\beta}) - \beta$

$= E(2\bar{X}) - \beta$

$= \frac{2}{n} \sum E(X_i - \beta)$

Bias $= 0$, hence unbiased

MSE $= V(\hat{\beta}) + (\text{Bias})^2$

$= V(\hat{\beta})$

$= V(4\bar{X})$

$= 4(V\bar{X})$

$\Rightarrow 4V\left[\frac{\sum x_i}{n}\right]$

$= \frac{3\beta^2}{n}$

As n increases, MSE decreases.

Hence = MSE in $\hat{\beta}$ is consistent

(iii) $\hat{\beta} = b\bar{X}$

$$E(\hat{\beta}_2) = E(b\bar{X})$$

$$= b\,E\left(\frac{\Sigma X_i}{n}\right)$$

$$= \frac{b}{n}\Sigma E(X_i)$$

$$= \frac{b}{n} \times \frac{\beta}{2} \times n$$

$$= \frac{b\beta}{2}$$

$$V(\hat{\beta}_2) = V(b\bar{X})$$

$$= b^2 V\left(\frac{\Sigma X_i}{n}\right)$$

$$= \frac{b^2}{n^2}\Sigma V(X_i)$$

$$= \cancel{b^2}\,\frac{3}{4}\frac{b^2\beta^2}{n}$$

$$MSE = V(\hat{\beta}_2) + (Bias)^2$$

$$= \frac{3b^2\beta^2}{4n} + \left(\frac{b\beta - 2\beta}{2}\right)^2$$

$$= \frac{3b^2\beta^2 + nb^2\beta^2 + 4n\beta^2 - 4n\beta^2 b}{4n}$$

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$$M = \frac{\beta^2}{4n}(3b^2 + nb^2 + 4n - 4nb)$$

$$\frac{dM}{db} = \frac{\beta^2}{4n}(6b + 2nb - 4n)$$

$$= 0.$$

$$\frac{\beta^2}{4n}(6b + 2nb - 4n) = 0.$$

$$3b + nb - 2n = 0.$$

$$b(3+n) = 2n$$

$$b = \frac{2n}{3+n}$$

$$b = \frac{2}{1 + \frac{3}{n}}$$

$$\frac{d^2M}{db^2} = \frac{\beta^2}{4n}(6+2n)$$

$$= \frac{\beta^2}{4n}(6+2n) > 0$$

Hence, $\frac{d^2M}{db^2}$ is minimum.

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$b = \frac{2}{(1+3/n)}$ merimises the MSE

(iv) $E(\hat{\beta}_2) = \frac{b\beta}{2}$.

$= \frac{2}{(1+3/n)} \times \frac{\beta}{2}$

$= \frac{n\beta}{(n+3)}$

Bias $= E(\hat{\beta}_2) - \beta$

$= \frac{n\beta - \beta(n+3)}{n+3}$.

$= \frac{-3\beta}{n+3}$.

$MSE = \frac{\beta^2}{4n}(3b^2 + nb^2 + 4n - 4nb)$.

$= \frac{\beta^2}{4n}\left[b^2(n+3) + 4n(1-b)\right]$

$= \beta^2\left[\frac{4n^2}{n+3} + \frac{4n(3-n)}{n+3}\right]$

$= \beta^2\left[\frac{4n^2 + 12n - 4n^2}{n+3}\right]$

$$MSE = \frac{12n\beta^2}{n+3}.$$

As $n \to \infty$, MSE does not tend to zero
Hence it is inconsistent

Q.5) i) $X \sim U(a, b)$

$\rightarrow E(X) = \frac{a+b}{2}.$

$$\bar{X} = \frac{a+b}{2}.$$

$$a = 2\bar{X} - b \quad \text{—(i)}$$

$\rightarrow V(X) = \frac{(b-a)^2}{12}.$

$$= S^2 = \frac{(b-a)^2}{12}$$

$$= 12S^2 = (b-a)^2$$

$$= 2\sqrt{3}S = b-a.$$

$$= 2\sqrt{3}S = 2\bar{X} - a - a.$$

$$= 2\sqrt{3}S = 2(\bar{X} - a)$$

$$= \sqrt{3}S = \bar{X} - a.$$

$$\hat{a} = \bar{X} - \sqrt{3}\,S.$$

ii) $b = 2\bar{X} - a$.

$b = 2\bar{X} - \bar{X} + \sqrt{3}S$.

$\hat{b} = \bar{X} + \sqrt{3}S$.

iii) Sample of $obs^n = 1, 2, 8, 50, 150$

$\bar{X} = 42.2$.

$S = 63.57$.

$a = \bar{X} - \sqrt{3}S$

$= -67.906$

$b = \bar{X} + \sqrt{3}S$

$= 152.3064$.

Hence we can conclude ~~of~~ that $\hat{b}$ is approx very close to sample value but $\hat{a}$ is very far from the true sample value, so ~~it~~ it is a potential weakness of MOM.

(iv) $X \sim U(0, b)$

$L = \prod_{i=1}^{n} f(x_i)$

$= \left(\frac{1}{b}\right)^n$

$\log L = -n \log b$

$\frac{dl}{db} = \frac{-n}{b}$.

$\frac{dl}{db} = 0$, gives $b = \infty$ which is not possible

Also

$$\frac{d^2 l}{db^2} = \frac{n}{b^2}$$

$\frac{n}{b^2} > 0$, which we have a minimum livelihood estimate

Thus $\hat{b} = max(x_i)$

Q.6) $H_0 : p = 0.1$
$H_1 : p > 0.1$

i) P(Type I error) = P(rejecting H_0 | H_0 is true)

Let X be the no. of hits in first 12 missiles and Y be the number of hits in 12 missiles

$P(\text{Type I Error}) = P(X \geq 3 | p = 0.1) + P(X = 0 | p = 0.1)$
$P(Y \geq 5 | p = 0.1) + P(X = 1 | p = 0.1) P(Y \geq 4 | p = 0.1)$
$+ P(X = 2 | p = 0.1) P(Y \geq 3 | p = 0.1)$

$= (1 - 0.8891) + 0.2824(1 - 0.9957)$
$+ (0.6590 - 0.2824)(1 - 0.9744)$
$+ (0.8891 - 0.6590)(1 - 0.8891)$
$= 0.15727 \approx 15\%$

ii) probablity of regretting null hypothesis. p= $p=0.3$

$= P(X \geq 3 \mid p=0.3) + P(X=0 \mid p=0.3)\, P(X \geq 5 \mid p=0.3) + P(X=1 \mid p=0.3)\, P(X \geq 4 \mid p=0.3) + P(X=2 \mid p=0.3)\, P(\cancel{X}Y \geq 3 \mid p=0.3)$

$= (1-0.2528) + 0.0138\,(1-0.7237) + (0.0850-0.0138)(1-0.4925) + (0.2528-0.0850)(1-0.2528)$

$= 0.9125 \approx 91\%$

iii) Probablity of type II error

Accepting H_0 where H_0 is false.

$= 1 - 0.91253$

$= 0.08747$

$\approx 9\%$ (where $p = 0.3$)

iv) 10 Space agencies fired 120 missile and recorded H_0 Hits.

$$\frac{X/n - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$$

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$n = 120, X = 40$

$\hat{p} = \frac{40}{120} = 0.333333$

$z_{10\%} = 1.2816$

Lower bound of 90% right tailed confidence interval for p is

$\hat{p} - 1.2816\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.3333 - 1.2816\sqrt{\frac{0.33(1-0.33)}{120}}$

$= 0.2782$

v) $p > 0.278$ at 10% LOS

$H_0 : p = 0.278$ vs $H_1 : p > 0.278$

One Sample binomial model.

$\frac{X - np_0}{\sqrt{np_0q_0}} \sim N(0,1)$ with continuity correction

$\frac{39.5 - 120(0.278)}{\sqrt{120 \times 0.278 \times 0.722}} = 1.2511$

insufficient evidence.

(vi) Lower bound of confidence interval implies there ~~is~~ a 10%. change of $p \leq 0.2782$ wh from the hypothesis test $\hat{p}_T$ could be less than or equal to 0.2780 with probablity of more than 10%.

vii) $H_0 : \mu_1 = \mu_2$ Vs $H_1 : \mu_1 \neq \mu_2$

$$T.S = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

Given

$\bar{X}_1 = 65$

$\bar{X}_2 = 70$

$S_1 = 54$

$S_2 = 70$

$n_1 = 12$

$n_2 = 15$

$$S_p^2 = \frac{1}{25}(11(2916 + 14(4900)) = 4027.04$$

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$$\frac{65-70}{65.451\sqrt{\frac{1}{12}+\frac{1}{15}}} = -0.2034.$$

There is $\pm 2.060 (= t_{25}, 2.5\%)$.

So we have insufficient evidence to reject H_0 at 5% level

Q. 7) ~~2 sided t-test~~

i) Public

$\bar{X}_1 = 30$

$S_1^2 = 64.3125$

Private

$\bar{X}_2 = 35.0555$

$S_2^2 = 67.40217$

ii) 2 sided t-test can be applied in case the samples come from populations with equal variance

We are testing $H_0 : \sigma_1^2 = \sigma_2^2$ Vs $H_1 = \sigma_1^2 \neq \sigma_2^2$

test statistics is $\frac{S_1^2 / \sigma_1^2}{S_2^2 / \sigma_2^2} \sim F_{n_1-1, n_2-1}$.

Value of statistics $\frac{64.31}{67.42} = 0.9542$

$F_{8,8}$ values at 5% levels are 0.22568 4.433.
Since the value of the test statistics b/w the above values, we have insufficient evidence to reject the hypothesis and ~~[illegible]~~

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iii) $H_0 : \mu_{Pr} = \mu_{Pu}$

$H_1 : \mu_{Pr} > \mu_{Pu}$

$$T.S = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{35.0555 - 30}{8.1152\sqrt{\frac{2}{9}}}$$

$= \cancel{1.312}\ 1.32151$

tabulated value $= t_{16, 5\%}$

$= 1.746$

insufficient evidence to reject H_0

Q8) i) Unbiased estimator

→ An unbiased estimator is an estimator where expected value is equal to the parameter.

→ Estimator is said to be unbiased if bias is equal to the parameters

ii) biased

→ A biased stimator is one which delivers an estimate which is consistently different from the parameter to be estimated.

iii) A biased is one which delivers are estimate which is consistency different from the parameter to be estimated.

iii) $MSE(\hat{\theta}) = Var(\theta) + Bias^2$

iv) The estimater having less MSE is more consistent

v) They both would be considerred to be consistent when MSE merimises such that when, $n \to \infty$, $MSE \to 0$

vi) i) Method of Moments

ii) Maximum likelihood estimate

Q.9) i) Variable t_k is defined as $t_k \Rightarrow \frac{N(0,1)}{\sqrt{\chi^2_k / k}}$

where k denotes the degree of freedom and the random variables $N(0,1)$ and χ^2_k are independent

ii) For $k > 2$

$E(t_k) = 0$

$$V(t_k) = \frac{k}{k-2}$$

iii) a) $\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1}$

$$P\left(t_{9,0.5\%} < \frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{9,98.5\%}\right) = 0.99$$

$$P\left(\bar{x} - t_{9,0.5\%} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{9,0.5\%} \frac{s}{\sqrt{n}}\right) = 0.99$$

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$t_{9, 0.5\%} = 3.250$

99% confidence interval for μ =

$$= \left(50 - 3.25 \times \sqrt{\frac{48.667}{10}}, 50 + 3.25 \times \sqrt{\frac{48.667}{10}}\right)$$

$= \cancel{42.83}$ $42.83, 57.17$

b) From (ii),

$$t_{n-1} \sim N\left(0, \sqrt{\frac{n-1}{n-3}}\right)$$

$$t_{n-1} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$$

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$$

$$\frac{\bar{x} - \mu}{S/\sqrt{7n/9}} \sim N(0, 1)$$

Critical values of confidence 2.58

$CI = (43.54, 56.45)$

Q.10) $n = 1000$

$\lambda = 3$

$X \sim Poi(n\lambda)$

$X \sim Poi(3000)$

P(Type I error) = P(reject H_0 | H_0 is true)

$= P(X > 3100 \mid X \sim Poi(3000))$

$P(X > 3100) = P\left(Z \frac{3100 - 3000}{\sqrt{3000}}\right)$

$= 3.39\%$

ii) Type II error = event of accepting the hypothesis when it is false

$= P[X > 3100 \mid X \sim Poi(n\lambda) - N(n\lambda; n\lambda)]$

$= 1 - P\left(Z < \frac{3100 - n\lambda}{\sqrt{n\lambda}}\right)$

iv) $\hat{\mu}$ is estimator of μ, $\hat{\mu}$ follows $N\left(\mu, \frac{\hat{\mu}}{n}\right)$

Hence, $\frac{\hat{\mu}-\mu}{\sqrt{\hat{\mu}/n}} \sim N(0,1)$

CI for μ is $(2.7613, 3.0387)$

Q. 114 $n = 12$

$\Sigma X = 213,200$

$\Sigma X^2 = 4,919,860,000$

$\bar{X} = 17,767$

$S = 10,144$.

New Sample mean

$\Sigma X' = \Sigma X - 11,000 - 48,000 + 27500 + 31,500$

$= 2,13,200$

$\bar{X}' = \frac{213200}{12} = 17,767$

New Sample standard deviation.

$\Sigma X^{2'} = 4,919,860,000 - (11,000)^2 - (48,000)^2 + (27,500)^2 + (31,500)^2$

$= 4,223,360,000$

$$s'^2 = \frac{1}{n-1}\left(\Sigma x^{2\prime} - n(\bar{x}')^2\right)$$

$$= \frac{1}{11}\left(4,243,360,000 - 12(17,767)^2\right)$$

$$= 41,396,775.64$$

$$s' = 6434.032611$$

Q.12) i) The Cramer Rao lower Bound result holds under very general conditions except where the where the range of the distribution involves the parameters, such as uniform distribution in this case.

This is due to discontinuity.

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ii) $Y = \max X_i$

For $0 < y < \theta$

$$P[Y < y] = P[\max X_i < y]$$

$$= P\left[\prod_{i=1}^{n} (x_i < y)\right]$$

$$= \prod_{i=1}^{n} P(X_i < y) \quad [\because X_i\text{'s are independent}]$$

$$= \prod_{i=1}^{n} \left[\int_0^y \frac{dx}{\theta}\right]$$

$$= \left(\frac{y}{\theta}\right)^n$$

PDF of Y will be

$$g_Y(y) = \frac{d}{dy} P[Y < y] = \frac{n}{\theta} y^{n-1} \text{ for } 0 < y < \theta$$

Now for any non-ve real no. k

$$E[Y^k] = \int_0^\theta y^k \frac{n}{\theta^n} \cdot y^{n-1}\, dy$$

$$= \frac{n\theta^k}{n+k}$$

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iii) Bias of estimator $\hat{\theta}(c)$ is given as

$$\text{Bias}(\hat{\theta}(c)) = E(\hat{\theta}(c) - \theta)$$
$$= E(cY - \theta)$$
$$= c \cdot E(Y) - \theta$$
$$= c \cdot \frac{n\theta}{n+1} - \theta$$
$$= \left(\frac{cn}{n+1} - 1\right)\theta$$

Mean Square error

$$\text{MSE}[\hat{\theta}(c)] = E[\{\hat{\theta}(c) - \theta\}^2]$$
$$= E[\{cY - \theta\}^2]$$
$$= E[c^2Y^2 - 2c\theta Y + \theta^2]$$
$$= c^2 E(Y^2) - 2c\theta E(Y) + \theta^2$$
$$= c^2 \frac{n\theta^2}{n+2} - c \cdot \frac{2n\theta^2}{n+1} + \theta^2$$

iv) $\left(\frac{c_\mu n - 1}{n+1}\right) \cdot \theta = 0$

$$c_\mu = \frac{n+1}{n}$$

(v) In order minimize $MSE[\hat{\theta}(c)]$, we need

$$0 = \frac{d}{dc} MSE[\hat{\theta}(c)]$$

$$= \frac{d}{dc}\left[\frac{c^2 \cdot n\theta^2}{n+2} - \frac{c^2 n\theta^2}{n+1} + \theta^2 \right]$$

$$= \frac{2c\, n\theta^2}{n+2} - \frac{2n\theta^2}{n+1}.$$

thus $C_m = \frac{2n\theta^2}{n+1} \cdot \frac{n+2}{2n\theta^2} = \frac{n+2}{n+1}.$

$$\frac{d^2}{dc^2} MSE[\hat{\theta}(c)] = \frac{d}{dc}\left[\frac{2c\, n\theta^2}{n+2} - \frac{2n\theta^2}{n+1} \right] =$$

$$\frac{2n\theta^2}{n+2} > 0.$$

$\Downarrow$

minimum

(vi) As n becomes large $\hat{\theta}(C_{\mu}) = \frac{n+1}{n} Y \to Y$.

Similarly $\hat{\theta}(C_m) = \frac{n+2}{n+1} Y \to Y$

Thus 2 estimates become one and same as $n \to \infty$.