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Probability & Stats

Assignment → I

Ques① (i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

⇒ Mean = $\frac{4+7+9+9+11+15+18+18+18+25}{10}$

$\bar{x} = \frac{\text{Sum}}{n}$

$\bar{x} \Rightarrow 13.4$

⇒ Median → $\frac{n^{th}/2 + \frac{n^{th}}{2}+1}{2}$

⇒ $\frac{11+15}{2} \Rightarrow 13$

⇒ Mode = 18

⇒ IQR ⇒ $q_2 = 13$, $q_1 = 8$, $q_3 = 18$

$q_3 - q_1$

⇒ 18 − 8

= 10

(ii)

No. of household	0	1	2	3	4
No. of people	5	11	26	15	3

⇒ $\bar{x} = \frac{\Sigma f \cdot x}{\Sigma f} \Rightarrow \frac{0\times5 + 1\times11 + 2\times26 + 3\times15 + 4\times3}{5+11+26+15+3}$

⇒ $\frac{11+52+45+12}{60} \Rightarrow \frac{120}{60} = 2$

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⇒ **Median :** $n = 60$

$$\frac{\left(\frac{n}{2}\right)^{th} + \left[\left(\frac{n}{2}\right)^{th}+1\right]}{2} \Rightarrow \frac{(30)^{th}+31}{2} \Rightarrow 30.5$$

∴

So, median would be '2'.

⇒ **Mode : 2**

Ques ④

⇒ **IQR** : $q_2 = 2$

$q_1 = 1$

$q_3 = 3$

$IQR = q_3 - q_1$

$= 3 - 1 = (2)$.

IQR = 2

⇒ **Standard deviation**

x	f	$\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	5	2	4	20
1	11	2	1	11
2	26	2	0	0
3	15	2	1	15
4	3	2	4	12

$$S = \sqrt{\frac{\sum f(x-\bar{x})^2}{n-1}} \Rightarrow \sqrt{\frac{58}{59}} = 0.99149$$

Q1 (a) Standard deviation

x	$\bar{x}$	$(x-\bar{x})^2$
4	13.4	88.36
7	13.4	40.96
9	13.4	19.36
9	13.4	19.36
11	13.4	5.76
15	13.4	2.56
18	13.4	21.16
18	13.4	21.16
18	13.4	21.16
25	13.4	134.56

$$S.d = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

$$\Rightarrow \sqrt{41.648}$$

$$\Rightarrow \boxed{6.4535.}$$

Ques (2) ~~$X \sim \lambda 0.5$~~ $X \sim P(0.5)$.

(i) $P(X=0) \Rightarrow \frac{e^{-\lambda} \lambda^x}{x!}$

$$\Rightarrow \frac{e^{-0.5} (0.5)^0}{0!} = e^{-0.5} = 0.6065.$$

(ii) Let Y = no. of years with a claim

then $Y \sim$ binomial $(3, 0.3935)$.

$P(Y=1) \Rightarrow {}^3C_1 \times (0.3935)^1 (0.6065)^2 \Rightarrow$

$\Rightarrow 0.434.$

$5 \rightarrow \frac{2}{5}$

(ii) $X \sim \exp(0.5)$

$$P(X>2) = \int_2^{\infty} \lambda e^{-\lambda x} dx$$

OR

$$\Rightarrow 0.5 \int_2^{\infty} e^{-0.5x} \cdot \Rightarrow \frac{e^{-0.5x}}{-0.5} \times 0.5$$

$$\Rightarrow -\left[e^{-0.5x}\right]_2^{\infty} \Rightarrow -\left[e^{-\infty} - e^{-1}\right]$$

$$\Rightarrow -\left[0 - \frac{1}{e}\right)$$

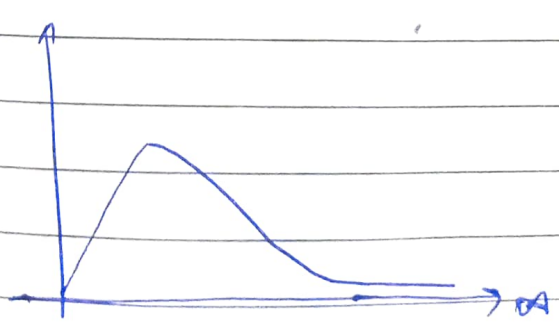
$$\Rightarrow \frac{1}{e} = 0.368$$

Ques (3) (i) $X \sim \text{Gamma}(2, 1/2)$

$$E[x] = \alpha/\lambda \Rightarrow \frac{2 \times 2}{1} = 4$$

$$V[x] = \alpha/\lambda^2 \Rightarrow 2 \times \frac{1}{1/4} = 8$$

(ii)



$$\text{Pdf} \Rightarrow \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\lceil \alpha}$$

increases in the beginning and then decreases when $\alpha = 2$, $\lambda = 1/2$.

(iii) Pdf $\Rightarrow \dfrac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\lfloor \alpha}$

Cdf : $\Rightarrow \displaystyle\int_0^x \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\lfloor \alpha}$

$\Rightarrow \dfrac{\lambda^{\alpha}}{\lfloor \alpha} \displaystyle\int_0^x \underset{II}{e^{-\lambda x}} \underset{I}{x^{\alpha-1}}$

$\Rightarrow \dfrac{\lambda^{\alpha}}{\lfloor \alpha} \times \dfrac{\lfloor \alpha}{\lambda^{\alpha}} \Rightarrow \dfrac{\lambda^{\alpha}}{\lfloor \alpha} \left[x^{\alpha-1} \right]_0^x$

$x^{\alpha-1} \displaystyle\int_0^x e^{-\lambda x} - \int_0^x (\alpha-1) \lfloor x^{\alpha-2} \times \int_0^x e^{-\lambda x} dx$

$\Rightarrow x^{\alpha-1} \times \left[\dfrac{e^{-\lambda x}}{-\lambda} \right]_0^x - \displaystyle\int_0^x (\alpha-1) x^{\alpha-2} \times \left[\frac{e^{-\lambda x}}{-\lambda} \right]_0^x$

$\dfrac{\lambda^{\alpha}}{\lfloor \alpha} x \Rightarrow x x^{\alpha-1} \times \left[\dfrac{e^{-\lambda x} - e^{0}}{-\lambda} \right] - \displaystyle\int_0^x (\alpha-1) \cdot x^{\alpha-2} \times \left[\frac{e^{-\lambda x} - 1}{-\lambda} \right]$

(iii) CDf of Gamma distribution.

$$\begin{cases} \int_0^x \frac{\lambda^\alpha x^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda x} & , x > 0 \\ 0 & , x < 0 \end{cases}$$

Ques (4) $P(L_1) = 0.4$, $P(L_2) = 0.5$, $P(L_3) = 0.1$

A ⇒ Package arriving late.

$P(A|L_1) \Rightarrow ?$ $A\{A|L_1$ $P(L_2|A) = ?$

$P(L_1|A) = P(A|L_1) = 0.2$

$P(A|L_2) = 0.4$

$P(A|L_3) = 0.1$

$$P(L_2|A) = \frac{P(L_2) \times P(A|L_2)}{P(L_1) \times P(A|L_1) + P(L_2) \times P(A|L_2) + P(L_3) \times P(A|L_3)}$$

$$\Rightarrow \frac{(0.5)(0.4)}{(0.4)(0.2) + (0.5)(0.4) + (0.1)(0.1)}$$

$$\Rightarrow \frac{0.20}{0.08 + 0.20 + 0.01} \Rightarrow \frac{0.20}{0.29} \Rightarrow \frac{20}{29}$$

$$\Rightarrow \boxed{0.6896}$$

Ques (6) $X \sim U(1,2)$ ⊛ Pdf of $Y = ?$

$$Y = \frac{3}{6-X}$$

$$f_Y(y) = P(Y \le y) = P\left(\frac{3}{6-X} \le y\right)$$

$$\Rightarrow P\left(\frac{3}{6-X} \le y\right) \Rightarrow P\left(3 \le y(6-X)\right)$$

Ques (6) $f_Y(y) = P(Y \le y)$

$$F_Y(y) = P\left(\frac{3}{6-X} \le y\right)$$

$$f_Y(y) = P\left(\frac{6-X}{3} \ge y\right)$$

$$\Rightarrow P(6-x \ge 3y)$$

$$\Rightarrow P(x-6 \le -3y)$$

$$= P(x \le 6-3y)$$

$$f_Y(y) \begin{cases} 0 & y > 2 \\ 6-3y & 1 \le y \le 2 \end{cases}$$

$$f'(y)\ y \begin{cases} 0 & y > 2 \\ -3 & 1 < y \le 2 \end{cases}$$

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Ques (7) (i) $f(3) = f(x \leq 3)$

$\Rightarrow 0.05 + 0.15 + 0.35$

$\Rightarrow 0.55$.

(2) $E(x) = 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$\Rightarrow \boxed{3.25}$

(3) $E(x^2) = 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$

$\Rightarrow 11.45$

$V(x) = E(x^2) - (E(x))^2 \Rightarrow 11.45 - (3.25)^2$

$\Rightarrow 0.8875$

(4) coefficient of skewness $\Rightarrow E(x-\bar{x})^3$.

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Ques (8) $X \sim B(15, 0.2)$

$P(X<3) \Rightarrow P(X=0) + P(X=1) + P(X=2)$

$\Rightarrow {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$1 \times (0.8)^{15} + 15 \times (0.2)(0.8)^{14} + 105 (0.2)^2 (0.8)^{13}$

$\Rightarrow 0.39802$

Ques (9) So, he won two prizes in 6 weeks.

So, $P(X=7) = {}^{6}C_2 (0.01)^3 (0.99)^4$.

$\Rightarrow 0.000014409$.

Ques (10) $X \sim P(0.4)$

$P(X>2) = 1 - P(X \leq 2)$

$\Rightarrow 1 - 0.99207$

$\Rightarrow 0.0793$

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Ques (11) P(Next three boys) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

Ques (12) $X \sim exp(10)$.

Req. $P(X < 0.1) \Rightarrow \int_0^{0.1} \lambda e^{-\lambda x} dx$

$\Rightarrow 10 \int_0^{0.1} e^{-10x} dx$

$\Rightarrow 10 \left[\frac{e^{-10x}}{-10} \right]_0^{0.1} \Rightarrow 10 \left[\frac{e^{-1} - 1}{-10} \right]$

$\Rightarrow 1 - e^{-1} = 0.6321$

Ques (13) $X \sim U(75, 120)$

$pdf \Rightarrow \int_{80}^{100} \frac{1}{b-a} dx \Rightarrow \frac{1}{b-a} [x]$.

Required Prob $\Rightarrow \frac{1}{45} [100 - 80]$

$\Rightarrow \frac{20}{45} = \frac{4}{9}$.

Ques (14)

$X \sim \text{Gamma}(5, 0.4)$

$\alpha = 5$

$\lambda = 0.4$.

$X \sim \text{Gamma}(\alpha, \lambda)$

then $2\lambda X \sim \chi^2_{2\alpha = r}$

$2 \times 0.4 \times X \sim \chi^2_{10}$.

$0.8X \sim \chi^2_{10}$.

$\Rightarrow P(X < 22.89)$.

$\Rightarrow P(2\lambda X < 22.89 \times 2\lambda)$

$\Rightarrow P(\chi^2_{10} < 22.89 \times 2 \times 0.4)$

$P(\chi^2_{10} < 18.312)$

So, 0.949898 lie

18.0 ⟶ 0.9450

18.5 ⟶ 0.9589

0.5 ⟶ 0.0079.

1 ⟶ $\frac{0.0079}{0.5}$

0.31 ⟶ $\frac{0.0079}{0.5} \times 0.31 \Rightarrow 0.004898$

18.50
18.3

18.31
18.00
0.31

So, 18.31 ⇒ 0.949898

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Ques 18 (1) Pdf $\Rightarrow f(x) = \frac{k}{(x+5)^4}$ $x > 0$

$$\int_0^\infty f(x)\,dx = 1$$

$$\Rightarrow k\int_0^\infty (x+5)^{-4}\,dx = 1$$

$$\Rightarrow \frac{k}{-3}\left[(x+5)^{-3}\right]_0^\infty = 1$$

$$\Rightarrow \frac{k}{-3}\left[\frac{1}{(x+5)^{-3}} - \frac{1}{(5)^{-3}}\right] = 1$$

$$\Rightarrow \frac{k}{-3}\left[0 - (5)^{-3}\right] = 1$$

$$\Rightarrow (5)^{-3}\,k = 3$$

$$k = \frac{3}{(5)^{-3}} \Rightarrow 3\times(5)^3 \Rightarrow 125\times 3 = 375.$$

(2) $f(10) = P(X \le 3) \Rightarrow \int_0^{10} f(x)\,dx.$

$$\Rightarrow 375\int_0^{10} (x+5)^{-4}\,dx \Rightarrow \frac{375}{-3}\times\left[(x+5)^{-3}\right]_0^{10}$$

$$\Rightarrow -125\left[(15)^{-3} - (5)^{-3}\right] \Rightarrow -125\left[\frac{1}{3375} - \frac{1}{125}\right]$$

$$\Rightarrow 0.962963$$

(iii) $E(x) \Rightarrow \int_0^\infty x f(x)\, dx$

$\Rightarrow 375 \int_0^\infty x \times (x+5)^{-4}\, dx \Rightarrow 375 \int_0^\infty x\,(x+5)^{-4}\, dx$

$(x+5) \rightarrow t$ $\quad$ let $x+5 \rightarrow t$

$dx \rightarrow dt$

$-4(x+5)^{-5}\, dx \rightarrow dt$

$dx \rightarrow$

$$\Rightarrow 375 \int_5^\infty (t-5)(t)^{-4}\, dt \Rightarrow 375 \int_5^\infty t^{-3} - 5t^{-4} \Rightarrow 375 \left[\frac{t^{-2}}{-2} - \frac{5t^{-3}}{-3} \right]_5^\infty$$

$$\Rightarrow 375 \left[-\left(\frac{5^{-2}}{-2} - 5\left(\frac{5^{-3}}{-3}\right) \right) \right] \Rightarrow 375 \left[-\left(\frac{1}{25(-2)} - \frac{1}{25(-3)} \right) \right]$$

$$\Rightarrow 375 \left[\frac{1}{25} \left(\frac{1}{2} - \frac{1}{3} \right) \right] \Rightarrow \frac{375}{25} \left[\frac{1}{6} \right]$$

$$\Rightarrow \frac{15}{6} \Rightarrow \frac{5}{2}$$

$\Rightarrow$ ~~0.25~~ 2.5.

$E(x) = 2.5$