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Q.1. Absolute Error = |approximate value - actual value|

$$= |12.65 - 12.5|$$

$$= 0.15.$$

Relative error = $\frac{\text{error value}}{\text{actual value}}$

$$= \frac{0.15}{12.5} = 0.012$$

Percentage error = Relative error × 100

$$= 1.2\%$$

Q.2 (a)

x	$f(x) = \log x$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6	0.7781513

Using lagranges interpolation $x_2 = 5$

let $x_0 = 4$ & $x_1 = 6$

$\therefore f(x_0) = 0.60206 \quad f(x_1) = 0.7781573$

$$f(x_2) = \frac{(x_2 - x_1)}{(x_0 - x_1)} \times f(x_0) + \frac{(x_2 - x_0)}{(x_1 - x_0)} \times f(x_1)$$

$$= \frac{(5 - 6)}{(4 - 6)} \times 0.60206 + \frac{5 - 4}{6 - 4} \times 0.7781573$$

$$= 0.69010565$$

Q.2(3) x $f(x)$

4.5 0.6532125814

5.5 0.7403862689

let $x_0 = 4.5$ $x_1 = 5.5$

$f(x_0) = 0.6532125814$ $f(x_1) = 0.7403862689$

$x_k = 5$

$$= \frac{(x_k - x_1)}{(x_0 - x_1)} \times f(x_0) + \frac{(x_k - x_0)}{(x_1 - x_0)} \times f(x_1)$$

$$= \frac{(5 - 5.5)}{(4.5 - 5.5)} \times 0.6532125814 + \frac{(5 - 4.5)}{(5.5 - 4.5)} \times 0.7403862$$

$= \cancel{0.96}$ 0.696787602

Q.3. $a = 1.5$; $b = 2$. Let $a = x_1$, $y = x_2$

$x^4 - x - 10$

$x_1 = 1.5$

$\therefore y_1 = (1.5)^4 - 1.5 - 10$

$y_1 = -6.4375$

$x_2 = 2$

$y_2 = (2)^4 - 2 - 10$

$y_2 = 4$

$x_3 = \frac{x_1 + x_2}{2} = \frac{3.5}{2} = 1.75$

$y_3 = (1.75)^4 - (1.75) - 10$

$= -2.37109375$

$x_4 = \frac{x_3 + x_2}{2} = \frac{1.75 + 2}{2} = 1.875$

$y_4 = 0.48461914$

~~$y_5 = x + y_1$~~ $x_5 = \frac{x_3 + x_4}{2} = \frac{1.75 + 1.875}{2}$

$x_5 = 1.8125$

$y_5 = -1.02024841$

$x_6 = \frac{x_5 + x_4}{2} = 1.8475$

$y_6 = -0.2847734032$

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21. $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
1	-1
2	5

Let $x_0 = 1$ $\quad f(x_0) = -1$, $f'(x_0) = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{(-1)}{2}$$

$$x_1 = 1 + \frac{1}{2} = \boxed{1.5}$$

$f(x_1) = 0.875$

$f'(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \frac{0.875}{5.75}$$

$x_2 = \boxed{1.347826081}$

$f(x_2) = 0.100682178$

$f'(x_2) = 4.449910286$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_3 = 1.325200947$

$f(x_3) = 0.00206000$

$f'(x_3) = 4.268247266$

6) $7A - (I+A)^3$

$= 7A - [I^3 + A^3 + 3A^2 I + 3AI^2]$

$= 7A - [I + A.A^2 + 3A^2 + 3A]$

$= 7A - [I + A.A + 3A + 3A]$ ... $\because A^2 = A$

$= 7A - [I + A^2 + 6A]$

$= 7A - [I + A + 6A]$ ... $(\because A^2 = A)$

$= 7A - [I + 7A]$

$= -I$

$\therefore 7A - (I+A)^3 = -I$

Q6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

Cofactor method:

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$|A| = 1(-6) + 1(-4 - 0) + 2(4 - 0)$

$= -6 - 4 + 8$

$= -2$

$C_{11} = -6 \quad C_{21} = 1 \quad C_{31} = -6$

$C_{12} = 4 \quad C_{22} = -1 \quad C_{32} = 2$

$C_{13} = 4 \quad C_{23} = -1 \quad C_{33} = 4$

$\therefore adj(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{(-2)} \begin{bmatrix} -6 & 1 & 6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 3 & -1/2 & -3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$

7. (a) $A = 8i - 9j$

$B = 7i - 9j$

$A \cdot B = |A| \cdot |B| \cos\theta$

$\therefore \cos\theta = \dfrac{A \cdot B}{|A| \cdot |B|}$

$A \cdot B = (8i - 9j) \cdot (7i - 9j)$

$= 56 + 81$

$= 137$

$|A| = \sqrt{(8)^2 + (-9)^2} = \sqrt{64+81}$

$= \sqrt{145}$

$= 12.0416$

$|B| = \sqrt{49+81} = \sqrt{130} = 11.4017$

$\cos\theta = \dfrac{137}{\sqrt{130} \times \sqrt{145}} = \dfrac{137}{137.2953022}$

$\cos\theta = 0.99784914$

$\therefore \theta = \cos^{-1}(0.99784914)$

$\theta = 3.75855°$

The angle between the vectors is $2.75856°$.

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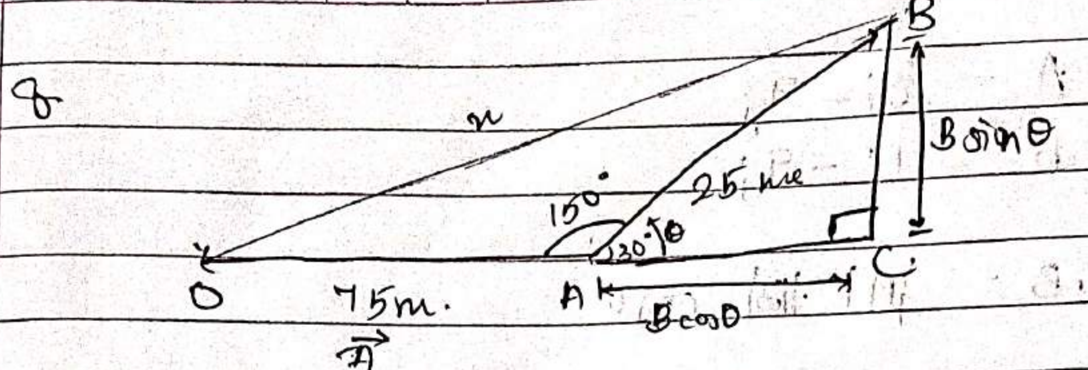

As per triangle law of vector addition.

$\therefore x^2 = (OA + B\cos\theta)^2 + (B\sin\theta)^2$

$x^2 = OA^2 + B^2 (\cos\theta^2 + \sin\theta^2) + 2AB\cos\theta$

$x^2 = OA^2 + B^2 + 2AB\cos\theta$

$\therefore x = \sqrt{OA^2 + B^2 + 2AB\cos\theta}$

$\therefore x = \sqrt{75^2 + 25^2 + 2(75)(25)\cos 30^\circ}$

$= \sqrt{6250 + 1875\times\sqrt{3}}$

$= \sqrt{9497.59526}$

$= 97.4556066$

9) Series of 3 digit number whose remainder is 2 when divided by 3.

101, 104, 107, 110, ... 998

This is an AP with $a = 101$, $d = 3$.

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$998 = 101 + 3n - 3$

$\therefore 3n = 900 \therefore \boxed{n = 300}$

Sum of number $= S_n$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{300}{2}[2\times101 + 299\times3]$

= 164850

Ans: The sum of all 3 digit numbers that leave a remainder of 2 when divided by 8 is 164850.

10. $t_6 = 32$. $t_8 = 128$.

The terms are in G.P.

$\therefore t_6 = ar^{6-1}$ $\qquad t_8 = ar^7$

$32 = ar^5$ $\qquad 128 = ar^7$

$\cancel{r = \frac{t_7}{t_6} = \frac{t_8}{t_7}}$

$\cancel{= \frac{ar^6}{32} = \frac{128}{ar^6}}$

$\frac{\cancel{a}r^{7}}{\cancel{a}r^5} = \frac{128}{32}$

$r^2 = 4$

$r = \pm 2$

$\therefore r$ is a ratio, $r = 2$.

Therefore, the common ratio of the G.P. is 2.

11 = $(4+3x)^5$

$$(a+b)^n = \sum_{y=0}^{n} {}^nC_y \, a^y \cdot b^{n-y}$$

$(4x+3x)^5 =$

$= {}^5C_0 (4)^0 (3x)^{5-0} + {}^5C_1 4^1 (3x)^{5-1} + {}^nC_2 (4)^2 (3x)^{5-2}$

$+ {}^5C_3 (4)^3 (3x)^{5-3} + {}^5C_4 (4)^4 (3x)^{5-4} +$

${}^5C_5 (4)^5 (3x)^0$

$= 1 \times 243x^5 + 5 \times 4 \times 81x^4 + 10 \times 16 \times 27x^3 +$

$10 \times 64 \times 9x^2 + 5 \times 256 \times 3x + 1024.$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024.$

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12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\lambda^3 - (2-5+3)\lambda^2 + (M_{11}+M_{22}+M_{33})\lambda - |A| I$

$|A| = \begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 2(-15) - (-3)(6) + 0(0)$

$= -30 + 18$

$= -12$

$M_{11} = -15$

$M_{22} = 6$

$M_{33} = -10 + 6 = -4$

$\lambda^3 - 0\lambda^2 + (-15+6-4)\lambda - (-12) I = 0$

$\lambda^3 - 13\lambda + 12 I = 0$

$\therefore$ ~~sum of~~ all = 0

$1 - 13 + 12 = 0$

$\therefore \lambda = 1$

1	1	0	-13	12
		1	1	-12
	1	1	-12	$\boxed{0}$

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda(\lambda+4) - 3(\lambda+4) = 0$

$(\lambda - 3)(\lambda + 4) = 0$

$\therefore \lambda = +3$ or $\lambda = -4$.

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$\therefore \lambda = 1, \quad \lambda = 3, \quad \lambda = 4$

Eigen vector for $\lambda = 1$ $\quad [A - \lambda I] x = 0$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-1 & -3-1 & 0-1 \\ 2-1 & -5-1 & 0-1 \\ 0-1 & 0-1 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 1 & -4 & -1 \\ 1 & -6 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$R_2 \longrightarrow R_2 - R_1$

$R_3 \longrightarrow R_3 + R_1$

$$\begin{bmatrix} 1 & -4 & -1 \\ 0 & -2 & 0 \\ 0 & -5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\lambda = 1, \quad \lambda = 3, \quad \lambda = -4.$

for $\lambda = 1$

Eigen vect = $[A - \lambda I] X = 0$

$$\therefore \left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 2-1 & -3 & 0 \\ 2 & -5-1 & 0 \\ 0 & 0 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \longrightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore \quad x_1 - 3x_2 = 0$

$2x_3 = 0$

$\therefore x_3 = 0.$

$x_1 = 3x_2$

$\therefore$ eigen vector for $\lambda = 3$

is $\begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} x_2$

$\lambda = 3.$

$$\left[\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right] X = 0$$

$$\begin{bmatrix} 2-3 & -3 & 0 \\ 2 & -5-3 & 0 \\ 0 & 0 & 3-3 \end{bmatrix} X = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \to R_2 + 2R_1$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore -x_1 - 3x_2 = 0$

$-2x_2 = 0$

$\therefore x_2 = 0$

$\therefore -x_1 = 0$

$\therefore x_1 = 0$

x_3 can not be determined $\therefore x_3 = \alpha.$

eigen vector for $\lambda = 3$ is $\begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$

$\lambda = -4$

$$\left[\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}\right] x = 0$$

$$\begin{bmatrix} 2+4 & -3 & 0 \\ 2 & -5+4 & 0 \\ 0 & 0 & 3+4 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 \to 3R_2 - R_1$, $R_1 \to R_1/3$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore \; 2x_1 - x_2 = 0$

$7x_3 = 0$

$\therefore x_3 = 0$

$x_2 = 2x_1$

Eigen vector for $\lambda = -4 = \begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} x_1$

13. $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

Using newton's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(x_0) = -13$

$f'(x_0) = 13$

$$x_1 = x_0 - \frac{-13}{13}$$

$x_1 = 5 - (-1)$

$x_1 = 6$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x_1) = 9$

$f'(x_1) = 32$

$$x_2 = 6 - \frac{9}{32}$$

$= 5.71875$

(14) $(1-x+x^2)^4$

let $(-x+x^2) = X$

$\therefore (1+X)^4 = (a+b)^4$

$\therefore$ Using binomial theorem.

$$(1+X)^4 = (1)^4 + {}^4C_1 (1)^3 (X)^1 + {}^4C_2 (1)^2 (X)^2 + {}^4C_3 (1)^1 (X)^3 + {}^4C_4 (1)^0 (X)^4$$

$$= 1 + 4 \times 1 \times X + 6 \times 1 \times X^2 + 4X^3 + X^4$$

$$= 1 + 4X + 6X^2 + 4X^3 + X^4$$

$$= 1 + 4(x^2-x) + 6(x^2-x)^2 + 4(x^2-x)^3 + (x^2-x)^4$$

$$= 1 + 4x^2 - 4x + 6(x^4 - 2x^3 + x^2) + 4(x^6 - 3x^5 + 3x^4 - x^3) + x^8 - 4x^7 + 6x^6 - 4x^5 + x^4$$

$$= 1 + 4x^2 - 4x + 6x^4 - 12x^3 + 6x^2 + 4x^6 - 12x^5 + 12x^4 - 4x^3 + x^8 - 4x^7 + 6x^6 + 4x^5 + x^4$$

$$= x^8 - 4x^7 + 10x^6 - 8x^5 + 19x^4 - 16x^3 + 10x^2 + 4x + 1$$

Q.15. $f(x) = 3\log(x) - e^{(x)}$

x	$f(x)$
1	-0.3678
2	0.767754.

$$x_3 = \frac{1+2}{2} = 1.5$$

$y_3 = 0.30514$

x	y
1	-0.3678
1.5	0.767754.

$$x_4 = \frac{1+1.5}{2} = 1.25$$

$y_4 = 0.0042252$

x	y
1	-0.3678
1.25	0.0042252

$$x_5 = \frac{1+1.25}{2} = 1.125$$

$y_5 = -0.1711949$

x	y
1.125	-0.1711949
1.25	0.0042252

$$x_6 = \frac{1.125+1.25}{2} = 1.1875$$

$y_6 = -0.081081914.$

x	y
1.1875	-0.081081
1.25	0.0042252

$$x_7 = \frac{1.1875+1.25}{2}$$

$= 1.21875$

$y_7 = -0.0378565547.$

$\therefore x \simeq -0.0378565547.$