

PRL-2 - Assignment 2

- ① Sensitivity analysis can be carried out at an individual policy level and at a portfolio level.
- At an individual policy level, sensitivity analysis allows the company to understand the impact of mis-estimating parameter values in the model
 - It can help show what the reductions would be in profits emerging, return on capital or other metrics targeted
 - This may help assess what margins may be included in the parameter values for the risks that are not borne out in reality.
 - At a portfolio level, sensitivity analysis can be used to assess the impact of shifts in mix of business.

- ② annual premium = 100,000 ; risk discount rate = 8.0%
 allocation % (1st year) = 90% ; allocation % (2nd year) = 10.5%
 interest on unit investments = 6.0% ; int. on non-unit reserves = 4.0%
 non charge = 0.50% ; BLO spread = 5.0%
 minimum death benefit = 5,00,000

	INR	% pfm	Total
Initial expense/commission			
renewal " / commission			

m

multiple decrement table:

X	$q_{x:n}^d$	$q_{x:n}^s$
40	0.000785	0.1
41	0.000962	0.05
42	0.001104	0.05
43	0.001208	0.05

x	$(a)^x$	$(2)^x$	$(a)^x$	$(2)^x$
40	0.000749	0.09996	0.899291	1.000
41	0.000938	0.04998	0.949026	0.899291
42	0.001076	0.04997	0.948951	0.853504
43	0.001178	0.004997	0.948352	0.809934

① option: a)

Unit fund (per policy at start of year)

value of units at	yr 1	yr 2	yr 3	yr 4
start of year	10000	90177	196809	309274
alloc	10000	101500	101500	101500
B/o	4500	5075	5075	5075
interest	5130	11196	11554	21500
mgmt. charge	453	989	17594	24342
value of units at yearend	90177	196809	309274	422891

② option: a)

cash flows (per policy at start of year)

	yr 1	yr 2	yr 3	yr 4
unlocated premium	10000	-1500	-1500	-500
B/o spread	4500	5075	5075	5075
expenses & commission's	12500	5000	5000	5000
interest	80	-57	-57	-57
mgmt charge	453	989	1554	2150.20
extra death benefit	307	289	205	85
end of year cash flow	2226	-777	-133	583

③

prop. in force

	1	0.899291	0.853504	0.809934
discount factor	0.945935	0.897338	0.84832	0.79829
expected NP of profit	1718.98			
premium signature	100000	83267.69	73174.5	64295.17
expected NP of premiums	326737.09			
profit margin	0.54%			

(iv)

option: a)

To calculate the expected provisions at the end of each year we have utilising the end of year cashflow figures & decrement tables in (i) above:

$$2V = 133 / 1.04 = 127.88$$

$$\text{cashflow in year 2} = -777 - (ap)_{41}(2V)$$

$$\Rightarrow \text{cashflow in year 2} = -777 - (ap)_{41}(127.88) = -898.37$$

$$\Rightarrow 1V = 898.37 / 1.04$$

$$\Rightarrow 1V = 863.82$$

These need to be adjusted as the 1st and 2nd for the values in respect of the beginning of the year. ∴ we have

$$\text{Year 2} : 127.88 (ap)_{41} = 121.37$$

$$\text{Year 1} : 863.82 (ap)_{40} = 776.83$$

(v)

option: a)

cashflow for years 2 & will be carried whilst year 1 will become:

$$2226 - 776.83 = 1449.17$$

	1449.17	0	0	583
prob. in force	1	0.899291	0.835509	0.809934
disc. factor	0.925925	0.857333	0.793832	0.750029
expected NPV profits	1688.90			

(3)

(i) The profit vector is the expected profit at the end of each policy year per policy in force at the beginning of the policy year eg $(-800, 1186, 178, 206, 219)$ could be the profit vector for a 5 year product.

(ii) The objective specified for expected level of profit is termed as "profit criterion".

eg NPV = 40% of initial sales commission

profit margin = 3% of EFTV of premium income

iii) risk discount rate = risk free rate + margin for risk
 $= 10\% + 3\% = 13\%$

EPV profit (Q.B.) = $-250v + 150v^2 + 200v^3 + 225v^4$
 $= 172.84$

1st year premium = P

EPV premium (Q.B.) = $P(1 + 0.4P_{42}v + 1.04^2 P_{42}v^2 + 1.04^3 P_{42}v^3)$
 $= P(1 + 0.9438 + 0.845094 + 0.77679)$
 $= 3.57186P$

profit margin = EPV profit / EPV premium

5% = $172.84 / 3.57186P$

$P = 976.17$

(i) Experience basis is used to calculate the expected future profits of a contract for comparison w/ a stated profit criterion. It represents the best estimate of expected future experience.

Pricing basis is used while setting premium.

Valuation basis is used to calculate the reserves to be held by an insurer.

(ii) While the prudent actuary is correct that reserves are held such that there is a low probability of insolvency occurring in the future it is incorrect to assume that everyone will die on the day after the valuation date. This is because a company whose reserving basis is extremely pessimistic will be holding extremely high reserves leading to large capital requirements. This will require higher profits resulting in higher premiums / charges for customer.

The reserving basis chosen has to satisfy any local legislation and preferential guidance.

(iii) we need to profit vectors of the policy in order to calculate

the non unit reserves. It can be divided by dividing the given profit signature w/ the prob. of surviving to the start of that year

$$= (-20, -35.79, 52.12, -30.65, -22.27, 104.4)$$

Let the reserves required at the end of policy year n be denoted as nV . No reserve is needed at end of 5th year $= 5V = 0$

$$4V = 22.27 / 1.06 = 21.01$$

$$3V = (30.65 + 21.01, P_{78} / 1.06) = 48.06$$

The cashflow of year 3 is sufficient to set up this reserve. Hence $2V = 0$, allowing for this, revised profit in 3rd year will be: $52.12 - 48.06$
 $= 5.54$

$$1V = 35.79 / 1.06 = 33.76$$

$$\text{revised profit for year 1} = -20 - 33.76, P_{70}) = -53.2$$

$$\therefore \text{revised profit vector} = (-53.2, 0, 5.54, 0, 0, 104.4)$$

⑤ i) Unit fund represents the value of unit of the policy holder at any time. It does not necessarily represent the amount that the policy holder is entitled to any point for eg. on surrender policy holder will get the surrender value which is the proportion of unit fund on death, maturity of surrender, the units held will be used to pay the benefit.

non-unit fund will arise from the following sources

a) premium less cost of allocation

b) expenses incurred by the insurer

c) interest earned on non-unit fund

d) FMC taken from unit fund.

ii) The life company sets up non-unit reserves if the overall cash flow in any month other than month one is negative w/o reserves.

Reserves are set up early so that money can be on $\pounds$ when required to eliminate the -ve cash flow.

Year	Age	Profit rester	q_n	P_n	$p(n-1)$	profit signature	PV
① 1	30	-250	0.00059	0.99941	1	(290)	(235)
2	31	-400	0.000602	0.999398	0.99941	(399)	(355)
3	32	-600	0.000617	0.999383	0.999888	(99)	(503)
4	33	1500	0.000636	0.999364	0.999192	(497)	1125

$$NPV @ 6\% = 91.18$$

② $2V = 600 / 1.03 = 582.52$

$$1V = 400 + 582.52(0.99941) / 1.03 = 953.57$$

$$\text{Year 1 revised cashflow} = -250 - 953.57(0.99941) = -1203$$

$$NPV = -1203 / 1.06 + 1185.99 = 51.08$$

③ The NPV after zeroising -ve cashflows to achieve a single financing phase is smaller. This is because the -ve cashflows have been accelerated hence being discounted less.

④ Decrement table

Age	Ind. prob. death	Ind. prob. surv.	sup. prob. surv.	sup. prob. mort.	prob. surv. at end	prob. surv. at end
61	0.006433	0.06	0.05981	0.00624	0.93375	1
62	0.009626	0.06	0.5971	0.00941	0.93089	0.93375
63	0.011344	0.06	0.5966	0.011	0.92934	0.92940
64	0.012716	0.06	0.5962	0.01233	0.92805	0.92944

Unit fund.

Year	1	2	3	4
Premium	30,000	30,000	30,000	30,000
Fund at start	—	11,740	38,723	69,525
Allocated premium	12,000	2,700	30,000	38,000
Less: bid offer spread	600	1,350	1,500	1,650
policy fee	—	—	—	—
Fund before interest	11,400	37,440	67,223	100,875
Plus: interest	570	1,875	3,361.14	5,048.76
Fund after interest	11,970	39,312	70,584	105,919
Less: FMC	180	590	1,059	1,589
Fund ended	11,790	38,723	69,525	104,330

Non-unit fund	18,000	30,000	—	(30,000)
unallocated prem	600	1,350	1,500	1,650
Add - b/o spread	—	—	—	—
policy fee	—	—	—	—
Less: commission	6,000	600	600	600
Less: expenses	5,000	200	2,000	2,000
Fund before interest	7,600	1,750	(1,100)	(3,950)
Add: Interest	266	61.25	(38.5)	(138.25)
Less: maturity benefit	—	—	—	4,841.16
Add: FMC	180	59	1,059	1,589
Less: additional death	550.43	576.32	315.34	—
non-unit cash flow	74,95.12	1824.12	(45.08)	(7340.63)

profit margin $\rightarrow 3.72\%$

Year	1	2	3	4
Profit	74,95.12	18,24.62	(415.08)	(7,340.63)
Pres. Surv at start	1	0.93299	0.86940	0.80772
Dis. factor	0.93457	0.87343	0.81629	0.762895
Pv profit	7,004.79	14,89.43	(294.55)	(4,524.73)
Dis. factor premium	1	0.93	0.87	0.81
Pv premium	30,000	26,185	22,781	19,280

ii) Profit margin could be higher/lower depending on the definition of the surrender benefit. If a sizeable penalty is imposed such that the unit price mostly covers the surrender benefit, profit margin could improve.

iii) To cover high initial expenses

⑧ Let P = annual premium payable.

$$P \ddot{a}_{61:4} = 500000 A_{61:4} + (100 + 0.25P)(\ddot{a}_{61:4} - 1) + 800$$

$$P(3.730) = 500000(0.85654) + (100 + 0.25P)(3.730 - 1) + 800$$

solving for P

$$P(0.975(3.730) + 0.025) = 500000(0.85654) + 100(2.730) + 800$$

$$P = 429343 / 3.662$$

$$P = 117250$$

Reserve required on the policy per unit sum assured

$$1V_{61:4} = 1 - \frac{\ddot{a}_{62:3}}{\ddot{a}_{61:4}} = 0.23240$$

$$2V_{61:4} = 1 - \frac{\ddot{a}_{63:2}}{\ddot{a}_{61:4}} = 0.47582$$

$$3V_{61:4} = 1 - \frac{\ddot{a}_{64:1}}{\ddot{a}_{61:4}} = 0.73183$$

Year	Premium	$t-1 V_{61:4}$	Disc. Fac.	NPV premium
1	117250.7681	1	1	117250
2	"	0.94389	0.925928	102443
3	"	0.88799	0.857339	89263
4	"	0.83903	0.793832	77629

$$\text{NPV of premium} = 386,618.04$$

$$\text{profit margin} = 2.40\%$$

⑨ (i) calculating the dependent probabilities:

n	$(aq)_n^A$	$(aq)_n^S$	$(aq)_n$	$n-42$	$(aq)_n$
42	0.000922	0.099909	0.899170	1	
43	0.001150	0.09855	0.8988965		0.899170
44	0.00127	0.000	0.998673		0.808323

calculating non-unit cashflow:

Year	unallocated premium	B/o	Exp	comm.	Interest	annual maint. charges	death cost	mortality cost
1	7000	75	2000	1000	298.25	39.85	1554	0
2	2000	200	750	300	80.5	149.17	945	0
3	750	281.25	750	300	-4.81	283.48	0	3354.37

Year	end of year cashflow	prob. in force	disc. factor	EPV of profit	Premium signature	EPV of premium	profit margin
1	4384.56	1	0.98498		1000		
2	1370.22	0.899170	0.783147	3083.46	8991.7	24287	12.7%
3	-3144.48	0.808323	0.69080		8083.23		

(ii) Reserves do not reduce profits. They only defer the release of profits so if the interest earned on reserves is equal to the risk discount rate then there is no effect of deferring the profit as the opportunity cost on money held as reserves is zero. Profit margin will be same as calculated earlier.

(iii)

Since the mortality rates & surrender rate is the same in the valuation basis & pricing basis, we will be able to use the dependent prob. derived in (i)

$$2V = 4532.25 / 1.07 = 4235.75$$

$$1V \times 1.07 - (aq)_{43} \times 2V = -(-160)$$

$$\therefore 1V = 4269.06$$

These will need to be adjusted

$$\text{Year 2} = 4235.75 (aq)_{43} = 3807.71$$

$$\text{Year 1} = 4269 \times (aq)_{42} = 3838.61$$

(10) (i)

policy year	premium allocated	cost of allocation	plus fund b/d	Fund before charge	Fund mgmt. charge	Policy admin fees	Fund c/d
1	17500	15750	15750	16852.70	84.26	500	16268.24
2	27500	24750	41018.24	43884.52	219.45	500	43190.07
3	27500	24750	67920.07	72674.47	363.37	500	71811.10
4	27500	24750	96561	103320.38	516.6	500	102303

policy year	expected profit on allocation	profit by projecting expenses	by projecting the non-unit interest	cashflows are: fund mgmt. charge	policy admin fees	non-unit death cost	profit in each year
1	9250	3500	402.5	84.26	500	730.98	6005.78
2	250	1100	-59.5	219.45	500	324.45	-517.5
3	250	1100	-59.5	363.37	500	0	-46.13
4	250	1100	-59.5	516.6	500	0	107.1

(ii) The non-unit reserves are:

start of year 3 : $46.13 / 1.07 = 43.11$
 end of year 2 : $43.11 (1.0785) + 517.5 = 559.96$
 start of year 2 : $559.96 / 1.07 = 523.33$
 end of year 1 : $523.33 \cdot 0.985 = 515.48$

revised profits are:

policy year	net unit reserve	int on reserve	re. in reserves	profit ignoring reserves	profit allowing for reserves
1	0	0	515.48	6005.78	5490.3
2	523.33	36.63	-480.87	-517.5	0
3	43.11	3.02	-43.11	-46.13	0
4	0	0	0	107.1	107.1

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