Calculus
Assignment - 2

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1) $I = \int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} d\omega$

Let ~~2/~~ $2/\omega = t$

$2\left(\frac{-1}{\omega^2}\right) d\omega = dt$

$\frac{d\omega}{\omega^2} = -\frac{dt}{2}$

When $\omega = 1/50$, $t = 100$
When $\omega = 2$, $t = 1$
Substituting in I

$I = -\frac{1}{2}\int_{100}^{1} e^t \, dt$

[Using property
$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$]

$I = \frac{1}{2} \int_1^{100} e^t \, dt$

$= \frac{1}{2} \left[e^t\right]_1^{100}$

$= \frac{1}{2} \left(e^{100} - e^1\right)$

$I = 1.34405857 \times 10^{43}$

2) $I = \int \frac{2t^3+1}{(t^4+2t)^3} \, dt$

~~Let~~

$\cancel{I = \int \frac{2t^3+1}{[t(t^3+2)]^3} \, dt}$

$$\text{Let } (t^4+2t) = y$$
$$(4t^3+2)dt = dy$$
$$2(2t^3+1)dt = dy$$
$$(2t^3+1)dt = \frac{dy}{2}$$

Substituting in I

$$I = \frac{1}{2}\int \frac{dy}{y^3}$$
$$= \frac{1}{2}\left[\frac{y^{-2}}{-2} + c\right]$$
$$I = \frac{-1}{4y^2} - \frac{c}{2}$$

3) $f'(x) = x^4 + 3x - 9$

Integrating $f'(x)$

$$f(x) = \int (x^4+3x-9)$$
$$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$$

4) $f(x) = \begin{cases} 6 & , x > 1 \\ 3x^2 & , x \leq 1 \end{cases}$

$I = \int_{-2}^{3} f(x)\,dx$

[Using property
$\int_a^c g(y)\,dy = \int_a^b g(y)\,dy + \int_b^c g(y)\,dy$]

$I = \int_{-2}^{1} f(x)\,dx + \int_1^3 f(x)\,dx$
$= \int_{-2}^{1} 3x^2\,dx + \int_1^3 6\,dx$
$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + [6x]_1^3$
$= (-2)^3 - 1 + 6(3) - 6$
$= -8 - 7 + 18$
$I = 3$

5) $I = \int 3(8y-1)\,e^{4y^2-y}\,dy$

Let $(4y^2 - y) = t$
$(8y-1)\,dy = dt$

Substituting in I

$I = 3\int e^t\,dt$
$I = 3(e^t + c)$
$I = 3e^{4y^2-y} + C$

6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$, $f(1) = -5/4$, $f(4) = 404$

Integrating $f''(x)$

$$f'(x) = \int (15(x)^{1/2} + 5x^3 + 6)\,dx$$

$$f'(x) = \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + C_1$$

$$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$$

Integrating $f'(x)$

$$f(x) = \int (10x^{3/2} + 5/4\, x^4 + 6x \cancel{dx} + C_1)\,dx$$

$$= \frac{10x^{5/2}}{5/2} + \frac{5x^5}{4\times 5} + \frac{6x^2}{2} + \cancel{x}\, C_1 x + C_2$$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1 x + C_2$$

At $x = 1$

$$f(1) = -5/4$$

$$-5/4 = 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + C_1(1) + C_2$$

$$-5/4 = 4 + \frac{1}{4} + 3 + C_1 + C_2$$

$$C_1 + C_2 = -\frac{5}{4} - \frac{1}{4} - 4 - 3$$

$$C_1 + C_2 = \frac{-6-28}{4}$$

$$C_1 + C_2 = \frac{-34}{4}$$

$$C_1 + C_2 = -\frac{17}{2} \rightarrow (1)$$

$$C_2 = -\frac{17}{2} - C_1 \rightarrow (1)$$

At $x = 4$

$f(4) = 404$

$404 = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + 4c_1 + c_2$

$4c_1 + c_2 = 404 - 128 - 256 - 48$

$4c_1 + c_2 = -28$

Substitute eq. ①

$4c_1 + (-17/2 - c_1) = -28$

$3c_1 = -28 + \frac{17}{2}$

$3c_1 = \frac{-56 + 17}{2}$

$c_1 = \frac{-39}{2 \times 3}$

$c_1 = \frac{-13}{2}$

Substitute in eq ①

$c_2 = \frac{-17}{2} - \left(\frac{-13}{2}\right)$

$c_2 = \frac{-4}{2}$

$c_2 = -2$

So, $f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13}{2}x - 2$

7) $\cancel{f(x+iy)+g(x\mp iy) = x+iy + x\mp iy} \quad 0$

~~Differentiat~~

7) $f(x+iy)+g(x-iy) = 0$

$f(x+iy) = -g(x-iy)$

$\cancel{f(x)+f(iy) = -g(x)+g(iy)} -g(-iy)$

$f(x)+g(x) = -g(-iy)-f(iy)$

Differentiating

$[f'(x)+g'(x)]dx = -[g'(iy)\cdot(i)+f'(iy)\cdot(i)]dy$

$[f'(x)+g'(x)]dx = -i[g'(iy)+f'(iy)]dy.$

8) $\frac{dr}{d\theta} = \frac{r^2}{\theta} \qquad r(1)=2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

Integrating both sides

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$\frac{1}{r} = \log|\theta| + c$

$\cancel{c = \frac{1}{r} - \log|\theta|}$

$r = [\log|\theta|+c]^{-1}$

$r(1) = 2$

$2 = [\log|1| + c]^{-1}$

$\cancel{c = 1/2 - \log|1|}$

$c = 1/2$

$r = [\log|0| + 1/2]^{-1}$

9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{dv}{9.8 - 0.196v} = dt$

Integrating both sides

$\int \frac{dv}{(9.8-0.196v)} = \int dt$

$\frac{\log|9.8-0.196v|}{-0.196v} = t + c$

$t = \frac{\log|9.8-0.196v|}{-0.196v} - c.$

10) $ty' + 2y = t^2 - t + 1 \qquad y(1) = 1/2$

$ty' = t^2 - t + 1 - 2y$

$y' = \frac{t^2 - t + 1 - 2y}{t}$

$y' = t - 1 + 1/t - 2y/t$

Let $\frac{y}{t} = v$

$y = vt$

$y' = v \cdot 1 + t v'$

$v + tv' = t - 1 + 1/t - 2v$

$tv' = t - 1 + \frac{1}{t} - 2v - v$

10) $ty' + 2y = t^2 - t + 1 \rightarrow (1) \quad y(1) = \frac{1}{2}$

$\cancel{t\frac{dy}{dt}} + \cancel{2y} =$

$$2y = t^2 - t + 1 - ty'$$

$$y(t) = \frac{t^2 - t + 1 - ty'}{2}$$

Substitute in eq. (1)

$$ty' + 2\left(\frac{t^2 - t + 1 - ty'}{2}\right) = t^2 - t + 1$$

$$2ty' + 2t^2 - 2t + 21 - 2ty' = (t^2 - t + 1)2$$

$$y(t) = \frac{t^2 - t + 1 - ty'}{2}$$

At $t = 1$

$$y(1) = \frac{\cancel{1} - \cancel{1} + 1 - (1)y'}{2}$$

$$\frac{1}{2} = \frac{1 - y'}{2}$$

$$1 = 1 - y'$$

$$y' = 1 - 1$$

$$y' = 0$$

Substituting in eq. (1)

$$t(0) + 2y = t^2 - t + 1$$

$$2y = (1) - 1 + 1$$

$$2y = 1$$

~~Integ~~

$$2y = t^2 - t + 1$$

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Integrating both sides

$$\int 2y\,dy = \int (t^2 - t + 1)\,dt$$

$$\frac{2y^2}{2} = \frac{t^3}{3} - \frac{t^2}{2} + t + c$$

$$y^2 = \frac{t^3}{3} - \frac{t^2}{2} + t + c$$

$$y(t) = \sqrt{\frac{t^3}{3} - \frac{t^2}{2} + t + c}$$

At $t = 1$

$$y(1) = \sqrt{\frac{1}{3} - \frac{1}{2} + 1 + c}$$

$$\frac{1}{2}\ \cancel{y(1)} = \sqrt{\frac{2 - 3 + 6}{6} + c}$$

$$\frac{1}{4} = \frac{5}{6} + c$$

$$c = \frac{1}{4} - \frac{5}{6}$$

$$c = \cancel{\frac{6-5}{24}}\ \frac{6-20}{24}$$

$$c = -\frac{14}{24}$$

$$c = -\frac{7}{12}$$

$$y(t) = \sqrt{\frac{t^3}{3} - \frac{t^2}{2} + t + -\frac{7}{12}}$$

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11) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $\qquad y(0) = -1$

$$\frac{y'}{y^3} = \frac{x}{\sqrt{1+x^2}}$$

Integrating both sides

$$\int y^{-3} y' = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

Let $(1+x^2) = t$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$$\frac{y^{-3+1}}{-3+1} = \int \frac{dt}{t^{1/2}}$$

$$\frac{y^{-2}}{-2} = \frac{t^{-1/2+1}}{-1/2+1} + c$$

$$\frac{y^{-2}}{-2} = \frac{(1+x^2)^{1/2}}{1/2} + c$$

$$\frac{1}{-2y^2} = 2\sqrt{1+x^2} + c$$

$$y^2 = [-\sqrt{1+x^2} - c/2]^{-1}$$

$$y = \cancel{\sqrt{\sqrt{1+x^2} - c}}$$

$$y^2 = [-\sqrt{1+x^2} + -c/2]^{-1}$$

$y(0) = 1$

$$(1)^{2} = [-\sqrt{1+0^2} - c/2]^{-1}$$

$$1 = -1 - c/2$$

$$c/2 = -1 - 1$$

$$c = -2$$

$$c = -1$$

$$y^2 = [-\sqrt{1+x^2} - 1]^{-1}$$

12) $f(x) = x^3 - 10x^2 + 6 \;,\; x = 3$

$n = 0 \;:\; f(x) = x^3 - 10x^2 + 6 \;,\; f(x=3) = -57$
$n = 1 \;:\; f'(x) = 3x^2 - 20x \;,\; f'(3) = -33$
$n = 2 \;:\; f''(x) = 6x - 20 \;,\; f''(3) = -2$
$n = 3 \;:\; f^{(3)}(x) = 6 \;,\; f^{(3)}(3) = 6$
$n \geq 4 \;:\; f^{(4)}(x) = 0 \;,\; f^{(4)}(3) = 0$

The Taylor expansion will be

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$x^3 - 10x^2 + 6 = \sum_{n=0}^{\infty} \frac{f^{(n)}(3)}{n!} (x-3)^n$$

$$x^3 - 10x^2 + 6 = (x-3)^0 \frac{(-57)}{0!} + (x-3)^1 \frac{(-33)}{1!} + (x-3)^2 \frac{(-2)}{2!} + (x-3)^3 \frac{(6)}{3!} + \sum_{n=4}^{\infty} \frac{0 \times (x-3)^n}{n!}$$

$$= -57 + -33(x-3) - (x-3)^2 + (x-3)^3 + 0$$

$$x^3 - 10x^2 + 6 = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

13) $f(x) = (1+x)^\mu$

$n = 0 \;:\; f(x) = (1+x)^\mu \;,\; f(0) = 1$
$n = 1 \;:\; f'(x) = \mu(1+x)^{\mu-1} \;,\; f'(0) = \mu$
$n = 2 \;:\; f''(x) = \mu(\mu-1)(1+x)^{\mu-2} \;,\; f''(0) = (\mu-1)\mu$
$n = 3 \;:\; f^{(3)}(x) = (\mu-2)(1+x)^{\mu-3} \;,\; f^{(3)}(0) = (\mu-2)(\mu-1)\mu$
$n = n \;:\; f^{(n)}(x) = (\mu-3)(1+x)^{\mu-4} \;,\; f^{(4)}(0) = \mu(\mu-3)(\mu-2)(\mu-1)$

Maclaurian series will be

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!}$$

$$(1+x)^\mu = 1 + \mu x + \frac{\mu(\mu-1)}{2!}x^2 + \frac{\mu(\mu-1)(\mu-2)}{3!}x^3 + \dots + \frac{\mu(\mu-1)(\mu-2)\dots(\mu-n+1)x^n}{n!}$$

14) $f(x) = \sqrt{1+x}$

$n=0$; $f(x) = \sqrt{1+x}$, $f(0) = 1$

$n=1$: $f'(x) = 1/2\,(1+x)^{-1/2}$, $f'(0) = 1/2$

$n=2$: $f^{(2)}(x) = -1/4\,(1+x)^{-3/2}$, $f''(0) = -1/4$

$n=3$: $f^{(3)}(x) = 3/8\,(1+x)^{-5/2}$, $f^{(3)}(0) = 3/8$

$n=4$: $f^{(4)}(x) = -15/16\,(1+x)^{-7/2}$, $f^{(4)}(0) = -15/16$

The Maclaurian series will be

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(0)\frac{x^n}{n!}$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x + \left(\frac{-1}{4}\right)\left(\frac{x^2}{2!}\right) + \left(\frac{3}{8}\right)\left(\frac{x^3}{3!}\right) + \left(\frac{-15}{16}\right)\left(\frac{x^4}{4!}\right) + \cdots$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots$$

15) $f(x) = e^{-6x}$, $x = -4$

$n=0$: $f(x) = e^{-6x}$, $f(-4) = e^{24}$

$n=1$: $f'(x) = -6e^{-6x}$, $f'(-4) = -6e^{24}$

$n=2$: $f''(x) = 36e^{-6x}$, $f''(-4) = 36e^{24}$

$n=3$: $f^{(3)}(x) = -216e^{-6x}$, $f^{(3)}(-4) = (-6)^3 e^{24}$

$n=4$: $f^{(4)}(x) = (-6)^4 e^{-6x}$, $f^{(4)}(-4) = (-6)^4 e^{24}$

$n=n$: $f^{(n)}(x) = (-6)^n e^{-6x}$, $f^{(n)}(-4) = (-6)^n e^{24}$

The Taylor series will be

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$$

$$= e^{24} + (-6e^{24})(x+4) + \frac{(-6)^2(e^{24})(x+4)^2}{2!} + \cdots + \frac{(-6)^n e^{24}(x+4)^n}{n!}$$

$$e^{-6x} = e^{24}\left(1 - 6(x+4) + 18(x+4)^2 + \cdots + \frac{(-6)^n(x+4)^n}{n!}\right)$$

Q16) $f(x) = \ln(3+4x) \quad , \quad x = 0$

$n = 0 : f(x) = \ln(3+4x) \quad , \quad f(0) = \ln 3$

$n = 1 : f'(x) = 4\left(\frac{1}{3+4x}\right) \quad , \quad f'(0) = 4/3$

$n = 2 : f''(x) = \frac{-4(4)}{(3+4x)^2} \quad , \quad f''(0) = -16/9$

$\cancel{n = 3 : f'''(x) = \frac{-(-1)^{3-1}(4)^3}{(3+4x)} \quad , \quad f^{(3)}(0)}$

$n = 3 ; f^{(3)}(x) = \frac{-4(-4)(4)(2)(3+4x)}{(3+4x)^4} \quad , \quad f^{(3)}(0) = 128/27$

The Taylor series will be

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$\ln(3+4x) = \ln 3 + \frac{4}{3}x - \frac{16}{9}\left(\frac{x^2}{2}\right) + \frac{128}{27}\left(\frac{x^3}{3!}\right) + \ldots$$

$$= \ln 3 + \frac{4}{3}x - \frac{8}{9}x^2 + \frac{64}{81}x^3 + \ldots$$