

* Numeric Algebra Methods.

Error, Dimensions & Interpolation.

I Lagrange's Interpolation

Q1 Given the known values

$$x = 9.5, \quad x_0 = 7, \quad x_1 = 8, \quad x_2 = 9, \quad x_3 = 10$$

$$y_0 = 3, \quad y_1 = 1, \quad y_2 = 1, \quad y_3 = 9$$

By using formula

$$y = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 \\ + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 \\ + \dots$$

$$\Rightarrow y = \left[\frac{(1.5)(0.5)(-0.5)}{(-1)(-2)(-3)} \right] 3 + \left[\frac{(2.5)(0.5)(-0.5)}{(1)(-1)(-2)} \right] 1 \\ + \left[\frac{(2.5)(1.5)(-0.5)}{(2)(1)(-1)} \right] 1 + \left[\frac{(2.5)(1.5)(0.5)}{(3)(2)(1)} \right] 9$$

$$\Rightarrow y = \frac{1}{1000} \left[\frac{75 \times 5}{2} \right] - \frac{1}{1000} \left[\frac{25 \times 25}{2} \right] + \frac{1}{1000} \left[\frac{25 \times 75}{2} \right] \\ + \frac{3}{1000} \left[\frac{25 \times 75}{2} \right]$$

$$\Rightarrow y = \frac{1}{16} \left[\frac{22}{88} \right] = 5.5$$

$\frac{1}{16}$
 y_2

So the answer is 5.5

Q2, x_i y_i

3	168
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$$\frac{120-168}{7-3} = -12$$

7	120
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$$\frac{-24-12}{9-3} = -2$$

$$\frac{72-120}{9-7} = -24$$

$$\frac{5+2}{10-3} = 1$$

9	72
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$$\frac{-9+24}{10-7} = 5$$

$$\frac{63-72}{10-9} = -9$$

10	63
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Here $n=6$

$$\begin{aligned} \Rightarrow f(6) &= 168 + (x-3)(-12) + (x+3)(x-7)(-2) + (x-3)(x-7)(x-9)(1) \\ &= 168 - 3 \times 12 + (3)(-1)(-2) + 3(-1)(-3)(1) \\ &= 168 - 36 + 6 + 9 = 147 \end{aligned}$$

Q3 $x=6$ $x_0=1$ $x_1=2$ $x_2=7$ $x_3=8$

$$f(x_0)=4 \quad f(x_1)=5 \quad f(x_2)=5 \quad f(x_3)=4$$

$$\begin{aligned} \Rightarrow f(6) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) \\ &+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) \\ &+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) \\ &+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3) \end{aligned}$$

$$\Rightarrow f(6) = \left[\frac{(4)(-1)(-2)}{(-1)(-6)(-7)} \right] 4 + \left[\frac{(5)(-1)(-2)}{(1)(-5)(-6)} \right] 5$$

$$+ \left[\frac{(5)(4)(-2)}{(6)(5)(-1)} \right] 5 + \left[\frac{(5)(4)(-1)}{(7)(6)(1)} \right] 4$$

$$\Rightarrow f(6) = \left[-\frac{8}{42} \times 4 \right] + \left[\frac{10}{30} \times 5 \right] + \left[\frac{-40}{-30} \times 5 \right] + \left[\frac{-20}{42} \times 4 \right]$$

$$= -\frac{32}{42} + \frac{50}{30} + \frac{200}{30} - \frac{80}{42}$$

$$= 0.76 + 1.66 + 6.66 - 1.90 \approx$$

$$\Rightarrow f(6) = 7.18$$

Q4. $n = 5$ $n_0 = 1$ $n_1 = 2$ $n_2 = 3$ $n_3 = 4$ $n_4 = 7$

$f(n_0) = 2$ $f(n_1) = 4$ $f(n_2) = 8$ $f(n_3) = 16$ $f(n_4) = 128$

$$\Rightarrow f(5) = \frac{(5-2)(5-3)(5-7)}{(1-2)(1-3)(1-7)} \times 2 + \frac{(5-1)(5-3)(5-7)}{(2-1)(2-3)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-7)}{(3-1)(3-2)(3-7)} \times 8 + \frac{(5-1)(5-2)(5-3)}{(7-1)(7-2)(7-3)} \times 128$$

$$= 35.6$$

$$\frac{\pi}{Q1} \quad x^3 + x - 1 = 0 \quad , [0.5, 1]$$

$$\begin{aligned} x_1 &= 0.5 \Rightarrow y_1 = [0.5]^3 + [0.5] - 1 \\ &= -0.375 \end{aligned} \quad \begin{aligned} x_2 &= 1 \\ y_2 &= 1^3 + 1 - 1 \\ &= 1 \end{aligned}$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{[0.5 \times 1] - [1 \times -0.375]}{1 - [-0.375]} = \frac{0.5 + 0.375}{1.375} = 0.6364$$

$$\text{Now, } x_1 = 0.6364 \quad x_2 = 1$$

$$\begin{aligned} y_1 &= [0.6364]^3 + 0.6364 - 1 \\ &= -0.105854 \end{aligned} \quad \begin{aligned} y_2 &= 1^3 + 1 - 1 \\ &= 1 \end{aligned}$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{[0.6364 \times 1] - [1 \times -0.105854]}{1 - [-0.105854]}$$

$$\Rightarrow \frac{0.6364 + 0.105854}{1.105854} = 0.6712043$$

$$\text{Now, } x_1 = 0.6712043 \quad x_2 = 1$$

$$\begin{aligned} y_1 &= [0.6712043]^3 + [0.6712043] - 1 \\ &= -0.026407 \end{aligned} \quad \begin{aligned} y_2 &= 1^3 + 1 - 1 \\ &= 1 \end{aligned}$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0.6712043 + 0.026407}{1 - [-0.026407]} = 0.6796634$$

$$\Rightarrow x_1 = 0.6796634 \quad x_2 = 1$$

$$\begin{aligned} y_1 &= [0.6796634]^3 + [0.6796634] - 1 \\ &= -0.0063713 \end{aligned} \quad \begin{aligned} y_2 &= 1^3 + 1 - 1 \\ &= 1 \end{aligned}$$

Now,

$$\Rightarrow 0.0063713 \rightarrow \text{Final answer.}$$

$$\textcircled{1} 2x^3 - 2x + 0.5 = 0$$

$$[0, 1]$$

$$x_1 = 0$$

$$y_1 = 0.5$$

$$x_2 = 1$$

$$y_2 = -0.5$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{[0 \times -0.5] - [1 \times 0.5]}{-0.5 - 0.5} = \frac{-0.5}{-1} = 0.5$$

$$\text{Now, } x_1 = 0.5$$

$$y_1 = -0.375$$

$$x_2 = 1$$

$$y_2 = -0.5$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{[0.5 \times -0.5] - [1 \times -0.375]}{-0.5 - [-0.375]} = \frac{-0.25 + 0.375}{-0.125} = -2$$

$$\textcircled{3} x^3 + 2x - 20 = 0$$

$$[2, 2.5]$$

$$x_1 = 2$$

$$y_1 = -8$$

$$x_2 = 2.5$$

$$y_2 = 0.625$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1.25 + 20}{8.625} = 2.4927536$$

$$\text{Now, } x_1 = 2.4927536$$

$$y_1 = 0.47503$$

$$x_2 = 2.5$$

$$y_2 = 0.625$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1.557971 - 1.187575}{0.14997} = 2.4698$$

$$\text{Now, } x_1 = 2.4698$$

$$y_1 = 0.00516$$

$$y_1 = 2.5$$

$$y_2 = 0.625$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1.543625 - 0.0129}{0.61984} = 2.4695$$

$$\text{Now, } x_1 = 2.4695$$

$$y_1 = -0.0009$$

$$x_2 = 2.5$$

$$y_2 = 0.625$$

$$\Rightarrow 2.4695 \rightarrow \text{Final answer.}$$

$$Q5 \ x^3 - 3x - 5 = 0$$

$$[2, 2.5]$$

$$x_1 = 2$$

$$y_1 = -3$$

$$x_2 = 2.5$$

$$y_2 = 3.125$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{6.25 - (-7.5)}{6.125} = 2.244897$$

$$\text{Now, } x_1 = 2.244897$$

$$y_1 = -0.421392$$

$$x_2 = 2.5$$

$$y_2 = 3.125$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{7.015303 + 1.05378}{3.546392} = 2.275209$$

$$\text{Now, } x_1 = 2.275209$$

$$y_1 = -0.04783$$

$$x_2 = 2.5$$

$$y_2 = 3.125$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{7.11002 + 0.119575}{3.17283} = 2.27859$$

$$\text{Now, } x_1 = 2.27859$$

$$y_1 = -0.00539$$

$$x_2 = 2.5$$

$$y_2 = 3.125$$

$$\Rightarrow 2.27859 \rightarrow \text{Final answer}$$

$$Q6 \ x \log_{10} x - 1.2 = 0$$

$$[2, 3]$$

$$\Rightarrow x_1 = 2$$

$$x_2 = 3$$

$$y_1 = \frac{-0.29691}{-0.59794}$$

$$y_2 = 0.23136$$

$$\frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0.46272 + 1.79382}{0.8293} = 2.72101$$

$$\Rightarrow x_1 = 2.72101$$

$$y_1 = -0.017094$$

$$x_2 = 3$$

$$y_2 = 0.23136$$

$$\frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{0.62953 + 0.051282}{0.248454} = 2.740193$$

$$\Rightarrow x_1 = 2.740193$$

$$y_1 = -0.00039$$

$$x_2 = 3$$

$$y_2 = 0.23186$$

$\rightarrow 2.740193 \rightarrow \text{Final answer}$

~~$$\textcircled{8} x^3 + 2x$$~~

$$\textcircled{7} x^3 - 4x + 1 = 0$$

$$\Rightarrow x^3 - 4[1] + 1 = -2 \quad -x$$

$$2^3 - 4[2] + 1 = 1 \quad +x$$

$\Rightarrow \text{Root lies b/w } [1, 2]$

$$\Rightarrow x_1 = 1$$

$$y_1 = -2$$

$$x_2 = 2$$

$$y_2 = 1$$

$$\frac{x_1 y_2 - y_1 x_2}{y_2 - y_1} = \frac{1 + 4}{3} = \frac{5}{3} = 1.6667$$

$$\text{Now, } x_1 = 1.6667$$

$$y_1 = -1.03689$$

$$x_2 = 2$$

$$y_2 = 1$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1.6667 + 2.07378}{2.03689} = 1.83636$$

$$\text{Now } x_1 = 1.83636$$

$$y_1 = -0.15283$$

$$x_2 = 2$$

$$y_2 = 1$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1.83636 + 0.30566}{1.15283} = 1.85805$$

$$\text{Now, } x_1 = 1.85805$$

$$y_1 = -0.01756$$

$$x_2 = 2$$

$$y_2 = 1$$

$$\Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{1.85805 + 0.03512}{1.01756} = 1.86049$$

$$\text{Now, } x_1 = 1.86049$$

$$y_1 = -0.002017$$

$$x_2 = 2$$

$$y_2 = 1$$

$1.86049 \rightarrow \text{Final answer}$

$$Q10 \quad x^3 - x - 4 = 0$$

$$\Rightarrow f(1) = 1^3 - 1 - 4 = -4$$

$$f(2) = 8 - 2 - 4 = 2.$$

Root lies b/w $[1, 2]$.

$$\Rightarrow \begin{aligned} x_1 &= 1 \\ y_1 &= -4 \end{aligned}$$

$$\begin{aligned} x_2 &= 2 \\ y_2 &= 2 \end{aligned}$$

$$\frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{2 + 8}{6} = \frac{10}{6} = 1.6667.$$

$$\Rightarrow \begin{aligned} x_1 &= 1.6667 \\ y_1 &= -1.03679 \end{aligned}$$

$$\begin{aligned} x_2 &= 2 \\ y_2 &= 2. \end{aligned}$$

$$\frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{3.3334 + 2.07358}{3.03679} = 1.78049$$

$$\Rightarrow \begin{aligned} x_1 &= 1.78049 \\ y_1 &= -0.13607 \end{aligned}$$

$$\begin{aligned} x_2 &= 2 \\ y_2 &= 2 \end{aligned}$$

$$\frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{3.56098 + 0.27214}{2.13607} = 1.79447$$

$$\Rightarrow \begin{aligned} x_1 &= 1.79447 \\ y_1 &= -0.01605 \end{aligned}$$

$$\begin{aligned} x_2 &= 2 \\ y_2 &= 2 \end{aligned}$$

$$\frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{3.58894 + 0.0321}{2.01605} = 1.796106$$

$$\Rightarrow \begin{aligned} x_1 &= 1.796106 \\ y_1 &= -0.00187 \end{aligned}$$

$$\begin{aligned} x_2 &= 2 \\ y_2 &= 2 \end{aligned}$$

$$\frac{x_1 y_2 - y_2 x_2}{y_2 - y_1} = \frac{3.592212 + 0.00374}{2.00187} = 1.79629$$

$$\Rightarrow \begin{aligned} x_1 &= 1.79629 \\ y_1 &= -0.00027 \end{aligned}$$

$$\begin{aligned} x_2 &= 2 \\ y_2 &= 2 \end{aligned}$$

1.79629 $\rightarrow$ Final answer