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ASSIGNMENT-1

:: NUMERICAL METHODS OF ALGEBRA ::

Q.1. Radius = 12.65 cm.
Actual length = 12.5 cm.

(i) Absolute error :

Area of circular plate $= \pi r^2$

$A'(r) = 2\pi r$

Change in Area $= A'(12.5)(0.15)$

$= 3.75\pi \text{ cm}^2$.

Exact calculation of change in Area

$= A(12.65) - A(12.5)$

$= 160.0225\pi - 156.25\pi$

$= 3.7725\pi \text{ cm}^2$.

(i) Absolute error = Actual value − Approximate value.

$= 3.7725\pi - 3.75\pi$

$\boxed{\text{Absolute error} = 0.0225\pi \text{ cm}^2.}$

(ii) Relative error $= \dfrac{\text{Actual value} - \text{Approximate value}}{\text{Actual value}}$

$= \dfrac{3.7725\pi - 3.75\pi}{3.7725\pi}$

$= \dfrac{0.0225\pi}{3.7725\pi}$

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$\boxed{\text{Relative error} = 0.006\ cm^2}$

(iii) Percentage error. $= \dfrac{\text{Relative error}}{} \times 100$

$= 0.006 \times 100$

$\boxed{\text{Percentage error} = 0.6\ \%}$

Q.2.

x	4	4.5	5.5	6.0
$f(x) = \log x$	0.60206	0.6532125	0.7403627	0.778151

(a) Interpolate log(5) using the points $x=4$ & $x=6$.

→ Linear Interpolation.

$$f(x) = f(x_0) + (x - x_0) f[x_1, x_0]$$

$x_0 = 4,\quad x_1 = 6 \longrightarrow f[x_1, x_0] = \dfrac{[f(6) - f(4)]}{6-4}$

$= 0.0880046$

$\therefore f(5) \approx f(4) + (5-4)\, 0.0880046 = 0.690106$

$\boxed{f(5) = 0.690106}$

(b) Interpolate log(5) using the points $x = 4.5$ & $x=5.5$

→ Linear Interpolation.

$\cancel{f[}\ x_0 = 4.5 \quad x_1 = 5.5$

$f[x_1, x_0] = \dfrac{[f(5.5) - f(4.5)]}{5.5 - 4.5} = 0.0871502$

$f(5) \approx f(4.5) + (5 - 4.5)\, 0.0871502$

$\boxed{f(5) = 0.696788}$

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Q.3. Find the root of $x^4 - x - 10 = 0$ approximately upto 5 iterations using Bisection Method.

Let $a = 1.5$ & $b = 2$.

→

$$f(x) = x^4 - x - 10$$
$$f(a) = (1.5)^4 - (1.5) - 10.$$
$$f(1.5) = -6.4375$$

$$f(b) = (2)^4 - 2 - 10$$
$$f(2) = 4.$$

1st:

$$c_1 = \frac{1.5 + 2}{2}$$

$$c_1 = 1.75$$
$$f(c_1) = (1.75)^4 - (1.75) - 10$$
$$= -2.37109375.$$

$b = 2.$ $f(b) = 4.$
$c_1 = 1.75$ $f(c_1) = -2.37109375$

2nd:

$$c_2 = \frac{1.75 + 2}{2}.$$

$$c_2 = 1.875$$
$$f(c_2) = (1.875)^4 - (1.875) - 10.$$
$$= 0.4846191406.$$

$c_2 = 1.875$ $f(c_2) = 0.4846191406$
$c_1 = 1.75$ $f(c_1) = -2.37109375.$

3rd:

$$c_3 = \frac{1.875 + 1.75}{2}$$

$$c_3 = 1.8125.$$

$$f(c_3) = (1.8125)^4 - (1.8125) - 10$$
$$= -1.020248413$$

$c_2 = 1.875$ $\quad$ $f(c_2) = 0.484691406$

$c_3 = 1.8125$ $\quad$ $f(c_3) = -1.020248413$

4^{th}:

$$c_4 = \frac{1.875 + 1.8125}{2}$$

$$c_4 = 1.84375$$
$$f(c_4) = (1.8435)^4 - (1.8435) - 10$$
$$= -0.2874840317$$

$c_2 = 1.875$ $\quad$ $f(c_2) = 0.484691406$

$c_4 = 1.84375$ $\quad$ $f(c_4) = -0.2874840317$

5^{th}:

$$c_5 = \frac{1.875 + 1.84375}{2}$$

$$c_5 = 1.859375$$
$$f(c_5) = (1.85935)^4 - (1.859375) - 10$$
$$= 0.09340312662$$

$\therefore$ Root of $x^4 - x - 10 = 0$ is approximately $\boxed{1.859375}$

Q.4. Find a root of an equation $f(x) = x^3 - x - 1$ using Newton Raphson method

$\rightarrow$

$$f(x) = x^3 - x - 1$$
Differentiate w.r.t. x
$$f'(x) = 3x^2 - 1$$

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x	$f(x)$
0	-1
1	-1
2.	5.

$x_0 = 2.$; $f(x_0) = 5$; $f'(x_0) = 11.$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$x_1 = 2 - \frac{5}{11}$$

$$x_1 = 1.5454545454.$$

$f(x_1) = 1.145755077.$ ~~$f'(x_1) =$~~
$f'(x_1) = 6.165289206.$ ~~$f'(x_1) =$~~

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

$$x_2 = 1.545454454 - \frac{1.145755077}{6.165289206}$$

$$x_2 = 1.359614908.$$

$f(x_2) = 0.1537048985.$
$f'(x_2) = 4.545658094.$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 1.359614908 - \frac{0.1537048985.}{4.545658094}$$

$$x_3 = 1.325801345.$$

$f(x_3) = 0.00462491$
$f'(x_3) = 4.2132476.$

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$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}.$$

$$x_4 = 1.325801345 - \frac{0.00462491}{4.2732476}.$$

$$x_4 = 1.324719051$$

$$f(x_4) = 0.0000046644.$$
$$f'(x_4) = 4.258928.$$

~~Root of $x^3 - x - 1$ is~~

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}.$$

$$x_5 = 1.324719051 - \frac{0.0000046644}{4.26464169}$$

$$x_5 = 1.324717957$$

$$f(x_5) = -0.000\ldots$$

$\boxed{\therefore \text{Root of } x^3 - x - 1 \text{ is } 1.324717957}$

Q.5. ~~Find a root of an equation $f(x)=$.~~ If A is a square matrix such that $A^2 = A$, then find the value of $7A - (I+A)^3$. here I is an identity matrix.

→ $A^2 = A$

$$\begin{aligned} 7A - (I+A)^3 &= 7A - (I^3 + A^3 + 3I^2A + 3IA^2) \\ &= 7A - (I + A^3 + 3A + 3A^2) \\ &= 7A - (I + A^2 \cdot A + 3A + 3A^2) \\ &= 7A - (I + A \cdot A + 3A + 3A) \end{aligned}$$

$$= 7A - (I + A \cdot A + 6A) \quad \left[\because I^2 = I,\ A^2 = A\right]$$
$$= 7A - (I + A^2 + 6A)$$
$$= 7A - (I + A + 6A)$$
$$= 7A - (I + 7A)$$
$$= 7A - I - 7A$$
$$= -I$$

$$\boxed{\therefore 7A - (I+A)^3 = -I}$$

Q.6. Find out the inverse of :-

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

Sol^n:

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{vmatrix}$$

$$|A| = 1(-6) + 1(-4) + 2(4)$$
$$= -6 - 4 + 8$$
$$= -10 + 8$$
$$= -2$$

Since $|A| \neq 0$, A^{-1} exists.

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$M_{11} = -6$ $\quad M_{21} = -1$ $\quad M_{31} = -6$

$M_{12} = -4$ $\quad M_{22} = -1$ $\quad M_{32} = -2$

$M_{13} = 4$ $\quad M_{23} = 1$ $\quad M_{33} = 4$

Now,

$A_{11} = (-1)^{1+1} \cdot M_{11} = -6$

$A_{12} = (-1)^{1+2} M_{12} = 4$

$A_{13} = (-1)^{1+3} M_{13} = 4$

$A_{21} = (-1)^{2+1} M_{21} = 1$

$A_{22} = (-1)^{2+2} M_{22} = -1$

$A_{23} = (-1)^{2+3} M_{23} = -1$

$A_{31} = (-1)^{3+1} M_{31} = -6$

$A_{32} = (-1)^{3+2} M_{32} = 2$

$A_{33} = (-1)^{3+3} M_{33} = 4$

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj}(A))$$

$$= \frac{1}{-2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\boxed{\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}}$$

Q.7 Using the definition of the scalar product, find the angle between the following vectors (Find the smallest non negative value).

a) $A = 8\hat{i} - 9\hat{j}$ & $B = 7\hat{i} - 9\hat{j}$

Sol^n-

The vectors are.

$\bar{A} = 8\hat{i} - 9\hat{j}$ & $\bar{B} = 7\hat{i} - 9j$.

We know,

$$\theta = \cos^{-1}\left[\frac{\bar{A}\cdot\bar{B}}{|A|\,|B|}\right]$$

$$= \cos^{-1}\left[\frac{(8\hat{i} - 9\hat{j})\cdot(7\hat{i} - 9\hat{j})}{|(8\hat{i} - 9\hat{j})|\;|(7\hat{i} - 9\hat{j})|}\right]$$

$$= \cos^{-1}\left[\frac{(8)(7) + (-9)(-9)}{\sqrt{(8)^2 + (-9)^2}\;\sqrt{(7)^2 + (-9)^2}}\right]$$ 137.

$$= \cos^{-1}\left[\frac{137}{(12.04)\,(11.40)}\right]$$

$$= \cos^{-1}\,\cancel{KIN}\,(0.9979)$$

$$= 3.71^\circ.$$

$$\therefore \theta = 3.71^\circ$$

$\therefore$ Thus, the angle between the two vectors is 3.71°.

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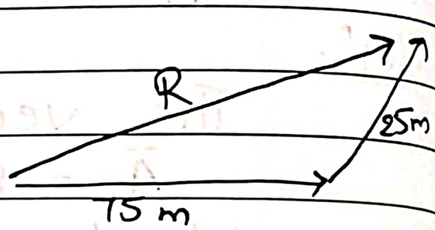
Q.8. Sabrina walked 75 meters to the east. Then she turned 30 degrees to the left & walked 25 meters. Determine the magnitude of Sabrina's displacement vector.

Soln :

R is the resultant vector.



$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{75^2 + 25^2 + 2\times75\times25\cos30^\circ}$

$= \sqrt{9497.595}$

$\boxed{\therefore R = 97.45 \text{ meter.}}$

Q.9. What is the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3?

Soln –

The lowest 3 digit number is 101 & highest number is 998

$\therefore$ The AP is 101, 104, 107, ..., 998

where,

- first term = a = 101
- ~~las~~ common difference = d = 3.
- nth term = a_n = 998

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We know,

$$a_n = a + (n-1)d$$
$$998 = 101 + (n-1)3.$$
$$998 - 101 = 3n - 3.$$
$$900 = 3n$$
$$\therefore n = 300$$

We also know sum to nth term in an A·P given by,

$$S = \frac{n}{2}(1st \text{ term} + \text{last term}).$$
$$S = \frac{300}{2}(101 + 998)$$
$$S = 164850$$

Hence, the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3 is 164,850

Q.10. The 6^{th} term of a G·P is 32 & its 8^{th} term is 128, then find the common ratio of the G.P.

Soln:

$$\left.\begin{aligned} T_6 &= 32 \\ T_8 &= 128 \end{aligned}\right\} \text{Given.}$$

We know,

$$T_n = a \cdot r^{n-1}$$
$$\therefore T_6 = a \cdot r^{6-1}$$
$$T_6 = a \cdot r^5$$
$$32 = a \cdot r^5 \quad \text{—} \quad (1)$$

and, $T_8 = a \cdot r^{8-1}$

$128 = ar^7$ — (2)

Dividing (2) by (1)

$$\frac{\cancel{a} \cdot r^7}{\cancel{a} \cdot r^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$\therefore r = 2$$

$$\boxed{\therefore \text{Common Ratio} = 2}$$

Q.11. Use the Binomial Theorem to expand $(4+3x)^5$.

Sol^n. According to ~~The~~ Binomial Theorem states ~~that~~,

$$\cancel{(a+b)^n} = \sum_{k=0}^{n} {}^nC_k \, \cancel{a^{n-k} b^k}.$$

$$\cancel{= a^n + n \cdot a^{n-1} b + \frac{n(n-1)}{2!} a^{n-2} b^2 +}$$

$$\cancel{\frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 +}$$

$$(a+b)^5 = a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5$$

$$\therefore (4+3x)^5 = (4)^5 + 5(4)^4(3x) + 10(4)^3(3x)^2 + 10(4)^2(3x)^3 + 5(4)(3x)^4 + (3x)^5.$$

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$\therefore (4+3x)^5 = 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

Q.12. Find all eigen values and corresponding eigen vectors for the matrix A if

$$\begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Solⁿ: Let $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\lambda^3 - (0)\lambda^2 + (-15+6)\lambda + 22 = 0$$
$$\lambda^3 - 9\lambda + 22 = 0.$$

Q.13. Use Newton's Method to determine x_2 for $f(x) = x^3 - 7x^2 + 8x - 3$ if $x_0 = 5$

Sol^n

$$f(x) = x^3 - 7x^2 + 8x - 3$$

Differentiate w.r.t x

$$f'(x) = 3x^2 - 14x + 8$$

Given that, $x_0 = 5$

$$\therefore f(x_0) = -13$$
$$f'(x_0) = 13$$

We know that,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \left\{\frac{-13}{13}\right\}$$

$$\therefore x_1 = 6.$$
$$f(x_1) = 9$$
$$f'(x_1) = 32$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 6 - \frac{9}{32}$$

$$\boxed{\therefore x_2 = 5.71875}$$

Q.14. Expand the following $(1-x+x^2)^4$.

Sol^n -

Put $(1-x) = y$

Then, $(1-x+x^2)^4 = (y+x^2)^4$.

$$(y+x^2)^4 = {}^4C_0\, y^4 (x^2)^0 + {}^4C_1\, y^3 (x^2)^1 + {}^4C_2\, y^2 (x^2)^2 + {}^4C_3\, y\, (x^2)^3 + {}^4C_4\, (x^2)^4$$

$$= y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8.$$

$$= (1-x)^4 + 4(1-x)^3x^2 + 6(1-x)^2x^4 + 4(1-x)x^6 + x^8.$$

$$= 1 - 4(1)^3(x) + 6(1)^2(x)^2 - 4(1)^1(x)^3 + x^4 + 4(1 - 3(1)^2(x) + 3(1)^1(x)^2 - x^3)x^2 + 6(1 - 2(1)(x) + x^2)x^4 + 4x^6(1-x) + x^8$$

$$= 1 - 4x + 6x^2 - 4x^3 + x^4 + 4x^2 - 12x^3 + 12x^4 - 4x^5 + 6x^4 - 12x^5 + 6x^6 + 4x^6 - 4x^7 + x^8.$$

$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8.$$

$$\boxed{\therefore (1-x+x^2)^4 = 1 - 4x + 10x^2 - 16x^3 + 19x^4 + 16x^5 + 10x^6 - 4x^7 + x^8.}$$

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Q.15 Find the appropriate value of x till 4 iterations for $e^{-x} = 3\log x$ using Bisection method.

Soln:

$$f(x) = e^{-x} - 3\log x$$

x	y
0.5	1.5096
1	0.36787
1.5	−0.30514

$\therefore x_0 = 1$ $\quad y_0 = 0.36787$

$x_1 = 1.5$ $\quad y_1 = -0.30514$

1st Iteration :

$$\therefore x_2 = \frac{1+1.5}{2} = 1.25$$

$$y_2 = -0.00422$$

$x_0 = 1$ $\quad y_0 = 0.36718$

$x_2 = 1.25$ $\quad y_2 = -0.00422$

2nd Iteration :

$$x_3 = \frac{1+1.25}{2} = 1.125$$

$$y_3 = 0.17119$$

$x_3 = 1.125$ $\quad y_3 = 0.17119$

$x_2 = 1.25$ $\quad y_2 = -0.0042$

3rd Iteration :

$$x_4 = \frac{1.125+1.25}{2} = 1.1875$$

$$y_4 = 0.08108$$

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$x_4 = 1.1875$ $y_4 = 0.08108$

$x_2 = 1.25$ $y_2 = -0.0042$

4^{th} Iteration :

$$x_5 = \frac{1.1875 + 1.25}{2} = 1.21875$$

$$y_5 = 0.037855$$

Hence we stop the iterations after 4
$\therefore$ The approximated value of x is 1.21875.