FM - Assignment

1) $d^{(4)} = 8\%$ p.a

$\frac{d^{(4)}}{4} = 2\%$ p.a

$i = (1-2\%)^{-4} - 1$

$= 8.416\,978473\%$

(i) $\Rightarrow \delta = \ln(1+i)$

$\delta = 8.081082927\%$

(ii) $i = 8.4165\%$

(iii) $(1+8.4165\%)(1+i) = \left(1 - \frac{d^{(12)}}{12}\right)^{-12}$

$d^{(12)} = \left(1 - (1+8.4165\%)^{-1/12}\right)12$

$d^{(12)} = 8.053933945\%$

2) $\delta(t) = 0.004t + 0.0002t^2$

i) $PV = 1000\, e^{-\int_0^{10} 0.004t + 0.0002t^2 dt}$

$= 1000\, e^{-\left[0.002t^2 + 0.0002\frac{t^3}{3}\right]_0^{10}}$

$= 1000\, e^{-[0.2 + 0.0667]} = 765.92$

ii) $(1+i)^{10} = e^{0.2667}$

$(1+i) = \sqrt[10]{1.305648693}$

$(1+i) = 1.02708827$

$i = 2.708827\%$

3) i) $d = iv$ $\quad v = \frac{1}{1+i}$

we know, $A = \frac{1}{1+i}$

$$\boxed{d = \frac{i}{1+i}}$$

ii) $\frac{d^{(4)}}{4} = 2.25\%$ $d^4 = 9\%$.

$d = 1 - (1 - 2.25\%)^4$

$= 8.7\%$

$d^{(4)} = 2(1 - (1 - 8.7\%)^{1/2}$

$= 8.89875\%$

4.i) d = iv we know, $V = \frac{1}{A} = \frac{1}{1+i}$

$d = \frac{i}{1+i}$

3 iii) 91-day government bill $= \frac{91}{365}$

4.i) d = iv we know, $V = \frac{1}{A} = \frac{1}{1+i}$

$d = \frac{i}{1+i}$

ii) $i = 0.08$

a) $d^{(12)} = (1.08)^{-1/12} = \left(1 - \frac{d^{(12)}}{12}\right)$

$d^{(12)} = 7.671477614\%$

b) $i^{(365)} = 365[(1+0.08)^{1/365} - 1]$

$= 7.688691554\%$

c) $\delta = \ln(1+i) = \ln(1.08) = 7.696104114\%$

d) $i^{(0.5)} = \left(1 + \frac{i}{0.5}\right)^{0.5}$ $i^{0.5} = 8.32\%$

$= 0.5((1+0.08)^{0.5} - 1]$

5) i) a) $d^4 = 6\%$

$d = 1 - \left(1 - \frac{6\%}{4}\right)^4$

$d = 5.866\%$

$i = (1-d)^{-1} - 1$

$i = 6.2319\%$

$i^{(2)} = 2[(1+6.2319\%)^{1/2} - 1]$

$= 6.137751552\%$

b) $d^{(12)} = 12[(1+6.2319\%)^{1/12} - 1]$

$= (1.062319)^{-1/12} = \left(1 - \frac{d^{(12)}}{12}\right)$

$d^{(12)} = 6.030230\%$

7) $PV = 20000$ $AV = 26000$

$n = \frac{17}{12}$ yrs

$26000 = 20000(1+i)^{17/12}$

$26(1+i)^{17/12} = 26$

$(1+i)^{17/12} = 1.3$

$(1+i) = 1.203457067$

$i = 20.3457067\%$

ii) $i^{(4)}$ $\Rightarrow$ $26000 = 20000\left(1+\frac{i^{(4)}}{4}\right)^{68/12}$

$\left(1+\frac{i^{(4)}}{4}\right)^{68/12} = 1.3$

$\sqrt[68/4]{1.3}$

$i^{(4)} = 18.9552546\%$

iii) $26000 = 20000\left(1+\frac{17i}{12}\right)$

$1.3 = \left(1+\frac{17}{12}i\right)$

$0.3 = \frac{17i}{12}$

$i = 21.1764706\%$

9) if $i^{(2)} = 0.0775$

$i = \left(1+\frac{0.0775}{2}\right)^2 - 1$

$= 7.90015625\%$

$\delta = \ln(1+7.9001)$

$= 7.604\%$

$(1+7.9\%) = \left(1-\frac{d^{(12)}}{12}\right)^{-12}$

$(1.079)^{-1/12} = 1-\frac{d^{(12)}}{12}$

$d^{(12)} = \left(1-(1.079)^{-1/12}\right)12$

$= 7.579\%$

$i^{(0.5)} = 0.5\left[(1+7.9\%)^{1/0.5}-1\right]$

$= 8.212\%$

10) $PV(t \geq 10) = e^{-\int_0^5 0.08}\left(e^{-\int_5^{10} 0.06}\right) e^{-\int_{10}^t 0.04}$

$= e^{-[0.08t]_0^5} e^{-[0.06t]_5^{10}} e^{-[0.04]_{10}^t}$

$= e^{-[0.07][0.06-0.3][0.04t-0.4]}$

$= e^{-[0.4+0.3+0.04t-0.4]}$

$= e^{-[0.04t+0.3]}$ $t \geq 10$

11) a) $n = \frac{9}{12}$ year $AV = 50{,}000$ $d = 18\%$

$PV = AV \times DF = 50000\left(1 - \frac{3}{4} \times 0.18\right)$

$= 43250$

b) $\delta = 6\%$

$\delta = \ln(1+i)$

$e^{\delta} = 1+i$

$e^{0.06} - 1 = i$

$i = 6.18365470\%$

$i^{(12)} = \left(1 - (1.06183654)^{-1/12}\right)12$

$= 5.985025\%$

$d^{(4)} = 6\%$

$d = 1 - \left(1 - \frac{6\%}{4}\right)^4$

$= 5.866344$

$i = (1-d)^{-1} - 1$

$= 6.231937\%$

12) Let AV be $= 2a$ $PV = a$ $i = 10\%$

$2a = a(1+i)^n$

Taking log on both sides

$\log 2 = n \ln 1.1$

$n = \frac{\log 2}{\ln 1.1}$

$n = 7.272$

ii) $AV = 2a$ $PV = a$ $d^{(12)} = 10\%$

$$2a = a\left(1 - \frac{d^{(12)}}{12}\right)^{-12n}$$

Taking log on both sides

$$\ln 2 = -12n \ln 0.99167$$

$$n = 6.905 \text{ years}$$