1) $X \sim Poi(\theta)$ $\qquad \frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$ $\qquad X = 2.0$

$\therefore \theta = x \pm 1.96 \left(\sqrt{\theta / n}\right)$

2) $X_i \sim N(\mu, \sigma^2)$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$E(\bar{X}) = \frac{E(\Sigma X_i)}{n} = \frac{n\mu}{n} = \mu$

$Var(\bar{X}) = \frac{Var(\Sigma X_i)}{n} = \frac{1}{n^2}(n\sigma^2) = \boxed{\frac{\sigma^2}{n}}$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ $\qquad E(S^2) = \frac{(n-1)\sigma^2}{(n-1)} = \sigma^2$

$Var(S^2) = \frac{2(n-1)\sigma^4}{(n-1)^2} = \boxed{\frac{2\sigma^4}{n-1}}$

$(50 < S^2 < 150)$ $\left(\frac{9 \times 50}{100} < \chi^2_9 < \frac{9 \times 150}{100}\right)$

$4.5, 13.5$ (Ans)

$n=4$ $\quad Bin(4,P)$ $\quad$ 180 Policy

3) i)

no. of Claim	0	1	2	3	4
no. of Policy	86	75	16	2	1

$$L(P)=\left((1-p)^4\right)^{86}\left(4P(1-P)^3\right)^{75}\left(6P^2(1-P)^2\right)^{16}\left(4P^3(1-P)\right)^2\left(P^4\right)^1$$
$$=(1-P)^{344}P^{75}(1-P)^{225}P^{32}(1-P)^{32}P^6(1-P)^2P^4$$
$$=(1-P)^{603}P^{117}\text{ Const}$$

$$\ln L(P)=603\ln(1-P)+117\ln P$$

$$\frac{d}{dP}\ln L(P)=\frac{-603}{1-P}+\frac{117}{P}=0$$

$117-117P=603P$

$q=.8375$ $\quad \hat{P}=0.1625$

ii) $P(X=0)=(1-P)^4=0.492$ $\quad P(X=2)=6P^2(1-P)^2=0.111$

$P(X=1)=4P(1-P)^3=0.3818$ $\quad P(X=3)=4P^3(1-P)=0.0144$

$P(X=4)=0.0007$

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.98	2.59	0.126

By clubbing;

	0	1	2+
O_i	86	75	19
E_i	88.56	68.72	22.71
$C_i=\frac{\Sigma(O_i-E_i)^2}{E_i}$	0.0757	0.5739	0.6061

$\Sigma C_i=1.2557\sim\chi^2_1$

$\therefore 1.2557<3.841$

$\therefore$ donot reject H_0

5) $U(a,b)$ $\quad E(X)=\frac{a+b}{2}$ $\quad var(X)=\frac{(b-a)^2}{12}$

show this: $a=\bar{x}-\sqrt{3}S$ $\quad a=2\bar{x}-b$ $\quad$ ~~$2\sqrt{3}S$~~ $a=b-2\sqrt{3}S$

$2\bar{x}-b=b-2\sqrt{3}S$

$b=\bar{x}+\sqrt{3}S$ $\quad a=\bar{x}-\sqrt{3}S$

Sample $=2,4,6,8,10$ $\quad \bar{x}=6$ $\quad S=3.16$

$a=6-\sqrt{3}(3.16)$ $\quad b=6+\sqrt{3}(3.16)=11.47$

$=0.527$

$U(0,b)$ $\quad f(x)=\frac{1}{b}$ $\quad L=\frac{1}{b^n}$ $\quad \log L=-n\log b$

$\frac{d}{dL}\log L=\frac{-n}{b}$ $\quad \therefore b\to\alpha$

6) $H_0\ P=0.1$ $\quad H_1\ P>0.1$ $\quad$ P: Pool of hit of missiles

12 missiles fired $\quad H_0$: 3 or more hits observed

otherwise $\quad \frac{1}{9}+\frac{1}{9}$

24 missiles fired total $\quad H_0$: 5 or more hits observed

7) i) $\bar{X}_{Pub} = 30$ $\quad \bar{X}_{Pri} = 35.056$

$S^2_{Pub} = 64.3125$ $\quad S^2_{Pri} = 67.403$ $\quad S_P^2 = 58.123$

$$\frac{(\bar{X}_{Pub} - \bar{X}_{Pri}) - (\mu_{Pub} - \mu_{Pri})}{S_P / \left(\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)} \sim t_{n_1+n_2-2}$$

ii) $H_0: \mu_{Pub} = \mu_{Pri}$ $\quad H_1: \mu_{Pub} \neq \mu_{Pri}$

$$\frac{(30-35.056)}{\sqrt{58.123}\sqrt{\frac{2}{9}}} = -1.407 \quad \therefore \text{accept } H_0$$

~~iii) $H_0: \mu_{Pri}$~~

iii) Private hospital donot result in higher claim

9) i) $t_k = \frac{N(0,1)}{\sqrt{\frac{t_k}{k}}}$ $\quad \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

ii) $\mu(t_k) = 0$ $\quad$ ~~μ~~ $Var(t_k) = \frac{k}{k-2}$

iii) $n=10$ $\quad \bar{X} = 50$ $\quad S^2 = 48.667$ $\quad CI = 99\%$

a) $t_{0.005,9} = 3.25$

$$\therefore \mu = \bar{X} \pm t_{0.5\%,9} \frac{S}{\sqrt{n}} = 50 \pm 3.25\sqrt{\frac{48.667}{10}}$$

$(57.17, 42.83)$

b) $t_{0.005,9} = 2.5758$ $\quad \mu = 50 \pm 2.5758\left(\sqrt{\frac{48.667}{10}}\right) = (55.68, 44.32)$

10) $n = 1000$ $\quad X \sim Poi(3)$ $\quad X \sim N(3000, 3000)$

if $P(X < 3100)$ then $X \sim Poi(3)$

else X donot follow $Poi(3)$

i) Type I error - $P(H_0$ is reject$/ H_0$ is true$)$

$= P(X > 3100, \cancel{\theta}\ \lambda = 3)$ $\therefore$ $X \sim N(3000, 3000)$

$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right) = P(Z > 1.826) = \boxed{3.39\%}$

ii) Type 2 error $\Rightarrow P(H_0$ accept $/ H_0$ is false$)$

iii) Power $= P($reject H_0 / H_0 false$) = 1 - \cancel{\Phi}\left(\left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)\right)$

11) $n = 12$ $\quad \Sigma x = 213200$ $\quad \Sigma x^2 = 4919860000$

$\bar{x} = 17767$ $\quad S = 10144$

$X_1 = 11000$ $\quad X_2 = 48000$ $\quad X_1' = 27500$ $\quad X_2' = 31500$

$\Sigma x = 213200 = A + 11000 + 48000$ $\quad A$ (sum of 10 terms)

$A = 154200$

$\Sigma x' = A + x_1' + x_2' = 213200$ $\quad \boxed{\bar{X}' = 17767}$

$\Sigma X^2 = 4919860000 = \cancel{B} \; B + 11000^2 + 48000^2$ $\quad B = 2494860000$

$\Sigma' X'^2 = B + x_1'^2 + x_2'^2 = 4243360000$

$S = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2} = 6434.03$

8) i) unbias when $\boxed{E(\hat{\theta}) = \theta}$

ii) bias is when $\boxed{E(\hat{\theta}) - \theta \neq 0}$

iii) MSE is when $\quad \theta = (E(\theta) - \theta)^2$ Var + bias^2