1) X = claim size

$X \sim N(900, 100^2)$

Y = Claim size on a car insurance

$Y \sim N(1200, 300^2)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$P(Y_1 + Y_2 + Y_3 - X_1 - X_2 - X_3 - X_4 > 800)$

$Y_1 + Y_2 + Y_3 - X_1 - X_2 - X_3 - X_4 \sim N(3(1200) - 4(800), 3 \times 300^2)$

$\therefore \sim N(400, 310000)$

$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right) = P(Z > 0.718)$

$= 1 - P(Z < 0.718)$

$= 0.23576.$

2) N = No. of claims.

$N \sim$ negative biomial di's $(3, 0.9)$

X = claim amount

$X \sim Gamma(6, 2)$

Y = total claim amount.

$M_Y(t) = M_N \ln(M_X(t))$

$M_X(t) = (1 - t/2)^{-6}$

$M_Y(t) = M_N(\log(1 - t/2)^{-6})$

$= M_X(t) = \left(\frac{pe^t}{1 - qe^t}\right)^k$

$$M_y(t) = \left(\frac{(0.9)(1-t/2)^{-6}}{1-(0.1)(1-t/2)^{6}} \right)^3$$

$$E(N) = \frac{0.3}{0.9} = \frac{10}{3} = 3.33$$

$$V(N) = \frac{k(0.1)}{(0.9)^2} = 3.33 \times \frac{1}{9}$$

$$= 0.39$$

$$E(X) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3$$

$$V(X) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = 1.5$$

$$V(Y) = E(N)V(X) + E(X)^2 V(N)$$

$$= 5 + \frac{10}{3} = \frac{25}{3} = 8.33$$

$$SD(Y) = \sqrt{\frac{25}{3}} = 2.88$$

(3) $$f_X(x) = \lambda e^{-\lambda(x-\lambda)}$$

$$M_X(t) = E(e^{tx})$$

$$= \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$$

$$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{x(t-\lambda)} dx$$

$$M_x(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda}\left(0 - e^{-\lambda\alpha} e^{t\alpha}\right)$$

$$= \frac{\lambda e^{t\alpha}}{\lambda - t}$$

ii) $$M'_x(t) = \lambda(e^{t\alpha})(\lambda - t)^{-2}(-1)(-1) + (\lambda)(e^{t\alpha})(\alpha)$$

$$= \frac{\lambda e^{t\alpha}}{(\lambda - t)^{2}} + \frac{\lambda e^{t\alpha}\alpha}{\lambda - t}$$

$$E(X) = M'_x(0) = \frac{\lambda(1)}{\lambda^2} + \frac{\lambda\alpha}{\lambda}$$

$$= \frac{1 + \lambda\alpha}{\lambda}$$

$$E(X^2) = M''_x(0) = \frac{2\lambda(1)}{\lambda^3} + \frac{\lambda\alpha}{\lambda^2} + \frac{\lambda\alpha^2}{\lambda}$$

$$= \frac{2}{\lambda} + \frac{2\alpha}{\lambda} + \alpha^2$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1 + \lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2}\right)$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \alpha^2 - \frac{2\alpha}{\lambda}$$

$$V(X) = \frac{1}{\lambda^2}$$

Q4) $G_V(t) = \left(\dfrac{p}{1-qt}\right)^K$, $p+q=1$

$\therefore$ Distribution is $-ve$ binomial

$$P(V=y) = \frac{\sqrt{K+y}}{\sqrt{5}\,\sqrt{K}}\, p^y q^y$$

Q5) $f(x,y) = ax^2 + axy$ $\quad 0 < x < 5$

$\quad 10 < y < 20$

$$1 = \int_{10}^{20}\int_{0}^{5} ax^2 + axy \, dx \, dy.$$

$$1 = \int_{10}^{20} \frac{1250}{3} + \frac{25}{2} ay \, dy.$$

$$a = \frac{3}{6875}$$

$$f_X(x) = a\int_{10}^{20} x^2 + xy \, dy.$$

$$= \frac{3}{6875}\left(10x^2 + 150x\right)$$

6) $X \to Poi(\lambda)$ $\quad Y \sim Poi(-\mu)$

$M_X(t) = e^{\lambda(e^t - 1)}$

$M_Y(t) = e^{\mu(e^t - 1)}$

$M_{X-Y}(t) = E\left(e^{t(X-Y)}\right)$

$= E\left(e^{tX - tY}\right)$

$$= \frac{E(e^{tx})}{E(e^{ty})} = \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$$

$$= e^{\lambda-\mu(e^t-1)}$$

$$M_{x+y}(t) = E(e^{t(x+y)})$$

$$= E(e^{tx})\, e^{(ty)}$$

$$= e^{\lambda+\mu(e^t-1)}$$

Q7) i) $Var(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2))^2$

$$= \frac{1^2(0) + 2^2(0.3) + 3^2(0.1) + 4^2(0.2)}{0.6} - [1(0) + 2(0.3) + 3(0.$$

$$= \frac{29}{36}$$

ii) $f(u,v) = \frac{48}{67}(2uv - v^2)$

$$E(U/V=v) = \int_0^1 u\, f(v/u=v)\,du$$

$$= \int_0^1 \frac{v\, f(u, v=u) \times v}{f(v=u)} \quad -(1)$$

$$f_v(v=u) = \int_0^1 \frac{48}{67}(2vu - v^2)\,dv$$

$$= \frac{48}{67}\left(v - \frac{1}{3}\right)$$

$$= \frac{48}{67}\left(\frac{3v-1}{(3)}\right)$$

Q8) $\alpha = 3 \quad \lambda = 2$

$$X \sim Gamma(3, 2)$$

$$E(Y / X = x) = 3x + 1$$

$$E(Y) = E(E(Y / X = x))$$
$$= E(3x + 1)$$
$$= 3E(X) + 1$$
$$= 3(3/2) + 1 = 11/2$$

$$V(Y) = E(2x^2 + 5) + V(3x + 1)$$

$$= 2(3) + 5 + 9(3/4)$$

$$= 11.75$$ Ans.

Q9) $Z = x + y$

$$E(X) = 3 \qquad E(Y) = \cancel{3}\,4$$
$$\sigma_x = \sqrt{V(X)} = 2 \qquad \sigma_y = \sqrt{V(Y)} = 1.$$

$$cov(X, Z) = cov(x, x + y)$$

$$= cov(x, x) + cov(x, y)$$
$$= V(X) + (-0.6)$$
$$= 3.4$$

$$var(z) = var(X + Y)$$
$$= var(X) + V(Y) + cov(X, Y)$$

$$= 3.8$$ Ans.

Q10) $f(x, y) = k x^{-\alpha} e^{-y/\beta}$

$$1 = k \int_{1}^{\infty} \int_{1}^{\infty} x^{-\infty} e^{-y/\beta} \, dx \, dy$$

$$1 = k \left(\frac{1}{\infty + 1} \right) \left(-\beta e^{-1/\beta} \right)$$

$$k = \frac{-1 + \alpha}{\beta e^{-1/\beta}}$$ Ans.