Assignment 1. Chapter - 1, 2, 3 and 4 (unit 1 & 2)

Ques1 $\lim_{h\to 0} \frac{2(-3+h)^2-18}{h} = \lim_{h\to 0} \frac{2(9+h^2-6h)-18}{h}$

$= \lim_{h\to 0} \frac{\cancel{18}+2h^2-12h-\cancel{18}}{h}$

$= \lim_{h\to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}} = \lim_{h\to 0} 2(h-6)$

$= 2(0-6) = 2\times -6 = -12.$

Ques2

$g(x) = \begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$

① At $x=4$

$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^-} 2x = 2(4) = 8$

② $\lim_{x\to 4^+} g(x) = \lim_{x\to 4^+} 2x = 2(4) = 8.$

$g(4) = 2(4) = 8.$

LHL = RHL = $g(4)$ ∴ hence $g(x)$ is continuous at $x=4$.

② At $x=6$

$\lim_{x\to 6^-} g(x) = \lim_{x\to 6^-} 2x = 2(6) = 12.$

$\lim_{x\to 6^+} g(x) = \lim_{x\to 6^+} x-1 = 6-1 = 5$

LHL ≠ RHL

Limit does not exist → hence discontinuous at $x=6$, Ques3 $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} = \lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$

$\Rightarrow \lim_{x \to 9} - \frac{\cancel{(-x+9)}(3+\sqrt{x})}{\cancel{(9-x)}} = \lim_{x \to 9} -(3+\sqrt{x})$

$= -(3+\sqrt{9}) = -$

$= -1(3+3)$

$= -6.$

Ques4 $f(x) = \frac{4x+5}{9-3x}.$

① At $x = -1.$

$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}.$

Continuous at $x = -1.$

② $x = 0$

$f(x)\ \frac{5}{9}$

continuous at $x = 0$

③ At $x = 3$

$f(3) = \frac{4(3)+5}{9-3(3)} =$ N.D

hence discontinuous at $x = 3.$

Ques5: $\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}.$

$$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - \cancel{2e^{2x}} + 3e^{-x}} = \cancel{\lim_{x\to\infty} \frac{24e^{4x} + 2e^{-2x}}{32e^{4x} - 2e^{2x} - 3e^{-x}}}$$

$$= \lim_{x\to\infty} \frac{\dfrac{6e^{4x} - e^{-2x}}{e^{4x}}}{\dfrac{8e^{4x} - e^{2x} + 3e^{-x}}{e^{4x}}}$$

$$= \lim_{x\to\infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + e^{-5x}}$$

$$= \frac{6 - e^{-\infty}}{8 - e^{-\infty} + e^{-\infty}} = \frac{6}{8} = \frac{3}{4}$$

Ques 6

$$g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$$

(1) $\lim_{y\to 6} g(y) = \lim_{y\to 6} 1 - 3y = 1 - 3(6) = -17.$

(2) $\lim_{y\to 2} g(y)$

$\lim_{y\to -2^-} g(y) = \lim_{y\to -2^-} y^2 + 5 = 9$

$\lim_{y\to -2^+} g(y) = \lim_{y\to -2^+} 1 - 3y = 7.$

Ques 7 $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (8t - 1)(t^3 - 8t^2 + 12) + (4t^2 - t)(3t^2 - 16t).$

$= 8t^4 - 8t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 64t^3$

$- 3t^3 + 16t^2.$

64

-12

$$\boxed{\cancel{f'(t)} = 20t^4 - 76t^3 + 24t^4 - 96t - 12}$$

Ques 8

$$R(\omega) = \frac{3\omega + \omega^4}{2\omega^2 + 1}.$$

$$R'(\omega) = \frac{(2\omega^2+1)(3+4\omega^3) - (3\omega+\omega^4)(4\omega)}{(2\omega^2+1)^2}.$$

$$= \frac{6\omega^3 + 8\omega^5 + 3 + 4\omega^3 - (12\omega^2 + 4\omega^5)}{(2\omega^2+1)^2}$$

$$= \frac{8\omega^5 + 4\omega^3 + 6\omega^2 + 3 - 12\omega^2 - 4\omega^5}{(2\omega^2+1)^2}.$$

$$= \frac{4\omega^5 + 4\omega^3 - 12\omega^2 + 3}{(2\omega^2+1)^2}.$$

$$= \frac{\cancel{4\omega^5 + 4\omega^3 - 12\omega^2 + 3}}{\cancel{4\omega^4 + 1 + 4\omega^2}}.$$

$e^x = a^x \log a$

$$\boxed{R'(\omega) = \frac{4\omega^5 + 4\omega^3 - 12\omega^2 + 3}{(2\omega^2+1)^2}}$$

Ques 9 $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \log 8$

$= 2e^x - 8^x \log 2^3$

$= 2e^x - 8^x \times 0.903$

$$\boxed{f'(x) = 2e^x - 0.903\, 8^x}$$

Ques10 $y = z^5 - e^z \ln(z)$.

$$\cancel{y' = 5z^4 - \left(e^z \ln z \times \frac{e^z}{z}\right)}.$$

$$= \cancel{5z^4 - e^z\left(\ln z \times \frac{1}{z}\right)}.$$

$$y' = 5z^4 - \left(e^z \ln z + \frac{e^z}{z}\right).$$

$$\boxed{y' = 5z^4 - e^z\left(\ln z + \frac{1}{z}\right)}$$

Ques11 (a) $f(x) = (6x^2 + 7x)^4$

$$\boxed{f'(x) = 4(6x^2 + 7x)^3 (12x + 7)}$$

(b) $f(t) = 5 + e^{4t + t^7}$

$$\boxed{f'(t) = e^{4t + t^7}(4 + 7t^6)}$$

Ques12: (a) $z = \ln(7 - x^3)$.

$$z' = \frac{1}{7 - x^3} \times -3x^2$$

$$\boxed{z' = \frac{-3x^2}{7 - x^3}}$$

(b) $g(v) = \dfrac{2}{(6 + 2v - v^2)^4}$

$$g'(v) = \frac{-2 \times 4 \cancel{(6 + 2v - v^2)^3} (2 - 2v)}{\cancel{(6 + 2v - v^2)^8}\,5}.$$

$$\boxed{g'(v) = \frac{-16(1 - v)}{(6 + 2v - v^2)^5}}$$

(c)

$f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2} \times 2t$

$$\boxed{f'(t) = \frac{2t}{1+t^2}}$$

Q13) $y = x^3 - 3x^2 - 9x + 12$

$y' = 3x^2 - 6x - 9$

$y' = 3(x^2 - 2x - 3)$

For maxima & minima

$y' = 0$

$3(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0$; $x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x+1)(x-3) = 0$

$\boxed{x = -1}$ or $\boxed{x = 3}$.

$y'' = 3(2x - 2)$

$= 6(x - 1)$

At $x = -1$

$y'' = 6(-1-1)$

$\boxed{y'' = -12}$

$y'' < 0$

-ve

$\therefore \boxed{x = -1}$ is point of maxima.

At $x = 3$

$y'' = 6(3-1) = 6(2)$

$\boxed{y'' = 12}$

$y'' > 0$

$x = 3$ is point of minima

Lastly

~~$4x^3-18x^2+24x-7$~~.

~~$4(1)^3-18(1)^2+24(1)-7$~~ $y=4x^3-18x^2+24x-7$

$$y' = 12x^2 - 36x + 24$$

$$y' = 12(x^2 - 3x + 2).$$

for maxima & minima.

$y' = 0.$

$12(x^2-3x+2) = 0.$; $x^2-3x+2=0.$

$x^2-2x-x+2=0$

$x(x-2)-1(x-2)=0$

$(x-1)(x-2)=0.$

$\boxed{x=1}$ or $\boxed{x=2}$.

$$y'' = 12(2x-3).$$

At $x=1$

$y'' = 12(2-3)$

$= 12(-1)$ $\boxed{y''=-12}$.

$y''<0$

$\therefore$ $\boxed{x=1}$ is point of maxima.

At $x=2$

$y'' = 12(4-3)$

$= 12(1)$

$\boxed{y''=12}$

$y''>0$

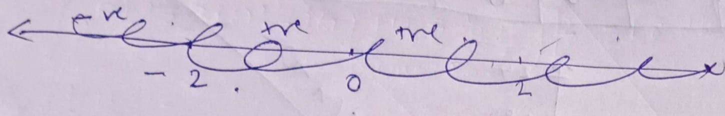
$\therefore$ $\boxed{x=2}$ is point of minima.

Ques 15 $f(x) = x^2(x+3)$. $\therefore f(x) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$.

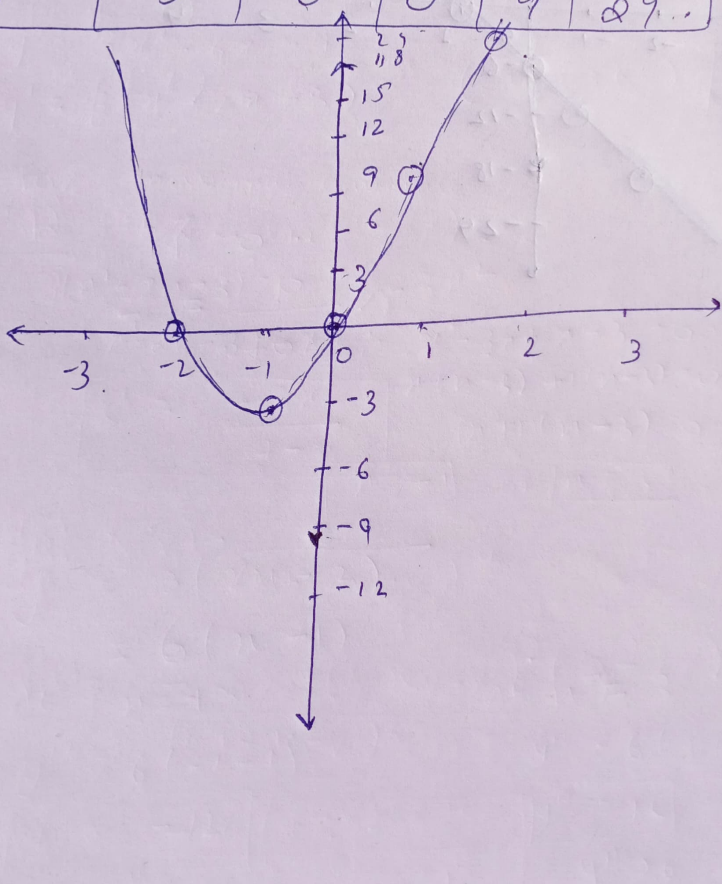
$= 3x(x + 2)$ $\quad -3\,(1)$

$\boxed{x=0}$ or $\boxed{x=-2}$



~~At~~ $x = -1$

x	-1	-2	0	1	2
$f(x)$	-3	0	0	9	24..



Ques 16

$f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$.

x	-2	-1	0	1	2
$f'(x)$	-18	-12	-6	0	6