

i) ~~80~~  $\sum x^2 = 92500$

(a)  $\bar{x} = 850/10 = 85$

$$\sum y^2 = 37277$$

$$\bar{y} = 173.7$$

$$\sum xy = 182560$$

$$S_{xy} = 182560 - n\bar{x}\bar{y} = 34915$$

$$S_{xx} = 20250$$

$$S_{yy} = 71000.1$$

$$\beta = \frac{S_{xy}}{S_{xx}} = 1.724197531$$

$$\alpha = \bar{y} - \beta\bar{x} = 27.1432$$

$\therefore$  equation of regression line

$$y_i = \alpha + \beta x_i$$

$$\therefore y_i = 27.1432 + 1.724197 x_i$$

(b) It can be hence concluded that the money spent by Arand's girlfriend is affected by the amount of cash he withdrew at his previous visit to the ATM

2.

$$(i) \quad \beta = \frac{\sum xy}{\sum x^2} = \frac{38.14 \times 1.41 \times 10^{-12} \times 38.75.2 \times 10^{-12}}{12 \times 12}$$

$$= \sum xy = 875.16$$

$$\sum x^2 = 301.66$$

$$\sum y^2 = 6407.7125$$

$$\beta = \frac{\sum xy}{\sum x^2} = 2.901146967$$

$$\alpha = \bar{y} - \beta \bar{x} = 2.901146 \times \bar{x} = 3.748181$$

$$y_i = 3.748181 - 2.901146 x_i$$

$$(ii) \quad (S.e.) \beta = \frac{\sqrt{\left[ \frac{\sum y^2 - \frac{(\sum y)^2}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} \right] (n-2)}}{\sum x^2}$$

$$= 1.182761292$$

$$\text{confidence interval } \beta \pm t_{(n-2), \frac{\alpha}{2}} \times 1.182761292$$

$$0.8773547, 5.424738162$$

(iii) mean 1B4 share price

$$S.e(w) = \sqrt{\frac{1 + \frac{(n_0 - n)^2}{\sum x^2}}{n}} \times \beta$$

$$= \sqrt{0.290221353} \times \beta$$

$$= 3.096042514$$





$$\mu_0 = 119.79406$$

$$CI = 112.9406377, 126.6474627$$

Q.2 (i) The <sup>time</sup> difference in commutation of city A seems the highest meaning the time taken to commute during peak hours is drastically different than the time taken to commute during non-peak hours.

The time difference in commute for city B is high but it is lower than the time difference of city A.

The time difference in commute for city C is lower than city B. Variation of city C seems the lowest. The time difference in commute of city D is the least among the four cities. Variation seems higher.

(ii) i) Assumption of homoscedasticity i.e. the population variance is the same.

(ii) The observations are independent

(iii) They must follow normal distribution.

(iii)  $H_0$ : The mean difference is the same for all cities.

$$SS_{\text{total}} = 1495 = \frac{103^2}{28}$$

$$= 1116.11$$

$$SS_{\text{reg}} = \frac{64^2 + 44^2 + 8^2 + (13)^2}{7} - \frac{103^2}{28}$$

$$= 516.11$$

$$SS_{\text{res}} = SS_{\text{tot}} - SS_{\text{reg}}$$

$$= 1116.11 - 516.11$$

$$= 600$$

ANOVA table:

| <del>SS</del> | df | SS      | MSS    |
|---------------|----|---------|--------|
| Regression    | 3  | 516.11  | 172.04 |
| residual      | 24 | 600     | 25     |
| total         | 27 | 1116.11 |        |

$$F_{\text{test}} \text{ calculator} = \frac{172.04}{25} = 6.88$$

which is much greater than

3.009 the 5% critical point of F distribution

$\therefore$  We have enough evidence to reject  $H_0$

$\therefore$  We can conclude that there are underlying differences b/w the cities.  $\therefore$



Q.4. Mathematical one way analysis.

$$y_{ij} = \mu + \beta x_{ij} + e_{ij}$$

$$j = 1, 2, 3, \dots, n_i$$

$k$  = number of treatments

$n_i$  = number of responses from the  $i$ th treatment.

$y_{ij}$  = is the  $j$ th response from the  $i$ th treatment

$e_{ij}$  = is the error term.

$$e_{ij} \sim N(0, 1)$$

c)  $H_0$ : Salaries are independent

$H_1$ : Salaries are dependent.

| $O_i$       |     |     |      |       |       |
|-------------|-----|-----|------|-------|-------|
| Paper Clerk | 3-5 | 5-8 | 8-10 | 10-12 | Total |
| 0-3         | 45  | 20  | 6    | 5     | 76    |
| 4-6         | 7   | 20  | 9    | 6     | 42    |
| 7-9         | 5   | 8   | 15   | 12    | 40    |
| Total       | 57  | 48  | 30   | 23    | 137   |

$E_i$

|       | 3-5   | 5-8   | 8-10  | 10-12 | Total |
|-------|-------|-------|-------|-------|-------|
| 0-3   | 27.42 | 23.09 | 14.43 | 11.06 | 76    |
| 4-6   | 15.15 | 12.76 | 7.97  | 6.11  | 42    |
| 7-9   | 14.48 | 12.15 | 7.57  | 5.82  | 40    |
| Total | 57    | 48    | 30    | 23    | 158   |

$\phi$

$$\frac{\sum (O_i - E_i)^2}{E_i} = 49.919$$

$$df = (4-1)(3-1) = 26$$

$$\chi^2_{\text{tab}} = 12.59$$

$$\chi^2_{\text{cal}} > \chi^2_{\text{tab}}$$

$\therefore$  We have sufficient evidence to reject  $H_0$

$\therefore$  we conclude that salary is dependent on the number of papers cleared.



Q. 4. Q. 5 (i)

Assumptions are:

The population must be normal

The observations must be independent

The assumption of homoscedasticity.

The sample variances are different hence the assumption that population variance is same will not hold true.

(ii) new sample mean of rate 1

$$\frac{1}{3} (\sqrt{29} + \sqrt{13} + \sqrt{21}) = 4.52$$

New sample variance for rate 2

$$\frac{(\log_e 180 - 4.827)^2 + (\log_e 90 - 4.827)^2 + (\log_e 120 - 4.827)^2}{3} = 0.1213$$

(iii) For this assumption the sample variance is same hence the assumption of homoscedasticity holds true. hence this transformation is viable. The scientist was correct in asserting that loge transformation must be done.

Q.6.

$$(i) S_{xx} = \sum x^2 - n\bar{x}^2$$

$$= 54.90$$

$$S_{yy} = \sum y^2 - n\bar{y}^2$$

$$= 8923.60$$

$$S_{xy} = \sum xy - n\bar{x}\bar{y}$$

$$= 661.20$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = 0.945$$

$$(ii) \beta = \frac{S_{xy}}{S_{xx}} = \frac{661.20}{54.9} = 12.04$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 9.23$$

$$\therefore \hat{y} = \hat{\alpha} + \hat{\beta}x = 9.23 + 12.04x$$

$$(iii) SS_{total} = S_{yy} = 8923.6$$

$$SS_{res} = S_{yy} - \frac{S_{xy}^2}{S_{xx}}$$

$$= 8923.6 - \frac{(661.2)^2}{54.9} = 960.30$$

$$SS_{reg} = 8923.6 - 960.3 = 7963.3$$



$$(iv) \text{COP} = R^2 = u^2 = (0.945)^2 = 0.8927$$

$$Q.7. (i) S_{xx} = 4789.42$$

$$S_{yy} = 1189.21$$

$$S_{xy} = 2176.84$$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{2176.84}{4789.42} = 0.4545$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x} = 10.79$$

$$\therefore y = -10.79 + 0.4545x$$

$$(ii) \hat{\sigma}^2 = \frac{1}{n-2} \left( S_{yy} - \frac{S_{xy}^2}{S_{xx}} \right)$$

$$= \frac{1}{9} \times \left( 1189.21 - \frac{(2176.84)^2}{4789.42} \right) = 2.22$$

$$S.e.(\hat{\beta}) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}} = \sqrt{\frac{2.22}{4789.42}} = 0.0681$$

$$H_0: \beta = 0, \quad H_1: \beta \neq 0$$

$$\frac{\hat{\beta} - 0}{S.e.(\hat{\beta})} = \frac{0.4545}{0.0681} = 6.674$$

The errors of the regression are iid and follow  $N(0, \sigma^2)$ ;  $\beta$  has  $t$  distribution with  $n-2$  dof.

at dof 9 the critical value of  $t$  dist is 2.68

the calculated value is more hence there is sufficient evidence to reject  $H_0$ .

Hence there is no relation b/w  $x$  &  $y$

(iii) 
$$r = \frac{\sum xy}{\sqrt{\sum xx \sum yy}}$$

$$= 0.91$$

(iv) 
$$n^2 = 25$$

$$y = 10.79 + 0.4545(x - 25)$$

$$= 22.15$$

$$\hat{r}^2 = 22.2$$

$$V(\mu_0) = \left[ \frac{1}{n} + \frac{(\mu_0 - \bar{x})^2}{\sum xx} \right] \hat{\sigma}^2$$

$$= 2.41 \times 3 = 7.23$$

\* variance of individual response

$$\left[ 1 + \frac{1}{n} + \frac{(\mu_0 - \bar{x})^2}{\sum xx} \right] \times \hat{\sigma}^2 = 7.2461$$



95% CI for mean  $\mu_{01}$

$$22.20 \pm 2.262 \times \sqrt{2.41}$$

$$= (16.7, 25.7)$$

95% CI for individual response

$$22.2 \pm 2.262 \times \sqrt{24.61}$$

$$= (11, 33.4)$$

(v) The residual plot shows a pattern. The correlation coefficient is high but the model does not seem to be a good fit because of this.

Q.9. (i) The marks scored and the number of hrs spent on social media seem to be negatively correlated according to the scatter plot.

$$(ii) S_{xx} = \sum x^2 - n\bar{x}^2 = 75$$

$$S_{yy} = \sum y^2 - n\bar{y}^2 = 2482.4$$

$$S_{xy} = \sum xy - n\bar{x}\bar{y} = -249.6$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = -0.686$$

$\therefore$  It is confirmed that they are negatively correlated.

(iii)  $H_0: \rho = 0$   
 $H_1: \rho < 0$

$$u = \frac{1}{2} \ln \left( \frac{1 - 0.686}{1 + 0.686} \right) = -0.8404$$

$$W \sim N \left( 0, \frac{1}{7} \right)$$

$$t_s = -0.8404 - 0 = -2.22$$

$$t_s = \frac{-0.8404}{\sqrt{\frac{1}{7}}} = -2.22$$

which is higher than the tabulated value  
hence we have ~~stat~~ enough evidence to  
reject  $H_0$ . it means a number of hrs spent on  
social media is negatively correlated.

(iv)  $\beta = \frac{r_{xy}}{r_{xx}} = -3.9467$

$$a = \bar{y} - \hat{\beta} \bar{x} = 82.16$$

$$y = 82.16 - 3.947x$$

(v)  $R^2 = u^2 = 0.4706$

47.06% of the total variation  
of the model is explained.

(vi) For every additional hr spent of one per day  
the total marks by reduce is by  
3.95 basis of fitted equation



§ 10.

$H_0$ : No difference among

$H_1$ : at least one industry differs significantly from the mean.

$$SS_r = \frac{5^2 + 10^2 + 8^2}{11} = 35.91$$

$$\bar{M} = \text{mean of resignation} = \frac{27 + 36 + 30}{3} = 31$$

$$SS_B = 20 \left( (27 - 31)^2 + (36 - 31)^2 + (30 - 31)^2 \right)$$

$$SS_B = 840$$

$$F_{2, 29} = \frac{840/2}{35.91/29} = 6.66$$

at 1% los of F dist with dof 2, 29, its critical value is 4.977

$\therefore$  calculated value is higher than this we reject  $H_0$ .

$\therefore$  resignation is diff rate is different across different countries.