Financial engineering

Assignment - 2

Roll No. 410.

1) given: $S_0 = £65$ · $\sigma = 25\%$ Pa.

$r = 2\%$ P.a · $K = £55$

$T = 6$ months $\Rightarrow 0.5$

$C_0 = ?$

(i) $C_t = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln\left(\frac{65}{55}\right) + \left(2\% + \frac{20^2}{2}\right)0.5}{20\% \sqrt{0.5}}$$

$d_1 = 1.08996$

$d_2 = d_1 = \sigma\sqrt{t}$

$d_2 = 0.91318$

$\therefore\ C_T = 65\,\Phi(1.09) - 55e^{-0.08\times0.5}\,\Phi(0.91318)$

$C_t = 11.4646.$

(ii) $\Delta = \dfrac{\text{Change in Price of call}}{\text{Change in price of underlying.}}$

(iii) $\Delta_c = \Phi(d_1)$

$\Delta p = -\Phi(-d_1).$

$\Delta c = 0.86214.$

(IV) $\Delta p = -0.13786.$

2(i) $\Delta = \dfrac{\text{Change in Price of derivative}}{\text{Change in Price of underlying asset}}$

rate of change of option Price with respect to change in price of underlying asset.

$V = \dfrac{\text{Change in price of option}}{\text{Chang in volatility of underlying asset.}}$

rate of change of option Price w.r.t change in volatility of underlying asset.

(ii) Put call Parity:

$$C - P = S - Ke^{-rT}.$$

$$\frac{d(C-P)}{d\sigma} = \frac{d(S - Ke^{-rt})}{d\sigma}$$

$V_C - V_P = 0$. ($\because$ S & K are constants)

$$\boxed{\Rightarrow \nu_C = \nu_P .}$$

(iii) $S_0 = \$55$

$X = \$50$

$\sigma = 25\%$ Pa.

$r = 5\%$.

$T = 1$

$$d_1 = \frac{\ln\left[\frac{S_0}{X}\right] + \left(r + \frac{1}{2}\sigma^2\right)T}{}$$

$$= \frac{\ln\left[\frac{55}{50}\right] + \left(5\% + \frac{1}{2} \times 25\%^2\right)}{25\%}$$

$$\boxed{d_1 = 0.70624}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$= 0.70624 - 25\%$

$d_2 \Rightarrow 0.4562$

$C_t = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2)$

$= 55(0.76115) - 50e^{-0.05}(0.67724)$

$C_t = \$0.65$

$C - P = S - Ke^{-rT}$

$\Rightarrow 0.65 - P = 55 - 50e^{-0.05}$

$P = \$2.211$

v) <u>Delta hedging</u>

A Portfolio for which the overall delta is equal to 0, is described as delta ~~heo~~ neutral. The Portfolio is immune to changes in the price of underlying asset.

<u>Vega hedging</u>

A portfolio for which the overall Vega is equal to 0 is described as Vega hedged or Vega neutral. The Portfolio is immune to small changes in the assumed level of volatility.

(v) X units of call option.
Y units of Put option
Z units of forward.

~~tatt~~	C.F @ PV	Δ	V
~~Put~~ call	$C = 0.65$	ΔC	V_c
Put	$P = 2.211$	ΔP	V_p
Forward	—	1	0

Δ of forward $= 1$
V of forward $= 0$

Vega hedging:

$x V_c + y V_p + z V_F = 0$

$(\because V_F = 0)$

$x V_c + y V_p = 0$

$(\because V_c = V_p)$

$V_c(x+y) = 0 \Rightarrow x + y = 0$

$\Rightarrow y = -x$

Delta Hedging

$x \Delta_c + y \Delta_p + z \Delta_F = 0$

$(\Delta_F = 1)$

$$\cancel{\Delta x +} \ x\Delta_c + y\Delta_P + Z = 0$$

$$(\because y = -x,\ \Delta_P = \Delta_c - 1)$$

$$\Rightarrow x\Delta_c - x(\Delta_c - 1) + Z = 0$$

$$\Rightarrow x + Z = 0$$

$$\Rightarrow \boxed{x = -Z} \quad \text{(ii)}$$

from (i) & (ii)

$$x = -y = -Z$$

Portfolio = 1000.

$$x.C + y.P + z.F = 1000$$

$$(F = 0)$$

$$\Rightarrow x.C + y.P = 1000$$

$$\Rightarrow x.9.6525 + y.2.2190 = 1000$$

$$= 0.6525x - 2.2190K = 1000$$

$$7.438x = 1000$$

$$\boxed{x = 134.43}$$

$$y = Z = -134.43$$

∴ portfolio.

1) long position on 134 call options

2) short position on 134 put options

3) short position on 134 forward.

Q3).

(i) $C_t = \mathcal{E}\left(e^{-r(T-t)} C_T / F_t\right)$

$F_t \rightarrow$ Filteration @ $t = 0$

$C_T \rightarrow$ Pay off under derivative at T.

$C_t \rightarrow$ derivative value at time.

(ii) $S_0 = £50$.

$X = £49$.

$r = 5\%$ p.a.

$\sigma = 25\%$ p.a.

$T = 0.5$

$$C_t = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2).$$

C_t :

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln\left(\frac{50}{49}\right) + (5\% + 12.5\%)0.5}{25\% \sqrt{0.5}}$$

$= 0.399$

$d_2 = d_1 - 25\% \times \sqrt{0.5}$

$\boxed{d_2 = 0.1673}$

$\Phi(d_1) = 0.6346$

$\Phi(d_2) = 0.5664$

$C_T = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$

$= 50 \times 0.6346 - 49e^{-0.05 \times 50}(0.5669)$

$C_T = 4.66$

(iii) Same as european call $= 4.66$

(iv) $S_t + P = C + Ke^{-rT}$

$P = 4.66 + 49e^{-0.05 \times 5} - 50$

$P = 2.95$.

(v) If the stock pays a dividend then the price of the underlying would fall.

This would lead to a fall in the price of the european call, and a in price of the european put.

Due to the potentially early exercise opportunity the American call would be more expensive than european call.

Q4 $Z_t \sim$ SBM under P

$C_t \rightarrow$ presivible process.

$\therefore$ There exists a measure Q equivalent to P where,

$\tilde{Z}_t = Z_t + \int_0^T \gamma_s ds$

$\tilde{Z}_t \rightarrow$ SBM under Q

If Z_t is a SBM under P and if Q is equivalent to P then there exists a Previsible process γ_t such that

$$\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$$

$\tilde{Z}_t \rightarrow$ SBM under Q

(ii) The discounted value of asset prices are martingales.

Q5)

(i) $\Delta = \Phi(d_1)$

$= \dfrac{\text{Change in price of underlying asset}}{\text{Change in Price of call options}}$

(ii) $$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) \cdot T}{\sigma\sqrt{T}}$$

$$\Rightarrow 0.3 = \frac{\ln\left(\frac{40}{45.91}\right) + (0.02 + 0.5 \times \sigma^2) \cdot 5}{\sigma\sqrt{5}}$$

$\Rightarrow \sigma = 2.5\gamma^2 = 0.05637$

$$\cancel{2.5\sigma^2 = 0.67083 \pm \sqrt{0.67083^2 - 4 \times 2.5 \times (0.0378)}}$$

$$2.5\sigma^2 - 0.67083\sigma - 0.0378 \cdot 2$$

$$\sigma = \frac{0.67083 \pm \sqrt{0.67083^2 - 4 \times 2.5 \times (0.0378)}}{2 \times 2.5}$$

$$= \sigma = 32\%$$

(iii) $R_0 = \$30$

$$\sigma = \sqrt{15\%} \text{ p.a}$$

Option Payoff = C at Time T if

$$\frac{S_1}{S_0} > K_S, \quad \frac{R_1}{R_0} < R$$

$$\text{Payoff} = Ce^{-rT}\, g\left(\frac{S_1}{S_0} < K_S\right) \cdot g\left[\frac{R_1}{R_0} < KR\right]$$

(iv) Perfect corr →

$\left(\frac{R_1}{R_0} = \frac{S_1}{S_0}\right)$.

$$\text{Price} = Ce^{-rT} \, \mathbb{Q}(R_1/R_0 < K_s) =$$

$$Ce^{-rT} \, \mathbb{Q}(S_1/S_0 < K_s)$$

$$\therefore \text{Price} = Ce^{-rT} \, \mathbb{Q}\left(\frac{R_1}{R_0} < \min(K_s, K_R)\right)$$

$$= Ce^{-rT} \, \mathbb{Q}\left(\frac{S_1}{S_0} < \min(K_s, K_c)\right)$$

(V) $$\text{Price} = Ce^{-rT} \, \mathbb{Q}\left(\frac{S_1}{S_0} < K_s\right) \cdot \mathbb{Q}\left(\frac{R_1}{R_0} < K_R\right)$$

$$\Rightarrow 50e^{-0.02 \times 50} \, \mathbb{Q}(S_1 < 32) \cdot \mathbb{Q}(R_1 < 18)$$

$$\Rightarrow 91.61$$

6 (i) a) $\Delta = -\Phi(-d_1)$

$= \Phi(d_1) - 1$

b) $\Delta = 0$

$100{,}000\,\Delta + 24830 = 0$

$\Delta = \dfrac{-24830}{100{,}000}$

$\Delta = -0.2483$

(ii) $\Phi(-d_1) = 0.2483$

$1 - \Phi(d_1) = 0.2483$

$\Phi(d_1) = 0.7517$

$d_1 = 0.68$

$0.68 = \ln\left(\frac{0.64}{0.63}\right) + \left(0.03 + \frac{1}{2}\sigma^2\right)$

$\sigma = 7.1\%$

(iii) a) $Xe^{-rT}\Phi(d_2) - S_0[\Phi(-d_1)] = 0.2983$

$= 0.63 \times e^{-0.03} \times [1 - \Phi(0.609)] - 0.69 \times 0.2483$

$= 6.894$

b) Total cash holding

$= 2483 \times 0.69 + 100{,}000 \times 0.6884$

$= £165{,}806$

Q7 i) $\Delta = \frac{\partial f}{\partial S} = \frac{\partial f}{\partial s}(t, S_1)$

$f \rightarrow$ derivative

$S \rightarrow$ underlying

$\gamma = \frac{\partial^2 f}{\partial S^2}$

$\nu = \frac{\partial f}{d \sigma_S}$.

(ii) $X = 50$, $C = 17.91$, $S_0 = 60$, $\sigma = 25\%$ pa.

$T = 3$ years, $r = 3\%$ pa $\quad \nu = \$29$

$\Delta = 0.801$

(iii) $f(S, \sigma + \delta) = f(S, \sigma) + \frac{\delta \, df}{d\sigma}$

$= 17.91 + 29 \times 0.02$

$= 18.49$.

Q8 $\Delta = \frac{df}{dS}$

(ii) $S_0 = \$100$,

$r = 3\%$

$X = 109.42$

$t = 1$

$\Delta = 0.42079$

$\rightarrow \Phi(d_1) = 0.42074$

$d_1 = 0.2$,

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$= -0.2 = \ln\left(\frac{100}{109.42}\right) + \left(0.03 + \frac{\sigma^2}{2}\right)$$

$$\Rightarrow -0.2\sigma + 0.09 - 0.03 - \frac{\sigma^2}{2} = 0$$

$$\Rightarrow -\frac{\sigma^2}{2} - 0.2\sigma + 0.06 = 0$$

$\sigma = 0.2$.

Q9 (i) PDE for g:

$$\frac{dg}{dt} + (r-q)\, S_T \frac{dg}{dS_T} + \frac{1}{2}\sigma^2 S_t^2 \frac{d^2g}{dS_t^2} = rg$$

Boundary condition

$$D_T = g(T, S_T) = f(S_T)$$

(ii) Price of derivative @ time t & value for

$$D_T = g(t, S_t) = \frac{S_t^n}{S_0^{n-1}} \times e^{\mu(T-t)} \quad (HS\cdot 1)$$

Then, $D_T = g(T, S_T)$

$$f(S_T) = \frac{S_T^n}{S_0^{n-1}}$$

$$\frac{dg}{dt} = -\mu \cdot \frac{S_t^n}{S_0^{n-1}} e^{\mu(T-t)} = -\mu g$$

$$\frac{dg}{dS_t} = \frac{n S_t^{n-1}}{S_0^{n-1}} e^{\mu(T-t)} = \frac{n}{S_t} g$$

$$\frac{d^2 g}{dS_t^2} = \frac{n(n-1) S_t^{n-2}}{S_0^{n-1}} e^{\mu(T-t)} = \frac{n(n-1) g}{S_t^2}$$

$$-\mu_g + (r-q) S_t \frac{n}{S_t} g + \frac{\sigma^2}{2} S_t^2 \frac{n(n-1)}{S_t^2} g - r_g$$

$$-\mu + (r-q)n + \frac{1}{2}\sigma^2 n(n-1) = r$$

$$\mu = (r-q)n - r + \frac{1}{2}\sigma^2 n(n-1)$$

If $q = 0$

$$\mu = rn - r + \frac{1}{2}\sigma^2 n(n-1)$$

$$= r(n-1) + \frac{1}{2}\sigma^2 n(n-1)$$

$$= \left(r + \frac{1}{2}\sigma^2 n\right)(n-1)$$

Q_{10} (i) Put call Parity for ~~Stochastic~~ Stock Paying no dividends.

$$C_t + Ke^{-r(T-t)} = S_t + P_t$$

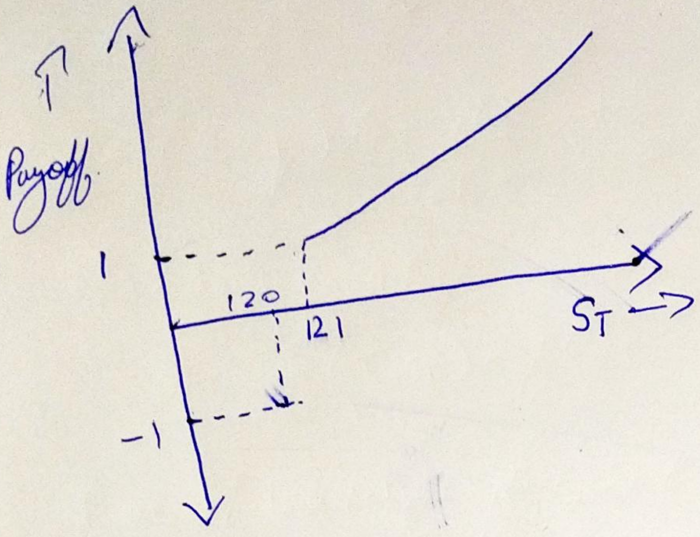
(ii) $X = 120$ $C = 10.09$ $S_0 = 910$

$T = 1$ $r = 2\%$ Pa

$$C = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2)$$

$$d_2 = d_1 - \sigma$$

(iii)



(iv) $S_1 - 121 \leq D \leq S_1 - 120$

$\Rightarrow S_0 - 121e^{-r} \leq V \leq S_0 - 120e^{-r}$

Q11. $x = 150$

$r = 2\%$ pa

$S_0 = 117.98$

Q12 $S_0 = 8$

$X = 9$

$r = 2\%$ pa

$\sigma = 20\%$

$T = 3/12$.