

# Probability & Statistics

## Assignment-1

Q1. (i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$$\rightarrow \text{Mean} = \frac{\sum x}{N} = \frac{134}{10} = \underline{\underline{13.4}}$$

$$\rightarrow \text{Median} = \frac{n+1}{2} = \frac{11}{2} = 5.5 \text{th term} = \frac{11+15}{2} = \frac{26}{2} = \underline{\underline{13}}$$

$$\rightarrow \text{Mode} = \underline{\underline{18}}$$

$$\rightarrow Q_1 = \frac{10}{4} = 2.5 \text{th term} = \frac{7+9}{2} = \frac{16}{2} = \underline{\underline{8}}$$

$$\rightarrow Q_3 = 3\left(\frac{10}{4}\right) = 7.5 \text{th term} = \frac{18+18}{2} = \underline{\underline{18}}$$

$$\rightarrow \text{IQR} = Q_3 - Q_1 = 18 - 8 = \underline{\underline{10}}$$

$$\rightarrow \text{SD} = \underline{\underline{14.73091}}$$

| (ii) (x - $\bar{x}$ ) | no. of H.H. | fx  | fx <sup>2</sup> | no. of people (f) | C.F. |
|-----------------------|-------------|-----|-----------------|-------------------|------|
| -2                    | 0           | 0   | 0               | 5                 | 5    |
| -1                    | 1           | 11  | 11              | 11                | 16   |
| 0                     | 2           | 52  | 104             | 26                | 42   |
| 1                     | 3           | 45  | 135             | 15                | 57   |
| 2                     | 4           | 12  | 48              | 3                 | 60   |
|                       |             | 120 | 298             | 60                |      |

$$\rightarrow \text{Mean} = \frac{\sum fx}{\sum f} = \frac{120}{60} = \underline{\underline{2}}$$

$$\rightarrow \text{Median} = \left(\frac{N+1}{2}\right) = \frac{61}{2} = 30.5 \text{th term} = \frac{2+2}{2} = \underline{\underline{2}}$$

$$\rightarrow \text{Mode} = \underline{\underline{2}}$$

$$\rightarrow Q_1 = \frac{N+1}{4} = \frac{61}{4} = 15.25 = 15^{\text{th}} + \frac{1(16)^{\text{th}}}{2} = \frac{1.25}{2} = 0.625$$

$$\rightarrow Q_3 = 3\left(\frac{N+1}{4}\right) = 45.75 = 45^{\text{th}} + \frac{3(46)^{\text{th}}}{2} = 3 + \frac{3(3)}{2} = \frac{5.25}{2} = 2.625$$

$$\rightarrow IQR = Q_3 - Q_1 = 2.625 - 0.625 = \underline{2}$$

$$\rightarrow S.D. = \sqrt{\frac{\sum (X - \bar{X})^2}{N}} = \sqrt{\frac{58}{60}} = \sqrt{0.966666667} = \underline{0.9831906}$$

Q5.

$$P(L1) = 40\% = .4$$

$$P(T/L1) = 20\% = .2$$

$$P(L2) = 50\% = .5$$

$$P(T/L2) = 40\% = .4$$

$$P(L3) = 10\% = .1$$

$$P(T/L3) = 10\% = .1$$

$$P(L2/T) = \frac{P(T/L2) \cdot P(L2)}{P(T/L1) \cdot P(L1) + P(T/L2) \cdot P(L2) + P(T/L3) \cdot P(L3)}$$

$$= \frac{(0.4)(0.5)}{(.4)(.2) + (.4)(.5) + (.1)(.1)}$$

$$= \frac{.20}{.08 + .20 + .01} = \frac{.20}{.29} = \underline{\underline{\frac{20}{29}}}$$

$$= 0.689655172$$

$$= \underline{\underline{68.9655172\%}}$$

Q4.

Male = 10

Female = 8

Qualified male = 6

Qualified female = 7

$$P(\text{At least 1 ~~qual~~ female}) = 8/18$$

$$P(2 \text{ qual. female} \cap \text{at least 1 female}) = 6/18$$

$$P(2 \text{ qual. female} / \text{at least 1 female}) = \frac{6/18}{8/18} = \frac{6}{8} = \frac{3}{4}$$

Q8

$$X \sim U(1, 2)$$

$$Y = \frac{3}{6-X}$$

$$6-X = \frac{3}{Y}$$

$$X = \frac{6Y-3}{Y}$$

$$f_X(x) = \frac{1}{2-1} = 1$$

$$f_{\frac{6Y-3}{Y}}\left(\frac{6Y-3}{Y}\right) = 1$$

$$f_Y(y) = \frac{6Y-3}{Y}$$



Q7

| X | $P(X=x)$ | $x \cdot P(X=x)$ | $x^2 \cdot P(X=x)$ | $x^3 \cdot P(X=x)$ |
|---|----------|------------------|--------------------|--------------------|
| 1 | .05      | .05              | .05                | .05                |
| 2 | .15      | .30              | .60                | 1.20               |
| 3 | .35      | 1.05             | 3.15               | 9.45               |
| 4 | .40      | 1.60             | 6.40               | 25.60              |
| 5 | .05      | .25              | 1.25               | 6.25               |
|   | 1.00     | 3.25             | 11.45              | 42.55              |

(i.)  $F(3) = P(X=1) + P(X=2) + P(X=3)$   
 $= .05 + .15 + .35$   
 $= \underline{\underline{0.55}}$

(ii.)  $E(X) = \sum x \cdot P(X=x) = \underline{\underline{3.25}}$

(iii.)  $V(X) = E(X^2) - [E(X)]^2$   
 $= 11.45 - (3.25)^2$   
 $= \underline{\underline{0.8875}}$

(iv.) co-eff. of skewness =  $E(X^3) - [E(X)]^3$   
 $= 42.55 - (3.25)^3$   
 $= \underline{\underline{8.221875}}$

Q9:  $P(W) = 0.01$   
 $P(L) = 1 - P(W) = 0.99$

$X \sim$  Winning raffle  
 $X \sim \text{Bin}(7, 0.01)$

$$P(X=3) = {}^7C_3 \cdot [P(W)]^3 \cdot [P(L)]^4$$

$$= 35 \cdot (0.01)^3 \cdot (0.99)^4$$

$$= \underline{\underline{0.000033621}}$$

Q11.

$$P(M) = .5$$

$$P(F) = .5$$

Event A = Next 3 child are boys

$$P(A) = 0.5 \times 0.5 \times 0.5$$

$$= \underline{\underline{0.125}}$$

Q13.

$$X \sim U(75, 120)$$

$$F_X(100) = \frac{100 - 75}{120 - 75} = \frac{25}{45} = 5/9$$

$$F_X(80) = \frac{80 - 75}{120 - 75} = \frac{5}{45} = 1/9$$

$$F_X(80 \leq x \leq 100) = F_X(100) - F_X(80)$$

$$= \frac{5}{9} - \frac{1}{9} = \underline{\underline{4/9}}$$

Q14.

$$X \sim \text{Gamma}(5, 0.5)$$

$$\alpha = 5$$

$$\lambda = .5$$

$$2 \lambda X = \chi^2_{10}$$

$$0.8 X = \chi^2_{10}$$

$$P(X < 22.89) = P(.8X < 18.312)$$

$$= P(\chi^2_{10} < 18.312) = \underline{\underline{0.9541788}}$$