NUMERICAL METHODS & ALGEBRA
Assignment

1. $r = 12.65$ cm
$r_{true} = 12.5$ cm

$A = \pi r^2 = 3.14(12.65)^2 = 502.47065 \text{ cm}^2$
$A_{true} = 3.14(12.5)^2 = 490.625 \text{ cm}^2$

(i) Absolute error $= A - A_{true}$
$= 502.47065 - 490.625$
$= 11.84565 \text{ cm}^2$

(ii) Relative error $= \dfrac{\text{Absolute error}}{\text{Actual value}}$
$= \dfrac{11.84565}{490.625}$

$= 0.024$

(iii) Percentage error $=$ Relative error $\times 100$
$= 2.4\%$

2.

x	4	4.5	5.5	6
$f(x) = \log x$	0.60206	0.6532125	0.7403627	0.7781513

(a) Using Lagrange's interpolation;

$$\log(5) = \frac{(5-4.5)(5-5.5)(5-6)}{(4-4.5)(4-5.5)(4-6)} \times 0.60206 +$$

$$\frac{(5-4)(5-5.5)(5-6)}{(4.5-4)(4.5-5.5)(4.5-6)} \times 0.6532125 +$$

$$\frac{(5-4)(5-4.5)(5-6)}{(5.5-4)(5.5-4.5)(5.5-6)} \times 0.7403627 +$$

$$\frac{(5-4)(5-4.5)(5-5.5)}{(6-4)(6-4.5)(6-5.5)} \times 0.7781513$$

$$= -0.100343 + 0.435475 + 0.493575 - 0.12969$$

Ans $\log 5 = \underline{0.699017}$

(b) $\log(5)$ using $x = 4.5$ & $x = 5.5$

$$\log(5) = \frac{(5-5.5)}{(4.5-5.5)} \times 0.6532125 +$$

$$= \frac{(5-4.5)}{(5.5-4.5)} \times 0.7403627$$

$$\log(5) = 0.326606 + 0.37018$$
$$= \underline{0.696786}$$

3. ~~f(x) x~~ $f(x) = x^4 - x - 10$

$a = 1.5,\ b = 2$

$f(a) = -6.4375,\ f(b) = 4$

Using Bisection method,

$\Rightarrow x_1 = \frac{a+b}{2} = \frac{1.5+2}{2} = \underline{1.75}$

$\Rightarrow f(x_1) = (1.75)^4 - 1.75 - 10 = -2.3711$

$\Rightarrow x_2 = \frac{1.75+2}{2} = \underline{1.875}$

$\Rightarrow f(x_2) = (1.875)^4 - 1.875 - 10 = 0.4846$

$\Rightarrow x_3 = \dfrac{1.75 + 1.875}{2} = 1.8125$

$\Rightarrow f(x_3) = (1.8125)^4 - 1.8125 - 10 = -1.02025$

$\Rightarrow x_4 = \dfrac{1.8125 + 1.875}{2}$

$= 1.84375$

$\Rightarrow f(x_4) = -0.2877$

$\Rightarrow x_5 = \dfrac{1.84375 + 1.875}{2} = 1.859375$

$\Rightarrow f(x_5) = 0.0934$

Ans $x = 1.859375$

4. $f(x) = x^3 - x - 1$

Using newton raphson method,

x	0	1	2
$f(x)$	-1	-1	5

$\Rightarrow x_0 = 1$

$\Rightarrow f(x_0) = -1$

$\Rightarrow f'(x_0) = 3x^2 - 1 = +2$

$\Rightarrow x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$x_1 = 1 - \left(\dfrac{-1}{+2}\right) = 1.5$

$\Rightarrow x_2 = 1.5 - \left(\dfrac{0.875}{5.75}\right) = 1.34783$

$x_2 = -0.5$

$f(x_2) = -0.625$

$f'(x_2) = -0.25$

$x_3 = -0.5 - \left(\frac{-0.625}{-0.25}\right) \quad -0.5 - \left(\frac{-0.625}{-0.25}\right)$

$x_3 = -3$

$\Rightarrow f(x_3) = -25$

$\Rightarrow f'(x_3) = 26$

$\Rightarrow x_4 = -3 - \left(\frac{-25}{26}\right)$

$x_4 = -2.03846$

$\Rightarrow f(x_4) =$

$x_2 = 1.34783$

$f(x_2) = 0.10069$

$f'(x_2) = 4.4499$

$\Rightarrow x_3 = 1.34783 - \left(\frac{0.10069}{4.4499}\right)$

$x_3 = 1.3252$

$f(x_3) = 0.002056$

$f'(x_3) = 4.2685$

$\Rightarrow x_4 = 1.3247$

$\Rightarrow f(x_4) = 0.0000765$

Ans Root of equation is $\underline{\underline{1.3247}}$.

5. $A \rightarrow$ square matrix

$A^2 = A$

$\Rightarrow 7A - (I+A)^3 = 7A - (I^3 + A^3 + 3I^2A + 3IA^2)$

We know that $IA = A$ & $I^3 = I$

$= 7A - [I + A^2 \cdot A + 3I(I \cdot A) + 3IA]$

$= 7A - [I + A^2 + 3IA + 3IA]$

$= 7A - [I + A + 6A]$

Ans $7A - (I+A)^3 = \underline{\underline{-I}}$

6. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1(0-6) - (-1)(-4) + 2(4)$

$= -6 - 4 + 8$

$= -2$

$\text{adj} A = \begin{bmatrix} +(-6) & -(-1) & +(-6) \\ -(-4) & +(-1) & -(-2) \\ +(4) & -(1) & +(4) \end{bmatrix}$

$= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj} A$

Ans $A^{-1} = \frac{1}{(-2)} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

7. $\vec{A} = 8i - 9j$

$\vec{B} = 7i - 9j$

Using scalar product

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

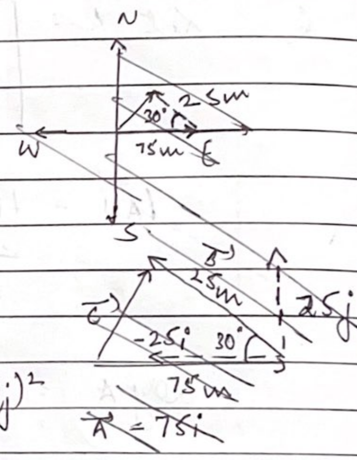
$(8i-9j)(7i-9j) = \sqrt{8^2+(-9)^2}\sqrt{7^2+(-9)^2}\cos\theta$

$56+81 = \sqrt{145}\sqrt{130}\cos\theta$

$\cos\theta = 0.997849$

Ans $\theta = 3.7587$

8. From the diagram;



~~Disp vector:~~ $\cancel{\vec{C} = \vec{A} + \vec{B}}$

$\cancel{\vec{C} = 75i + (-25i}$

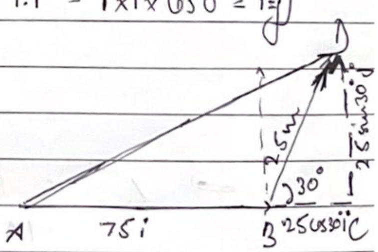
$|\vec{AD}|^2 = |\vec{AC}|^2 + |\vec{CD}|^2$

$|\vec{AD}| = \sqrt{(75i + 25\cos 30 i)^2 + (25\sin 30 j)^2}$

$= \sqrt{9341.345264 i^2 + 156.25 j^2}$

$= \sqrt{9497.595}$ $\quad [i^2 = i.i = 1\times 1\times \cos 0° = 1 = j^2]$

$|\vec{AD}| = 97.4556m$



Ans Magnitude of displacement vector is 97.4556m.

9. 3 digit no's that leave a remainder of 2 when divided by 3 form an AP :

AP :- 101, 104, 107, ... 998

Using $a_n = a + (n-1)d$

$\Rightarrow 998 = 101 + (n-1)3$

$\Rightarrow 897 = 3n - 3$

$\Rightarrow 900 = 3n$

$\Rightarrow n = 300$

$S_n = \frac{n}{2}(a + a_n)$

$= \frac{300}{2}(101 + 998)$

$= \underline{\underline{164850}}$

10. 6^{th} term of GP = 32

$\Rightarrow ar^5 = 32$ —— (i)

8^{th} term = 128

$\Rightarrow ar^7 = 128$ —— (ii)

(ii) ÷ (i)

$\Rightarrow \frac{\cancel{a}r^7}{\cancel{a}r^5} = \frac{\cancel{128}^{4}}{\cancel{32}}$

$\Rightarrow r^2 = 4$

Ans $\Rightarrow \boxed{r = 2}$

11. $(4+3x)^5$

Using Binomial Theorem :

$(4+3x)^5 = {}^5C_0 (3x)^5 4^0 + {}^5C_1 (3x)^4 (4) + {}^5C_2 (3x)^3 4^2$
$+ {}^5C_3 (3x)^2 4^3 + {}^5C_4 (3x) 4^4 + {}^5C_5 (3x)^0 4^5$

$= (3x)^5 + 5(3x)^4 \cdot 4 + 10(3x)^3 \cdot 4^2 + 10(3x)^2 4^3 + 5(3x)4^4 + 4^5$

Ans $(4+3x)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

For Eigen Value:

$\Rightarrow A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I| = (2-\lambda)\left[(-5-\lambda)(3-\lambda)\right] - (-3)\left[2(3-\lambda)\right]$

$= (2-\lambda)\left[-15+5\lambda-3\lambda+\lambda^2\right] + 3(6-2\lambda)$

$= (2-\lambda)(\lambda^2+2\lambda-15) + 18 - 6\lambda$

$= \cancel{2\lambda^2} + 4\lambda - 30 - \lambda^3 - \cancel{2\lambda^2} + 15\lambda + 18 - 6\lambda$

$0 = -\lambda^3 + 13\lambda - 12$

$\Rightarrow \lambda^3 - 13\lambda + 12 = 0 \quad \text{—(i)}$

Using Trial & Error

$\lambda = 1$ satisfies equ (i)

$(\lambda - 1)\overline{)\lambda^3 - 13\lambda + 12}\left(\lambda^2 + \lambda - 12\right.$

$\lambda^3 - \lambda^2$

$- \quad +$

$\lambda^2 - 13\lambda + 12$

$\lambda^2 - \lambda$

$- \quad +$

$-12\lambda + 12$

$-12\lambda + 12$

$\times$

$\Rightarrow (\lambda-1)(\lambda^2+\lambda-12)=0$

$\Rightarrow (\lambda-1)(\lambda^2+4\lambda-3\lambda-12)=0$

$\Rightarrow (\lambda-1)(\lambda+4)(\lambda-3)=0$

Ans $\Rightarrow \lambda = -4, 1, 3$

13. $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 3x^2 - 14x + 8 = 13$

Using newton's method

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$x_1 = 5 - \dfrac{(-13)}{13}$

$\Rightarrow x_1 = 6$

$f(x_1) = 9$

$f'(x_1) = 32$

$\Rightarrow x_2 = 6 - \dfrac{9}{32}$

Ans $\Rightarrow \boxed{x_2 = 5.71875}$

14. Expand : $(1-x+x^2)^4$

$(1-x+x^2)^4 = [(1-x)+x^2]^4$

$= (1-x)^4 + 4(1-x)^3(x^2) + 6(1-x)^2(x^2)^2$

$+ 4(1-x)(x^2)^3 + (x^2)^4$

$= 1 + 4(-x) + 6(-x)^2 + 4(-x)^3 + (-x)^4 +$

$4x^2[1-x^3-3x+3x^2] + 6x^4(1+x^2-2x)$

$+ 4x^6(1-x) + x^8$

$$= 1 - 4x + 6x^2 - 4x^3 + x^4 + 4x^2 - 4x^5 - 12x^3$$
$$+ 12x^4 + 6x^4 + 6x^6 - 12x^5 + 4x^6 - 4x^7 + x^8$$

Ans $\boxed{(1-x+x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1}$

15 $e^{-x} = 3 6 g x$

$e^{-x} - 3 6 g x = 0$

$\Rightarrow f(x) = e^{-x} - 3 6 g x$

x	1	2
$f(x)$	0.3679	-0.7678

Using Bisection method

$\Rightarrow x_1 = \frac{1+2}{2} = 1.5$

$\Rightarrow f(x_1) = -0.3051$

$\Rightarrow x_2 = \frac{1+1.5}{2} = 1.25$

$\Rightarrow f(x_2) = -0.004225$

$\Rightarrow x_3 = \frac{1+1.25}{2} = 1.125$

$\Rightarrow f(x_3) = 0.1712$

$\Rightarrow x_4 = \frac{1.125+1.25}{2} = 1.1875$

$\Rightarrow f(x_4) = 0.0811$

Ans $\boxed{x = 1.1875}$