PROBABLITY AND STATISTICS
ASSIGNMENT - 1

1] i 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$$\text{Mean} = \frac{134}{10} = 13.4$$

$$\text{Median} = \frac{(n/2)^{th}\text{ term} + (n+2/2)^{th}\text{ term}}{2}$$

$$= \frac{11+15}{2} = 13$$

$$Q_1 = \left(\frac{n+1}{4}\right)^{th}\text{ term} = 2.75^{th}\text{ term}$$

$$= 7 + 0.75(9-7) = 8.5$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}\text{ term} = 8.25$$

$$= 18 + 0.25(18-18) = 18$$

inter quartile range $= 18 - 8.5$
$= 9.5$

Mode $= 18$

ii)

No. of households	No. of ppl	fx	c.f
0	5	0	0
1	11	11	11
2	26	52	37
3	15	45	52
4	3	12	55
	60	120	

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = 2$$

$$\frac{n+1}{2} =$$

$$\text{Median} = \frac{60^{th}\text{ term} + 61^{st}\text{ term}}{2} = \frac{2+2}{2} = 2$$

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$$S.D = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$$

$$= 16.21$$

$Q_1 = \frac{(n+1)}{4}^{th}$ term $= \cancel{30.25^{th} \text{ term}}$ ~~30.5^{th} term~~ 15.25^{th} term

$= \cancel{2 + 0.25(2-2) = 2}$ 1

$Q_3 = \frac{3(n+1)}{4}^{th}$ term $=$ ~~90.75^{th} term~~ 45.75^{th} term

$= 3 + 0.75(3-3) = 3$

inter-quartile range $= Q_3 - Q_2$
$= 3 - 2 = 2$

Mode $= 2$

$$s.d = \sqrt{\frac{1}{n-1}\Sigma f x^2 - \bar{x}^2} \quad \sqrt{\frac{1}{n-1}\Sigma f(fx - \bar{x})^2}$$

$$= \sqrt{58/59} = 0.9915$$

2] i) No. of claims per year = 0.5

Poi $(\lambda = 0.5)$

$$P(X=0) = \frac{e^{-0.5} \times 0.5^0}{0!} = e^{-0.5} = 0.60653$$

ii) ~~[P(x ≥ 1) × P(X=0) . P(X=0)] × 3 if one or more cla in one year~~

If one or more claim in 1 year:

$$[P(X \geq 1) . P(X=0) . P(X=0)] \times 3$$

$$[(0.60653)^2 \times [1 - P(X \leq 0)]] \times 3$$

$$[(0.60653)^2 \times [1 - 0.60653]] \times 3$$

$$= 0.43425$$

iii) ~~As~~ In One claim has and we have to find time till the next claim. We will use exponential distribution

$$X \sim Exp(\lambda = 0.5)$$

$$P(X \geq 2) = 1 - P(X \leq 2)$$

$$= 1 - [1 - e^{-\lambda x}]$$

$$= e^{-\lambda x}$$

$$= 0.367879$$

3] $X \sim \text{Gamma}(\alpha = 2, \lambda = 1/2)$
$0 < x < \infty$

i) $E(X) = \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$

$V(X) = \frac{\alpha}{\lambda^2} = \frac{2}{(1/2)^2} = 8$

$S.D(X) = \sigma = \sqrt{8} = 2.8284$

ii) The distribudion seems to be skewed due to high variance of 8 and as the mean (=4), is so close to the range $0<x<\infty$ we can assume it to be posidively skewed.

iii) $F_X(x) = \int_0^x f_x(x)\, dx$ $\quad [F_X'(x) = f_X(x)]$

$= \int_0^x \frac{0.5^{(2}}{\sqrt{2}} x^{(2-1)} e^{-x/2} dx$

$= \frac{0.5^2}{\sqrt{2}} \int_0^x x \cdot e^{-x/2} dx$

$= \frac{0.5^2}{\sqrt{2}} \left[x \cdot \int_0^x e^{-x/2} dx - \int \left(\frac{d(x)}{dx} \int e^{-x/2} dx \right) dx \right]_0^x$

$= \frac{0.5^2}{\sqrt{2}} \left[x \cdot \frac{e^{-x/2}}{-1/2} - \int \frac{e^{-x/2}}{-1/2} dx \right]_0^x$

$= \frac{0.5^2}{\sqrt{2}} \left[-2x \cdot e^{-x/2} - 4e^{-x/2} \right]_0^x$

$= \frac{0.5^2}{\sqrt{2}} \left[(-2x \cdot e^{-x/2} - 4e^{-x/2}) - (-4) \right]$

$= \frac{0.5^2}{\sqrt{2}} \left[\frac{-2x}{e^{x/2}} - \frac{4}{e^{x/2}} + 4 \right]$

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$= 0.5^2 \left[\frac{-2x-4}{e^{x/2}} + 4 \right]$

$= \frac{-0.5x - 1}{e^{x/2}} + 1$

5] Given : $P(L_1) = 40\% = 0.4$ $\quad P(L/L_1) = 20\% = 0.2$

$P(L_2) = 50\% = 0.5$ $\quad P(L/L_2) = 40\% = 0.4$

$P(L_3) = 10\% = 0.1$ $\quad P(L/L_3) = 10\% = 0.1$

To find : $P(L_2/L)$

$$P(L_2/L) = \frac{P(L_2/L) \times P(L_2)}{\sum_{i=1}^{3} P(L_2/L_i) \cdot P(L_i)}$$

$$= \frac{0.4 \times 0.5}{(0.2 \times 0.4) + (0.4 \times 0.5) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$= 0.6896552$$

6] Given : $X \sim U(1,2)$

$Y = \frac{3}{6-x}$ $\quad \therefore x = 6 - \frac{3}{y}$

$F_Y(y) = P(Y \leq y) = P\left(\frac{3}{6-x} \leq y\right) = P\left(x \leq 6 - \frac{3}{y}\right)$

$= P(x \leq 6 - 3/y) = F_X(6 - 3/y)$

$F_Y(y) = F_X\left(6 - \frac{3}{y}\right)$

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Differentiating :

$$x = 6 - \frac{3}{y}$$

$$\frac{dx}{dy} = \frac{3}{y^2}$$

$$F'_Y(y) = f_Y(y)$$

$$F'_y(y) = f_x(x) \times \frac{dx}{dy}\left(6 - \frac{3}{y}\right)$$

$$= 1 \times \frac{3}{y^2}$$

$$\therefore f_Y(y) = \frac{3}{y^2}$$

7]

x	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

i) $F(3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(X) = \sum_{x=1}^{5} x \cdot P(X=x)$

$= 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$= 3.25$

iii) $V(X) = E(X^2) - [E(X)]^2$

$= \sum_{x=1}^{5} x^2 \cdot P(X=x) - \left[\sum_{x=1}^{5} x \cdot P(X=x)\right]^2$

$= [1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)] - [3.25]^2$

$= 11.45 - 10.5625$

$= 0.8875$

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iv) Coeffecient of skewness = $\frac{3(\text{Mean} - \text{median})}{6}$

$= \frac{3(3.25 - 3)}{\sqrt{V(X)}}$

$= 0.7961173$

8] Probablity (p) = 0.2

$n = 15$

$X \sim Bin(15, 0.2)$

$P(X < x) = {}^nC_r \cdot p^r q^{n-r}$

$P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 \cdot (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14}$

$+ {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= 0.03518 + 0.13194 + 0.2308$

$= 0.39792$

9] The man takes 7 trials and the last one is his 3^{rd} win with the probablity of 0.01.

$X \sim NBin(3, 0.01)$

$P(X = x) = {}^{n-1}C_{r-1} p^r q^{n-r}$

$P(X = 7) = {}^6C_2 (0.01)^3 q^4$

$= {}^6C_2 (0.01)^3 (0.99)^4$

$= 0.0000414709$

10] Rate = 2 per work week (5 days)

λ = 2 per 5 days

λ = 2/5 per day

$X \sim Poi(\lambda = 2.5)$

$P(x > 2) = 1 - P(X < 2)$

$= \cancel{1 + [e^{\lambda} / \lambda^x / x!]} = 1 - e^{-\lambda x}$

~~E~~ - $= 0.993262$

11] Since the probablity of having a boy or girl is independent of each other and previously having a boy or gir

Probablity of next 3 boys $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

12] Rate: 10 per day

Time between 2 claims is less than 0.1 days

$X \sim Poi(\lambda = 10)$

$P(X < 0.1) = 1 - e^{-\lambda x}$

$= 1 - e^{-10(0.1)}$

$= 0.63212$